Joint Estimation of DOA and Time-Delay in Underwater Localization

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Abstract

Joint estimation of direction of arrival (DOA) and time delay plays a great role in source localization, which attracts many researchers not only in the areas of radar, sonar, geological exploration but also in wireless communication. [1][2][3][4] M. Wax applies approximate MLE with iteration algorithm [1], which convert a 2-D search into two or three 1-D search. A.J.van der Veen use 2-D ESPRIT to conduct joint estimation [2]. Both of them show good performance at a cost of large computation. And both of them require deconvolution in frequency domain to transfer time-delay into phase. The deconvolution leads to two problems. One is blowing up noise, the other is leading to spurious peak if the emitted signal is non-minimum phase. In this paper, a simple method using 1-D ESPRIT is presented to complete joint estimation of DOA and time-delay, which requires no deconvolution. It is suitable for active underwater localization where non-minimum phase signal is frequently employed. The method can estimate parameters of three reflectors with big difference between amplitudes as large as 12dB. The statistical performance of new estimators and the probability of correct pairing are given by computer simulations. It shows that better performance of the new method can be achieved for multiple source localization even in low SNR.

1 Modeling

The emit signal is a sine wave with an envelop s(t). An M-element d-spaced uniform linear sensor array is employed as a receiver. Assume that there are *p* reflectors in water. If a single echo wave is sampled with N points in time domain, the complex envelop of received echo can be written as a N×M matrix:

$$X = \sum_{i=1}^{p} s(t-\tau_i) b_i \exp(j\varphi_i) a^T(\theta_i) + N$$
$$= S(\tau)\Psi A^T(\theta) + N(t) = \tilde{S}(\tau) A^T(\theta) + N \quad (1)$$

where

$A(\theta) = [a(\theta_1), a(\theta_2), \cdots, a(\theta_p)]$ is the direction matrix of sensor array;

$a(\theta_i) = \{1, \exp(-j\omega_i), \cdots, \exp[-j(M-1)\omega_i]\}^T$ is ith column of $A(\theta)$, which denotes the direction vector of the array;

$\omega_i = 2\pi \dfrac{d\sin(\theta_i)}{\lambda}, i = 1,2,\cdots,p$ is the spatial frequency of ith reflector, λ is the wave length of carrier signal;

$\Psi = diag[b_1 \exp(j\varphi_1), b_2 \exp(j\varphi_2), \cdots, b_p \exp(j\varphi_p)]$, here b_i and φ_i are the amplitude and initial phase of ith reflector;

$S(\tau) = [s(t-\tau_1), s(t-\tau_2), \cdots, s(t-\tau_p)]$ is the normalized time-delay matrix, $s(t-\tau_i)$ is the envelop time-delay ; vector of ith reflector;

$\tilde{S}(t) = S(t)\Psi$;

$N = [n_1(t), n_2(t), \cdots, n_i(t), \cdots n_M(t)]$ is the noise matrix, where $n_i(t)$ is the sequence of ith sensor.

ESPRIT method requires two identical sub-arrays. In the case of uniform linear array, the sub-arrays are usually constructed as following: Taking 1~(M-1)th elements as one sub-array, the data matrix of which can be written in

$$X_1 = \tilde{S}(\tau) A_-^T(\theta) \quad (2)$$

where $A_-(\theta)$ is a sub-matrix of $A(\theta)$ in (1). Compared with $A(\theta)$, the last row of $A_-(\theta)$ has been canceled. In the same way, taking 2~Mth elements as another sub-array, the data matrix of which can be written in

$$X_2 = \tilde{S}(\tau)\Phi A_-^T(\theta) \quad (3)$$

where $\Phi = diag[\exp(j\omega_1), \exp(j\omega_2), \cdots, \exp(j\omega_p)]$ contains the DOA information of reflectors.

2 The joint estimation algorithm of DOA and time-delay

(1) Principle

Conventional ESPRIT method apply following two covariance matrix :

$$R_0 = X_1^H X_1 = A_-^*(\theta)\tilde{S}^H(\tau)\tilde{S}(\tau)A_-^T(\theta) \quad (4)$$

$$R_1 = X_1^H X_2 = A_-^*(\theta)\tilde{S}^H(\tau)\tilde{S}(\tau)\Phi A_-^T(\theta) \quad (5)$$

Apparently, they meet

$$R_0 A_-^*(\theta)\Phi = R_1 A_-^*(\theta) \quad (6)$$

Employing a generalized eigen-decomposition to get Φ, the DOAs of sources can be obtained.

The key step of above-mentioned is to construct two covariance matrix whose generalized eigenvalue is Φ. In fact, the covariance matrix can be generated in another way:

$$Y_0 = X_1 X_1^H = \tilde{S}(\tau)A_-^T(\theta)A_-^*(\theta)\tilde{S}^H(\tau) \quad (7)$$

$$Y_1 = X_1 X_2^H = \tilde{S}(\tau)A_-^T(\theta)A_-^*(\theta)\Phi^H\tilde{S}^H(\tau) \quad (8)$$

They are all N×N non full rank matrix with rank of M, but they meet following equation too :

$$Y_0\tilde{S}(\tau)\Phi^H = Y_1\tilde{S}(\tau) \quad (9)$$

So Φ can be estimated by using a generalized eigen-decomposition on Y_0, Y_1. The difference between (6) and (9) lies in that former eigenvector denotes direction vector of sources, while latter eigenvector denotes time-delay vector of sources. Therefore the time-delay information is contained in eigenvectors, while DOAs can be obtained from main eigenvalues. That is to say, both DOA and time-delay parameters can be estimated via a single generalized eigen-decomposition, and parameters of a source are paired automatically since the eigenvalues are paired with eigenvectors.

(2) Procedure of the algorithm

The algorithm of this method is different from conventional ESPRIT. Since Φ is the eigenvalues of signal subspace in Y_0 and

Y_1, we take a eigen-decomposition on Y_0 firstly. That is

$$Y_0 = VEV^H \tag{10}$$

where V contains the eigenvectors, $E = diag(\lambda_1, \lambda_2, \cdots, \lambda_N)$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$ is the eigenvalue in descendant order.

The number of sources p can be determined by λ_i according to AIC[6].

Then construct a projection matrix of signal subspace using main components of V in (10) as

$$Y^{\#} = \sum_{i=1}^{p} \lambda_i^{-1} V_i V_i^H \tag{11}$$

$Y_1 Y^{\#}$ is the projection of Y_1 on the signal subspace of Y_0. It can be decomposed in

$$Y_1 Y^{\#} = U E_1 U^H \tag{12}$$

where each column of U (marked as U_i) is the generalized eigenvector in (9). The non-zero elements γ_i in $E_1 = diag(\gamma_1, \gamma_2, \cdots, \gamma_N)$ are the corresponding generalized eigenvalues in (9).

The directions of sources are given by the phases of γ_i, as

$$2\pi \frac{d}{\lambda} \sin(\theta_i) = \angle \gamma_i,\ \ i = 1, 2, \cdots, p \tag{13}$$

To obtain time-delay information from U_i, we calculate the cross-correlation of U_i and emitted signal s(t) :

$$r_i(\tau) = \frac{1}{N} s^T (t + \tau) U_i\ ,\ \ i = 1, 2, \cdots, p \tag{14}$$

The time-delay estimation $\hat{\tau}_i$ is given by maximizing $r_i(\tau)$.

In this algorithm, since every eigenvalue is corresponding to its eigenvector, direction and time-delay of every source are paired automatically. Time-delay parameters can be estimated by other methods, such as delay-MUSIC [7][8]. But they need additional pairing algorithm because two set parameters given by different methods are not paired.

(3) An example

Taking a sine wave modulated by a triangle as the emitted signal, the real envelope of which is shown in Figure 1. A 16-sensor linear uniform array with a half-wavelength spacing is employed to receive the echoes. In this simulation, there are 3 reflectors with equivalent strength, time-delay and direction parameter of which are shown in Table 1. The time-delay intervals between closer reflectors are chosen as a half of resolution of correlation method, while direction intervals are approximately 2/3 of beamwidth. Without high resolution method, these three reflectors can be resolved in neither time-delay domain nor spatial domain.

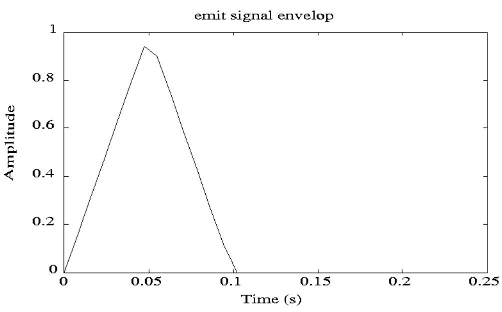


Figure 1. Real envelop of emitted signal

Table 1. True parameters of reflectors

	time-delay (second)	Direction(deg)
reflector 1	0.6213	-4.3513
reflector 2	0.6667	0.0000
reflector 3	0.7154	3.7784

A gate of duration is set to cover the echoes of these 3 reflectors, within which 30 snapshots are sampled uniformly. Then a 30×16 data matrix is formed.

This data matrix is divided into two sub-matrixes. Their auto-correlation and cross-correlation matrix are calculated. Having employed eigen-decomposition on the auto-correlation matrix, AIC curve is given in Figure 2. According to the curve, the number of sources can be determined correctly as 3, since AIC reach its minimum while p=3.

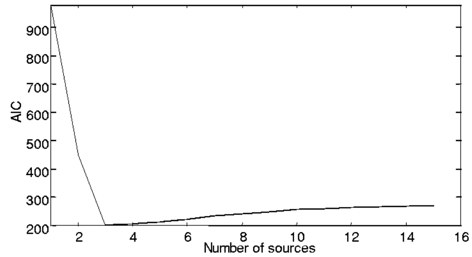


Figure 2. AIC curve via p

The generalized eigenvalues and eigenvectors can be obtained by (11) and (12). The directions of three sources can be estimated directly by the generalized eigenvalues. To estimate the time-delay parameters, the cross-correlation of 3 main generalized eigenvectors and emitted signal are calculated which are shown in Figure 3. The locations of peaks of 3 curves are the time-delay estimation. The precision in Figure 3 is the sampling interval. To improve the precision, finer research is required.

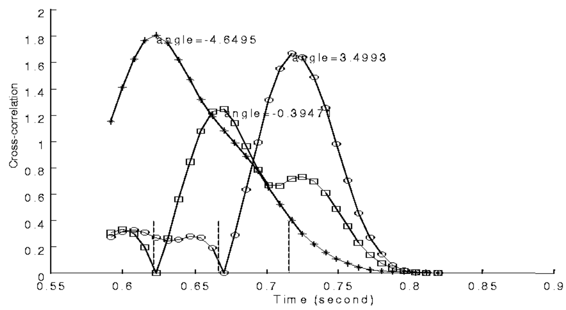


Figure 3. Time-delay estimation

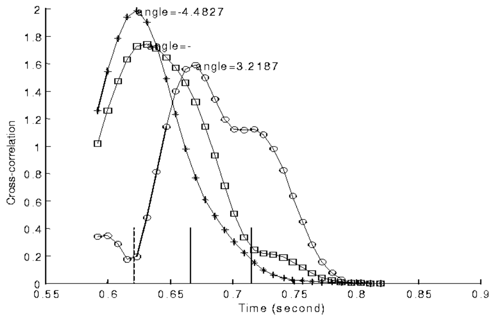
Table 2 gives the results of parameter estimation, which shows high resolution, perfect precision, and correct pairing.

Table 2. Estimation results of parameters

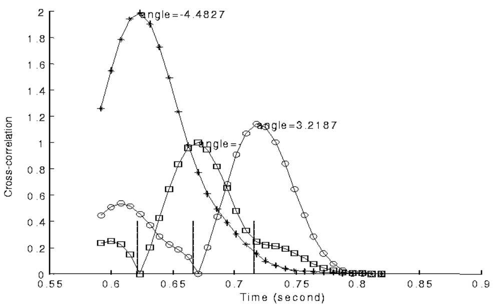
	Time-delay (rough)	Time-delay (fine)	Direction
Reflector1	0.9757 second	0.9804 second	-2.8353 deg
Reflector2	0.9443 second	0.9412 second	-0.2543 deg
Reflector3	0.8816 second	0.8831 second	3.2645 deg

3 Modification for different amplitude targets

In above-mentioned algorithm, the delay estimation is given by the cross-correlation between each eigenvector and emitted signal. There is a hypothesis here: each eigenvector contains only one time delay vector. In fact, each eigenvector is composed by all of time delay vectors. As an example, Fig. 3 shows that almost every cross-correlation curve has two or more peaks. This is a leakage phenomenon. It does not matter while the amplitude difference between sources is small. If the difference becomes larger, the leakage of stronger sources will affect the time delay estimation of weaker sources. Fig. 4(a) shows the time delay estimation for three sources with amplitude ratio equal to 2.5:1:0.4 while SNR=20dB. Obviously, taking the position of highest peaks of these curves can not obtain the time delay parameter of the weakest source.



(a) Before modification



(b) After modification

Figure 4 Time delay estimation

To keep the validity for the sources with large difference in amplitudes, a modification is presented. The way lays in following operation: Kick off the leakage of stronger source from the eigenvector corresponding to the weaker source, so that the modified eigenvector contains only the information of weaker sources. Take U_1, U_2 as the eigenvectors corresponding to the stronger and weaker sources respectively. The time delay estimation of stronger source can be given by original eigenvector U_1. To eliminate the leakage in U_2, let

$$U_2' = U_2 - C \cdot s(t - \hat{\tau}_1) \tag{15}$$

where the coefficient C is specified by minimizing the remain of the power:

$$C = \arg[\min_C |U_2 - C \cdot s(t - \hat{\tau}_1)|^2] \tag{16}$$

That is

$$C = \frac{U_2^H U_1 + U_1^T U_2^*}{2U_2^H U_2} \tag{17}$$

Above algorithm can be extended to the case of more sources. Suppose $U_1, U_2, \cdots, U_p$ are the generalized eigenvectors sorted by power in descending order. Except the strongest one U_1, the other eigenvectors should be modified by

$$U_i = U_i - \sum_{j=1}^{i-1} C_j \cdot s(t - \hat{\tau}_j), \quad i = 2,3,\cdots,p \tag{18}$$

where

$$C_j = \arg[\min_{C_j} \left| U_i - \sum_{k=1}^{j} C_j \cdot s(t - \hat{\tau}_k) \right|^2] \tag{19}$$

Holding the same condition in Fig. 4(a), Fig. 4(b) gives the cross-correlation curves with modification. The maximum peaks of all curves appear at the position close to their true time delays respectively.

4 Simulations

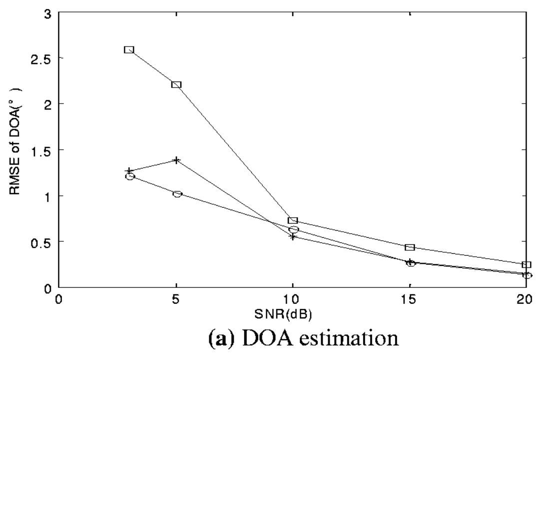
(1) Performance via SNR

Keeping the same simulation condition, the statistical performance of proposed method is evaluated in different SNR. Table 3 shows the probabilities of correct order determining, resolution, and correct pairing. All the statistical results following are given by 200 trials.

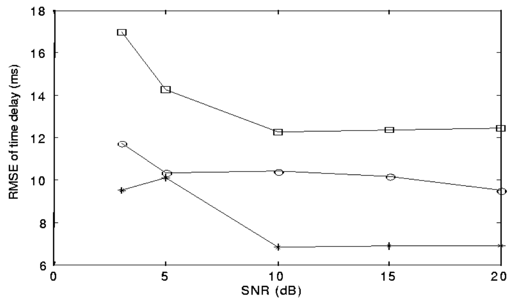
Table 3 Probabilities of correction for order-determining, resolution, and pairing

SNR (dB) / Probability (%)	20	15	10	5	3
order-determining	0.940	0.930	0.895	0.865	0.460
resolution	0.940	0.930	0.895	0.835	0.335
pairing	0.935	0.925	0.890	0.595	0.165

According to Table 3, if SNR is higher than 10dB, both the probability of correct resolution and the probability of correct pairing are bigger than 89%. Figure 5 shows the root mean square of direction and time-delay estimation. According to Figure 5, while SNR is higher than 8dB, the RMSE of direction estimation is no more than 1° (a fourth of beam width), and the RMS of time-delay estimation is no more than 10ms(a tenth of duration of emitted signal).



(a) DOA estimation



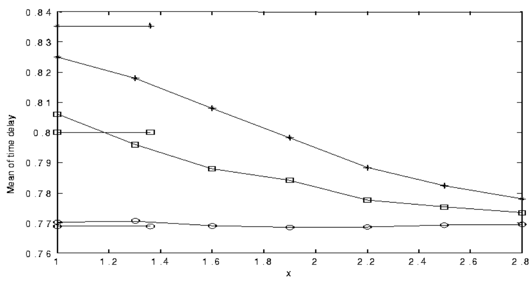
(b) Time-delay estimation

Figure 5. Statistical performance of joint parameter estimation for three sources by using new method
(Each curve denotes one reflector respectively)

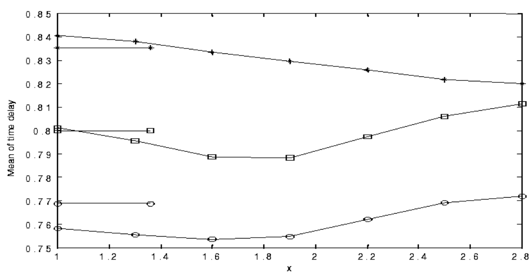
The simulations show that the proposed method can ultimately satisfy the requirement of underwater localization of multiple close-spaced reflectors.

(2) Performance via the amplitude difference

The other condition of simulation is held the same as above-mentioned, but the amplitude difference is changed. For convenience, the amplitude ratio of three reflectors is set as $x:1:1/x$, where x changes from 1 to 2.8 with step of 0.3. Accordingly, the power difference between the strongest and the weakest reflectors varies from 0dB to 17.9dB. The mean of time delay estimation via difference factor x is shown in Fig.6 when SNR=10dB (the median strength reflector). Fig. 6(a) shows the original case, where the estimates move forward to the value of strongest target while x increases. Then more errors appear to the time delay estimation of the weaker reflectors. Fig. 6(b) shows the case after modification, where the means of time delay estimations are around their respective true values regardless of x changing. This simulation indicates that the modification algorithm works well in all cases of various amplitude differences.



(a) Before modification



(b) After modification

Figure 6 Mean of time delay estimation

5 Conclusion

A new method based on 1-D ESPRIT is presented in this paper for joint estimation of DOA and time delay parameters of multiple underwater targets, which generates the covariance matrix different from traditional ESPRIT. Direction and time-delay parameters are estimated simultaneously by generalized eigenvalues and eigenvectors, and no additional algorithm is needed to pair the two kinds of parameters. The advantages of this method are: a) small computation; b) the superb ability of processing non-minimum phase emitted signal, and c) robust estimation in the case of large differences of amplitudes between multiple targets. Whole algorithm is based on a single echo wave. Simulations show that it can provide satisfied probability of the resolution and precision of estimates. By the aid of efficient array calibration, the proposed method shows great prospect in applications of underwater localization.

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