BLIND EQUALIZATION OF MULTIUSER CDMA CHANNELS: A FREQUENCY-DOMAIN APPROACH

Konstantinos I. Diamantaras

Department of Informatics
Technological Education Institute
GR-54101 Sindos, Greece
kdiamant@alpha.it.teithe.gr

Athina P. Petropulu

Dept. Electrical & Computer Eng.
Drexel University
Philadelphia, PA 19104, USA
athina@artemis.ece.drexel.edu

ABSTRACT

The blind estimation of mixing channels resulting from frequency selective fading and multipath in a multi-user CDMA system is an important problem in wireless communications. We present a novel frequency-domain approach using second order spectral statistics for recovering the unknown channels. Unlike other methods which are based on time-domain analysis we make no particular assumption about the support of the mixing channels except that they have finite length (FIR). The method is based on the fact that the source sequences obtain a known spectral color derived from the corresponding spreading code used in CDMA.

1. INTRODUCTION

Code Division Mulitple Access (CDMA) communication systems have attracted a lot of attention recently due to their efficient utilization of the available bandwidth and their flexibility in accommodating variable traffic patterns. In a CDMA channel all users share the same frequency band and different orthogonal spreading codes are designed and assigned to different users so as to minimize the cross-correlation between the transmitted signals. In addition to multi-user interference, CDMA suffers from inter-chip interference (ICI) induced from multipath and frequency selective channels. The multipath problem is most severe in indoor and urban environments which are of great interest to mobile communications applications. Channel equalization techniques are necessary in order to compensate for these problems. In general two approaches can be taken: (a) to use special training sequences which however reduce spectral efficiency, or (b) to use blind equalization methods.

Although the blind multi-user CDMA equalization problem looks similar to the blind MIMO deconvolution problem [6] it is in fact a lot simpler than that. The reason is the fact that the user signals are colored by known frequency shaping filters called spreading code sequences. Therefore, the recovery of the transmitted sources is not quite a blind process as some information about the sources is known to the receiver. The rich structure of these colored signals can be taken advantage of and various algorithms have been proposed for that particular problem [3, 4, 2, 5]. In [4] a zero-forcing receiver is proposed which completely removes Multi-User Interference (MUI) and ICI under certain conditions, using however knowledge of the signature waveforms. In [2] a subspace algorithm is developed which is applicable however when only a few users are active. In fact the method will fail if the code length L_c is less than 4 times the number of users. Other subspace methods have also been proposed [5, 3] meeting with success under certain conditions. All these methods however, are based on the time-domain analysis of the observed signals and they make explicit use of the mixing channel length L.

In this paper we propose a frequency-domain approach which recovers the spectra of the unknown channels regardless of their lengths – provided of course, that the observation sequence is long enough. The method uses the second order characteristics of the signal spectra and it is based on the fact that the source signals are colored with different shaping filters which are known to the receiver.

This work was supported by NSF under grant MIP-9553227

2. DATA MODEL AND PROBLEM FORMULATION

An n-user CDMA system with M receivers can be described by the following equation for the i-th receiver baseband signal:

$$x_i(t) = \sum_{j=1}^{n} \sum_{l=-\infty}^{\infty} g_{ij}(t - lT_s)s_j(l) \tag{1}$$

where j is the user index, $s_j(l)$ is the transmitted symbol sequence (each symbol usually taken from a finite alphabet, e.g. $\{+1, -1\}$), and T_s is the symbol duration. For user j each symbol is multiplied by the pre-assigned spreading code sequence $\{c_j(1), \cdots, c_j(L_c)\}$ at L_c times the symbol frequency T_s, so that the resulting chips have duration $T_c = T_s/L_c$. The signature $g_{ij}(t)$ couples the j-th user with the i-th receiver. By construction, $g_{ij}(t)$ incorporates the known sequence $c_j(k)$ and the unknown channel $h_{ij}(t)$ which represents the multipath fading environment between the j-th user and the i-th receiver. In particular we have

$$g_{ij}(t) = \sum_{m=1}^{L_c} h_{ij}(t - mT_c)c_j(m) \tag{2}$$

In general $h_{ij}(t)$ may be modeled by an FIR filter with finite support $[0\ LT_c]$. L is not known and we shall make no particular assumption about it.

The i-th receiver baseband signal $x_i(t)$ is sampled at the chip rate $1/T_c$ to obtain the following discrete time systems

$$x_i(k) = \sum_{j=1}^{n} \sum_{l=-\infty}^{\infty} g_{ij}(k-l)s_j(l) \quad (3)$$

$$g_{ij}(k) = \sum_{m=1}^{L_c} h_{ij}(k-m)c_j(m) \quad (4)$$

where we write $x_i(k)$, $g_{ij}(k)$, and $h_{ij}(k)$ instead of $x_i(kT_c)$, $g_{ij}(kT_c)$, and $h_{ij}(kT_c)$ respectively. Rewriting (3) and (4) into

$$\begin{aligned} x_i(k) &= \sum_{j=1}^{n} \sum_{m=-\infty}^{\infty} h_{ij}(k-m) \sum_{l=m-L_c}^{m-1} c_j(m-l)s_j(l) \\ &= \sum_{j=1}^{n} \sum_{m=k-L}^{k} h_{ij}(k-m)e_j(m) \quad (5) \end{aligned}$$

we readily find the sampled observation at the i-th receiver to be the multi-channel mixture of the following colored processes

$$e_j(k) = c_j(k) \star s_j(k) = \sum_{l=k-L_c}^{k-1} c_j(k-l)s_j(l) \quad (6)$$

(the symbol $\star$ denotes convolution).

Stacking the discrete observations in (5) for antennas $i = 1$ through M into a column vector $\mathbf{x}(k)$ we obtain

$$\mathbf{x}(k) = \sum_{l=0}^{L} \mathbf{H}(l)\mathbf{e}(k-l) \quad (7)$$

where $[h_{i1}(l), \cdots, h_{in}(l)]$ is the i-th row of $\mathbf{H}(l)$ and $\mathbf{e}(k-l) = [e_1(k-l) \cdots e_n(k-l)]^T$.

We assume that the symbol sequences $\{s_j(k)\}$ are unknown, wide-sense stationary, white sequences which are also pairwise uncorrelated

$$E\{s_i(k)s_j(k+l)\} = \begin{cases} \delta(l) & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

On the other hand, the sequences $e_j(k)$ are colored, and are pairwise uncorrelated as well:

$$E\{e_i(k)e_j(k+l)\} = \begin{cases} r_{e_i}(l) & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

The covariance function $r_{e_i}(l)$ is easily derived to be

$$r_{e_i}(l) = \sum_{l_1=1}^{L_c-l} c_i(l_1)c_i(l+l_1) \quad (10)$$

Since the spreading codes $c_i(k)$ for all the users are known to the receiver the covariance functions $r_{e_i}(l)$ are also known for all i.

Our problem is to find both the mixing filters $h_{ij}(k)$ and the transmitted symbols $s_j(k)$ given the M sampled observation sequences $x_i(k)$. Although the problem is reminiscent of the blind multi-channel separation problem the rich spectral structure of $\mathbf{e}(l)$ and the the fact that its statistics are known makes it a lot easier to solve. In order to simplify the discussion in the following we shall assume that $M = n$.

3. FREQUENCY-DOMAIN APPROACH

Our approach is based on the frequency domain analysis of the signals. If we take the length-N DFT[1] ($N > L$) on both sides of Eq. (7) we obtain

$$\mathbf{x}(\omega) \approx \mathbf{H}(\omega)\mathbf{e}(\omega) \quad (11)$$

where the element $H_{ij}(\omega)$ of the matrix $\mathbf{H}(\omega)$ is the DFT of the unknown filter $h_{ij}(k)$. The approximate equality above would be replaced with equality if the sequence $\mathbf{s}(k)$ is periodic with period N, or if the length of the sequence $\mathbf{x}$ is larger than L plus the length of $\mathbf{e}$.

Let us define the covariance matrix of the complex stochastic DFT process $\mathbf{x}(\omega)$ for a pair of frequencies ω_1, ω_2:

$$\begin{aligned} \mathbf{R}_x(\omega_1,\omega_2) &= E\{\mathbf{x}(\omega_1)\mathbf{x}(\omega_2)^H\} \\ &= \mathbf{H}(\omega_1)\mathbf{R}_e(\omega_1,\omega_2)\mathbf{H}(\omega_2)^H \quad (12) \end{aligned}$$

(the superscript ^{H} denotes the matrix Hermitian transpose). We have,

$$\begin{aligned} \mathbf{R}_e(\omega_1,\omega_2) &= E\{\mathbf{e}(\omega_1)\mathbf{e}(\omega_2)^H\} \\ &= \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} e^{-j\frac{2\pi(\omega_1 k-\omega_2 l)}{N}} E\{\mathbf{e}(k)\mathbf{e}(l)^T\} \end{aligned}$$

Since $E\{\mathbf{e}(k)\mathbf{e}(l)^H\}$ is diagonal, the matrix $\mathbf{R}_e(\omega_1,\omega_2)$ is also diagonal. The i-th diagonal entry is equal to:

$$\begin{aligned} [R_e]_{ii}(\omega_1,\omega_2) &= \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} e^{-j\frac{2\pi(\omega_1 k-\omega_2 l)}{N}} r_{e_i}(k-l) \\ &= \sum_{l=0}^{N-1} e^{-j\frac{2\pi(\omega_1-\omega_2)l}{N}} \sum_{\tau=-l}^{N-1-l} e^{-j\frac{2\pi\omega_1\tau}{N}} r_{e_i}(\tau) \quad (13) \end{aligned}$$

For $\omega_1 = \omega_2 = \omega$, Eq. (13) is simplified into the following expression

$$\begin{aligned} [R_e]_{ii}(\omega) &= \sum_{l=0}^{N-1}\sum_{\tau=-l}^{N-1-l} e^{-j\frac{2\pi\omega\tau}{N}} r_{e_i}(\tau) \\ &= N r_{e_i}(0) + \sum_{\tau=1}^{N-1} r_{e_i}(\tau) \sum_{l=\tau}^{N-1-\tau} e^{-j\frac{2\pi\omega\tau}{N}} \\ &\quad + \sum_{\tau=-(N-1)}^{-1} r_{e_i}(\tau) \sum_{l=-\tau}^{N-1+\tau} e^{-j\frac{2\pi\omega\tau}{N}} \\ &= N r_{e_i}(0) + 2\sum_{\tau=1}^{N-1} r_{e_i}(\tau)(N-2\tau)\cos(\frac{2\pi\omega\tau}{N}) \end{aligned} \quad (14)$$

where by a slight abuse of notation we write $\mathbf{R}_e(\omega)$ instead of $\mathbf{R}_e(\omega,\omega)$. $\mathbf{R}_e(\omega)$ is by definition positive definite and by Eq. (14) it is also real and diagonal.

[1] We use the following convention in which we define DFT: $x(\omega) = \sum_k x(k) exp\{-j\frac{2\pi\omega k}{N}\}$ and IDFT: $x(k) = \frac{1}{N}\sum_k x(\omega) exp\{j\frac{2\pi\omega k}{N}\}$.

Define the $n \times n$ complex matrix $\mathbf{V}(\omega)$ by the following pre-whitening operation

$$\mathbf{V}(\omega)\mathbf{R}_x(\omega,\omega)\mathbf{V}(\omega)^H = \mathbf{I} \quad (15)$$

$$\mathbf{V}(\omega)\mathbf{H}(\omega)\mathbf{R}_e(\omega)\mathbf{H}(\omega)^H\mathbf{V}(\omega)^H = \mathbf{I} \quad (16)$$

and let

$$\mathbf{y}(\omega) = \mathbf{V}(\omega)\mathbf{x}(\omega) = \mathbf{V}(\omega)\mathbf{H}(\omega)\mathbf{e}(\omega) \quad (17)$$

Furthermore, define

$$\mathbf{W}(\omega) = \mathbf{V}(\omega)\mathbf{H}(\omega)\mathbf{R}_e(\omega)^{1/2} \quad (18)$$

so

$$\mathbf{W}(\omega)\mathbf{W}(\omega)^H = \mathbf{I}, \quad \text{for all } \omega \in \mathbf{C}$$

Consider now the covariance matrix for different frequency pairs ω_1, ω_2

$$\begin{aligned}\mathbf{R}_y(\omega_1,\omega_2) &= E\{\mathbf{y}(\omega_1)\mathbf{y}(\omega_2)^H\} \\ &= \mathbf{V}(\omega_1)\mathbf{H}(\omega_1)\mathbf{R}_e(\omega_1,\omega_2)\mathbf{H}(\omega_2)^H\mathbf{V}(\omega_2)^H \\ &= \mathbf{W}(\omega_1)\mathbf{D}(\omega_1,\omega_2)\mathbf{W}(\omega_2)^H \quad (19)\end{aligned}$$

where

$$\mathbf{D}(\omega_1,\omega_2) = \mathbf{R}_e(\omega_1)^{-1/2}\mathbf{R}_e(\omega_1,\omega_2)\mathbf{R}_e(\omega_2)^{-1/2}. \quad (20)$$

The matrices $\mathbf{W}(\omega_1)$, $\mathbf{W}(\omega_2)$, are orthogonal whereas the matrix $\mathbf{D}(\omega_1,\omega_2)$ is diagonal. It is interesting to note that the diagonal entries of $\mathbf{D}$ are the complex correlation coefficients between $e_i(\omega_1)$ and $e_i(\omega_2)$:

$$\begin{aligned}[D]_{ii}(\omega_1,\omega_2) &= \rho(e_i(\omega_1), e_i(\omega_2)) \\ &= \frac{[R_e]_{ii}(\omega_1,\omega_2)}{([R_e]_{ii}(\omega_1)[R_e]_{ii}(\omega_2))^{1/2}} \quad (21)\end{aligned}$$

hence, $0 \leq |[D]_{ii}(\omega_1,\omega_2)| \leq 1$. Let us define

$$\mathbf{C}_y(\omega_1,\omega_2) = \mathbf{R}_y(\omega_1,\omega_2)\mathbf{R}_y(\omega_1,\omega_2)^H \quad (22)$$

so

$$\mathbf{C}_y(\omega_1,\omega_2) = \mathbf{W}(\omega_1)\mathbf{D}(\omega_1,\omega_2)\mathbf{D}(\omega_1,\omega_2)^H\mathbf{W}(\omega_1)^H \quad (23)$$

Equation (23) represents the eigenvalue decomposition of $\mathbf{C}_y(\omega_1,\omega_2)$. Once we estimate $\mathbf{C}_y(\omega_1,\omega_2)$ we can then perform a standard eigenvalue decomposition in order to estimate $\mathbf{W}(\omega_1)$. Notice that although the matrix $\mathbf{C}_y$ is a function of both ω_1 and ω_2 the eigenvector matrix $\mathbf{W}$ is only a function of ω_1. That gives us the luxury to verify the results for different ω_2 and also allows us to combine matrices $\mathbf{C}_y$ with same ω_1 but different ω_2 in order to obtain more robust eigenvector estimates. Another important point is the fact that the eigenvectors are unique (upto a permutation and scaling) only if the eigenvalues $|[D]_{ii}(\omega_1,\omega_2)|^2$ are distinct. In general the eigenvalues are distinct since the signals e_i have different colored spectra. Nevertheless, for some frequency pairs it is possible that the eigenvalues be very close to each other. In that case the eigenvector estimates are quite unreliable. We discuss this problem in more detail in [1]. Finally, the eigenvalues can be computed by substituting (13) into (21), so they can be estimated from the covariance sequences $r_{e_i}(l)$. Using these estimates we can resolve the eigenvector ordering ambiguity provided that the eigenvalue estimates are close enough to the real values.

4. BLIND MULTIUSER CDMA EQUALIZATION ALGORITHM

Our method is composed of two parts. In the first part, described in Section 4.1, we estimate the magnitudes of the mixing filters H_{ij} as well as their relative phases. In the second part, described in Section 4.2, the phase of the channels is recovered using any ARMA parameter estimation approach.

4.1. Estimating channel magnitude and relative phase

Once we have resolved the eigenvector ordering ambiguity we can then use the eigenvector estimates $\hat{\mathbf{W}}(\omega)$ in order to estimate the unknown channels $H_{ij}(\omega)$. There is still some ambiguity left in $\hat{\mathbf{W}}(\omega)$ as any matrix of the following form

$$\hat{\mathbf{W}}(\omega) = \mathbf{W}(\omega)\mathbf{\Phi}(\omega) \quad (24)$$

is an orthogonal eigenvector matrix of $\mathbf{C}_y(\omega_1,\omega_2)$, where $\mathbf{\Phi}(\omega) = \mathrm{diag}[e^{j\phi_1(\omega)} \cdots e^{j\phi_n(\omega)}]$, is a diagonal matrix with unknown, unit-norm, complex diagonal elements. Still we can use Eq. (24) in order to estimate $\mathbf{H}(\omega)$:

$$\hat{\mathbf{H}}(\omega) = \mathbf{V}(\omega)^{-1}\hat{\mathbf{W}}(\omega)\mathbf{R}_e(\omega)^{-1/2} \quad (25)$$

so we have $\hat{\mathbf{H}}(\omega) = \mathbf{V}(\omega)^{-1}\mathbf{W}(\omega)\mathbf{\Phi}(\omega)\mathbf{R}_e(\omega)^{-1/2}$. Since both $\mathbf{\Phi}(\omega)$ and $\mathbf{R}_e(\omega)^{-1/2}$ are diagonal they commute and we obtain $\hat{\mathbf{H}}(\omega) = \mathbf{H}(\omega)\mathbf{\Phi}(\omega)$, or

$$\begin{aligned}|\hat{H}_{ij}(\omega)| &= |H_{ij}(\omega)|, \quad (26)\\ \angle\hat{H}_{ij}(\omega) &= \angle H_{ij}(\omega) + \phi_j(\omega). \quad (27)\end{aligned}$$

From (26) follows that the magnitude of the channels is perfectly reconstructed. Eq. (27) shows that the phase estimate contains an unknown offset which is, however, the same for all channels H_{1j}, ..., H_{nj}, suggesting that the relative phase $\angle H_{ij}(\omega) - \angle H_{jj}(\omega)$ is perfectly recovered as well.

4.2. Source reconstruction

Based on (26) and (27) we get:

$$\frac{\hat{H}_{ij}(\omega)}{\hat{H}_{jj}(\omega)} = \frac{H_{ij}(\omega)}{H_{jj}(\omega)} \quad (j \neq i) \quad (28)$$

Thus, the ratio of $H_{ij}(\omega)$ and $H_{jj}(\omega)$ is can be estimated. Assuming that $h_{ij}(n)$, $h_{jj}(n)$ have no common zeros, they can be computed from their ratio using any ARMA parameter estimation method. Although so far we did not need the lengths of the channels, at this point, some ARMA methods will require the lengths of $h_{ij}(n)$, $h_{jj}(n)$.

The proposed method can be summarized as follows:

Algorithm 1 Let $\mathbf{x}(k)$ be the sampled vector observation sequence in the multiple receivers. Select a segment size N_f and perform the following steps:

1. Estimate $r_{e_i}(l)$ from (10) and $\mathbf{R}_e(\omega_1,\omega_2)$ from (13) for all pairs of discrete frequencies $\omega_1, \omega_2 = 0, \cdots, N_f - 1$.

2. Estimate $r_{x_i,x_j}(l)$ from ensemble data for all pairs of indices i, j. Also estimate $\mathbf{R}_x(\omega_1, \omega_2)$ using the following expression (obtained similar to equation (13))

$$[R_x]_{ij}(\omega_1, \omega_2) = \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} e^{-j\frac{2\pi(\omega_1 k - \omega_2 l)}{N}} r_{x_i,x_j}(k, l)$$

3. Let $\mathbf{V}(\omega)$ be the prewhitening operator for $\mathbf{R}_x(\omega, \omega)$:

$$\mathbf{V}(\omega)\mathbf{R}_x(\omega, \omega)\mathbf{V}(\omega)^H = \mathbf{I}$$

4. Estimate $\mathbf{R}_y(\omega_1, \omega_2)$ by $\mathbf{V}(\omega_1)\mathbf{R}_x(\omega_1, \omega_2)\mathbf{V}(\omega_2)^H$.
5. For each frequency $\omega_1 = 0, \cdots, N_f - 1$, select a frequency ω_2 (typically $\omega_2 = \omega_1 \pm 1$) and form $\mathbf{C}_y(\omega_1, \omega_2)$ as in (22). Compute the eigenvalue decomposition

$$\mathbf{C}_y(\omega_1, \omega_2) = \hat{\mathbf{W}}(\omega_1)\mathbf{\Sigma}(\omega_1, \omega_2)\hat{\mathbf{W}}(\omega_1)^H.$$

6. Use (21) to compute $[D]_{ii}(\omega_1, \omega_2)$. Theoretically for any frequency pair ω_1, ω_2, we should have $|[D]_{ii}|^2 = [\Sigma]_{\pi(i)\pi(i)}$ for some permutation function $\pi()$. Arrange the eigenvalues $[\Sigma]_{ii}$ in the same magnitude order as $|[D]_{ii}|^2$ and arrange the eigenvectors in the same order as well.
7. Compute $\hat{\mathbf{H}}(\omega)$ using (25) and form the ratios $G_{ij} = \hat{H}_{ij}/\hat{H}_{jj}$, for all $j \neq i$.
8. Estimate the channels $H_{ij}(\omega)$, $H_{jj}(\omega)$ from their ratio G_{ij} via any ARMA parameter estimation method.

5. PRELIMINARY RESULTS

In this section we include some preliminary simulation results for the case of 2-input 2-output system. Since the performance of the method will eventually depend on the ARMA parameter estimation approach required at step 8 of the algorithm, we here try to eliminate this dependence as follows. If $\hat{\mathbf{H}}$ is used instead of $\mathbf{H}$ for deconvolution, then based on (26),(27) we reconstruct $\mathbf{\Phi}^{-1}(\omega)\mathbf{e}(\omega)$. Thus the reconstructed signals $e_1(k)$ and $e_2(k)$ have the correct magnitude spectra, but their phases differ from the correct ones by $\phi_1(\omega)$ and $\phi_2(\omega)$, respectively. In the sequel we test the success of the proposed method in estimating $\hat{\mathbf{H}}$ by comparing the estimated magnitude spectra of $e_1(k)$ and $e_2(k)$ versus the correct ones.

We formed the signals $e_1(k)$ and $e_2(k)$ of length $N = 16384$ by convolving two white independent binary random processes (source signals) with the signatures
$c_1(k) = [0.5774, 0.0000, -0.5774, 0.5774]$ and
$c_2(k) = [0., -0.7071, 0., 0., 0.7071]$.
The channels were taken to be
$h_{11}(k) = [[1.00, 1.0270, 0.3338, 0.0339, 0]$,
$h_{12}(k) = [1.00, 1.6908, 1.3609, 1.6603, 1.6883]$,
$h_{21}(k) = [1.00, 1.5908, 1.6609, 1.7603, 1.7083]$,
$h_{22}(k) = [1.00. - 1.9000, 0.6500, -0.0770, 0.0030]$.

The matrix $\mathbf{R}_x(\omega_1, \omega_2)$ was estimated as shown in Step 2 of the algorithm. The frequency ω_2 was selected equal to $\omega+1$ for all ω_1. The prewhitening vector $\mathbf{V}(\omega)$ was obtained as

$$\mathbf{V}(\omega) = \mathbf{R}_x^{-1/2}(\omega) \tag{29}$$

Figure 1 illustrates the true versus the estimated magnitude spectrum of $e_1(k)$ and $e_2(k)$ in db, corresponding to 50 independent input realizations.

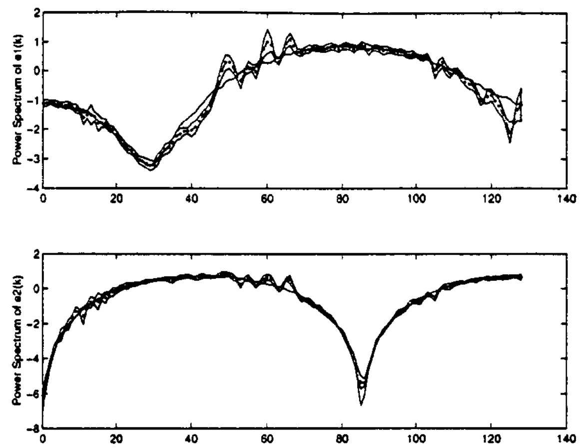

Figure 1: True (dotted line) vs. estimated magnitude spectra. Solid line indicate the estimated mean of 50 Monte Carlo simulations; gray area indicates standard deviation around the estimated mean.

6. REFERENCES

[1] K.I. Diamantaras and A.P. Petropulu, "Blind equalization of multiuser CDMA channels: a frequency domain approach", IEEE Trans on Signal Processing, submitted in 1998.

[2] Subspace-Based Channel Estimation for Code Division Multiple Access Communication Systems", S. E. Bensley and B. Aazhang, IEEE Tr. Communications, pp. 1009–1020, vol. 44, no. 8, Aug. 1996.

[3] E.G. Ström, S. Parkvall, S.L. Miller, and B.E. Ottersten, "Sensitivity Analysis of Near-Far Resistant DS-CDMA Receivers to Propagation Delay Estimation Errors", in Proc. IEEE Vehic. Technology Conf., pp. 757–761, 1994.

[4] M.K. Tsatsanis and G.B. Giannakis, "Multirate Filter Banks for Code-Division-Multiple-Access Systems", in Proc. ICASSP 95, Detroit, MI, May 1995.

[5] M. Torlak and G. Xu, "Blind Multiuser Channel Estimation in Asynchronous CDMA Systems", IEEE Tr. Signal Processing, pp. 137–147, vol. 45, no. 1, Jan. 1997.

[6] J. Xavier, V. Barroso, and J.M.F. Moura, "Closed Form Blind Identification of MIMO Channels", in Proc. ICASSP-98, vol. 6, pp. 3165-3168, Seattle WA, 1998.