

# MUTUALLY ORTHOGONAL TRANSCEIVERS FOR BLIND UPLINK CDMA IRRESPECTIVE OF MULTIPATH CHANNEL NULLS

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## ABSTRACT

Suppression of multiuser interference (MUI) and mitigation of multipath effects constitute major challenges in the design of third-generation wireless mobile systems. Most wideband and multicarrier uplink CDMA schemes suppress MUI statistically in the presence of unknown multipath. For fading resistance, they all rely on transmit- or receive-diversity and multichannel equalization based on bandwidth-consuming training, or, blind techniques. Either way, they impose restrictive and difficult to check conditions on the FIR channel nulls. Relying on symbol blocking, we design A Mutually-Orthogonal Usercode-Receiver (AMOUR) system for quasi-synchronous blind CDMA that eliminates MUI deterministically and mitigates fading irrespective of the unknown multipath and the adopted signal constellation. Analytic evaluation and preliminary simulations reveal the generality, flexibility, and superior performance of AMOUR over competing alternatives.

## 1. INTRODUCTION

Multiuser interference (MUI) and multipath-induced interchip interference (ICI) are critical performance limiting factors in the design of third-generation wireless systems because they define their capabilities in handling high data rates and interactive multimedia services. MUI and ICI suppression is thus of paramount importance in mobile wideband CDMA standards such as UMTS and IMT-2000 [6]. Multipath causes frequency-selective fading, destroys orthogonality of user codes, and when unknown, it precludes usage of linear zero-forcing (ZF), MMSE, or nonlinear (DF, ML) multiuser detectors for MUI suppression [11]. But even when multipath channel estimates are available (e.g., using bandwidth-consuming training sequences) it is well known that especially for multichannel uplink CDMA systems multiuser equalization is only possible under certain polynomial rank conditions on channel matrices that are difficult to check at the receiver [10].

Thanks to their versatility in handling variable rates, relaxed requirements for power control, and minimal cooperation among users, self recovering (blind) CDMA receivers are appealing for mobile radio and digital broadcasting systems. However, even for the constrained class

of equalizable channels blind receivers require subspace decompositions (see e.g., [5]), or, suppress MUI statistically (and thus asymptotically) when reduced complexity adaptive receivers are sought [10]. Antenna diversity trades off improved performance for receiver complexity and statistical MUI suppression [3]. Generalizing Orthogonal Frequency-Division Multiple Access (OFDMA), the recent, so called Lagrange - Vandermonde (LV) CDMA transceivers [4, 7], have low complexity and offer blind MUI elimination by judicious design of user codes. But similar to OFDMA and depending on the multipath channel, LV transceivers require extra diversity to ameliorate (but not eliminate) fading effects caused by channel nulls [7, 9]. User code hopping and maximal ratio combining diversities are also used to combat fading in the increasingly popular (albeit bandwidth expanding) multicarrier (MC) CDMA systems [1, 2].

Relying on symbol blocking, we develop in this paper A Mutually-Orthogonal Usercode-Receiver (AMOUR) structure for quasi-synchronous blind uplink CDMA that eliminates MUI deterministically and mitigates fading irrespective of the unknown multipath. The system encompasses LV-CDMA and MC-CDMA as special cases, can have low FFT-based complexity, and appears to offer considerable design flexibility. Based on the multirate block model of Section 2, we develop the AMOUR-CDMA system in Section 3, and test its performance in Section 4.

## 2. BLOCK SYMBOL MODELING

The block diagram in Fig. 1 represents the uplink channel of a CDMA system, described in terms of its discrete-time equivalent baseband model, where signals, codes, and channels are represented by samples of their complex envelopes taken at the chip rate (only transmitter and receiver filters for one, the  $m$ th, user are shown). Advance/delay elements and down/up-samplers (D/U) serve the purpose of blocking and inserting zeros, so that each of the  $M$  users maps successive blocks of  $K$  symbols of the information sequence  $s_m(n)$  to blocks of length  $P > K$ , each containing  $P - K$  trailing zeros (guard chips). The  $i$ th block is depicted in Fig. 1 with its  $\mathcal{Z}$  transform  $S_m(i; z) := \sum_{k=0}^{K-1} s_m(iK + k)z^{-k}$ . Before transmission through the

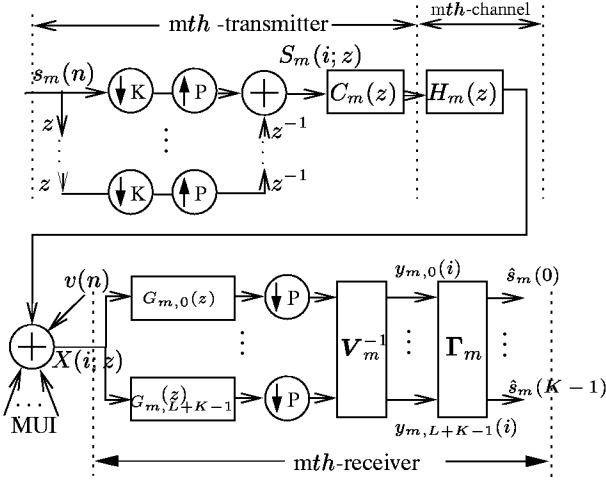


Figure 1: Discrete-time baseband AMOUR system

unknown FIR channel with transfer function  $H_m(z)$ , each of these  $P$ -long blocks is encoded with a code  $c_m(p)$  of length  $M(L+K) < P$  denoted by its  $\mathcal{Z}$ -transform  $C_m(z) := \sum_{p=0}^{M(L+K)-1} c_m(p)z^{-p}$ , where  $L$  stands for the maximum (over all  $m$ ) channel order. In addition to multipath,  $H_m(z)$  includes the spectral-shaping pulse and the  $m$ th user's asynchronism in the form of delay factors.

The  $i$ th received block  $\mathbf{x}(i) := [x(iP) \cdots x(iP + P - 1)]^T$  ( $T$  denotes transpose) is represented by its  $\mathcal{Z}$ -transform  $X(i; z) := \sum_{p=0}^{P-1} x(iP + p)z^{-p}$ , and consists of chips from the  $m$ th user of interest along with MUI chips from other users and AGN  $v(n)$ . The receive-filterbank,  $\mathbf{g}_m(p) := [g_{m,0}(p) \cdots g_{m,L+K-1}(p)]^T$ , performs vector filtering of the block  $\mathbf{x}(i)$  and after downsampling (to get back to the symbol rate) and multiplication by matrix  $\mathbf{V}_m^{-1}$  (to be specified in Section 3) we obtain the  $(L+K) \times 1$  vector  $\mathbf{y}_m(i) := [y_{m,0}(i) \cdots y_{m,L+K-1}(i)]^T$ . Our goal is to design user code polynomials  $\{C_m(z)\}_{m=0}^{M-1}$  and receive-filters  $\{G_{m,l}(z)\}_{l=0}^{L+K-1}$  capable of eliminating MUI deterministically, and thus enabling usage of single user equalizers denoted by the  $K \times (L+K)$  matrix  $\mathbf{\Gamma}_m$  to eliminate multipath effects, suppress noise, and recover the  $i$ th symbol block  $\hat{\mathbf{s}}_m(i) := [\hat{s}_m(0) \cdots \hat{s}_m(K-1)]^T$  from  $\mathbf{y}_m(i)$ .

Our system design parameters are as follows:

- d1)** Each input block is designed to contain  $K \gg L$  symbols, and each transmit-block size (at the chip rate) is chosen equal to  $P = (M+1)(L+K) - 1$ .
- d2)** Codes  $\{C_m(z)\}_{m=0}^{M-1}$  are selected to have order  $M(L+K) - 1$ , which creates  $L$  trailing zeros per block.
- d3)** Receive-filters  $\{G_{m,l}(z)\}_{l=0}^{L+K-1}$  will turn out to have order  $P, \forall m$ .

Note that except for an upper bound  $L$  on their orders, all uplink channels are allowed to be unknown. In quasi-synchronous (QS) CDMA systems, mobile users attempt to synchronize with the base-station's pilot waveform but their timing maybe off by 2–3 chips due to multipath and relative motion. Thus, our choice  $K \gg L$  does not entail very large  $K$ 's and thus it would not cause excessive

decoding delays. With each user transmitting  $K$  symbols per block, our system's spreading gain is [c.f. d1]):

$$\mathcal{R} := \frac{P}{K} = \frac{(M+1)(L+K) - 1}{K}, \quad (1)$$

which for sufficiently large  $K \gg L$  is  $\approx M$  (= to the number of users); hence, bandwidth is not over expanded. Thanks to the  $L$  trailing zeros [c.f. d2)], no interblock interference (IBI) is present in our received  $P$ -long blocks. Therefore, despite the presence of MUI and ICI that is allowed in our QS setup, one can focus on each block  $X(i; z)$  separately and express it in the  $\mathcal{Z}$ -domain as:

$$X(i; z) = \sum_{\mu=0}^{M-1} S_\mu(i; z)C_\mu(z)H_\mu(z) + V(i; z), \quad (2)$$

where  $V(i; z) := \sum_{p=0}^{P-1} v(iP + p)z^{-p}$ .

### 3. AMOUR FOR BLIND CDMA

Suppose we start with  $M(L+K)$  distinct points  $\rho_{m,l}$  on the complex plane and assign  $L+K$  of them to be roots common to all  $C_\mu(z)$  polynomials in (2), except the  $m$ th one. Our assignment amounts to

$$C_\mu(\rho_{m,l}) = 0, \quad \forall \mu \neq m, l \in [0, L+K-1], \quad (3)$$

which shows that evaluation of  $X(i; z)$  at  $z = \rho_{m,l}$  eliminates MUI from user  $m$  and yields

$$X(i; \rho_{m,l}) = S_m(i; \rho_{m,l})C_m(\rho_{m,l})H_m(\rho_{m,l}) + V(i; \rho_{m,l}). \quad (4)$$

Note that at most  $L$  of the  $L+K$  roots  $\{\rho_{m,l}\}_{l=0}^{L+K-1}$  can be roots of  $H_m(z)$ ; thus, we guarantee that  $X(i; \rho_{m,l}) \neq 0$  on at least  $K$  points, which is precisely the minimum number of values we need to know the  $(K-1)$ st-order polynomial  $S_m(i; z)$  in order to identify uniquely the  $m$ th user's  $i$ th block  $\mathbf{s}_m(i)$ . Roots  $\{\rho_{m,l}\}_{l=0}^{L+K-1}$  are "signature roots" of user  $m$  but are not roots of  $C_m(z)$ . The latter must contain signature roots of the remaining  $M-1$  users; hence,

$$C_m(z) = \mathcal{K}_m Q_m(z) \prod_{\mu=0, \mu \neq m}^{M-1} \prod_{\lambda=0}^{L+K-1} (1 - \rho_{\mu, \lambda} z^{-1}) \quad (5)$$

where  $\mathcal{K}_m$  is a constant controlling the  $m$ th user's transmit power, and  $Q_m(z)$  a code-normalizing polynomial of order  $L+K-1$  whose coefficients are chosen to satisfy:

$$Q_m(\rho_{m,l}) \prod_{\mu=0, \mu \neq m}^{M-1} \prod_{\lambda=0}^{L+K-1} (1 - \rho_{\mu, \lambda} \rho_{m,l}^{-1}) = 1, \quad (6)$$

for  $l \in [0, L+K-1]$ . Specification of the receive-filters  $\mathbf{g}_m(p)$  follows if we observe that

$$\begin{aligned} X(i; \rho_{m,l}) &= [1 \ \rho_{m,l}^{-1} \cdots \rho_{m,l}^{-P+1}] \mathbf{x}(i) \\ &:= \mathbf{v}^T(\rho_{m,l}) \mathbf{x}(i), \end{aligned} \quad (7)$$

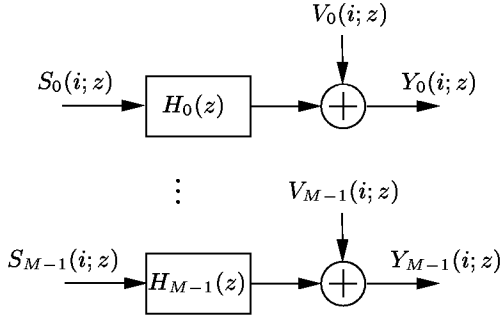


Figure 2: Equivalent parallel AMOUR-CDMA system

which amounts to convolution with the Vandermonde vector  $[\rho_{m,l}^{-P+1} \cdots \rho_{m,l}^{-1} 1]$ ; i.e.,  $g_{m,l}(p) = \rho_{m,l}^{-P+1+p}$ .

With the normalization in (6), eq. (5) yields  $C_m(\rho_{m,l}) = \mathcal{K}_m$  and along with definition (7) allows one to write the signal-only component in (4) as:  $\mathbf{v}^T(\rho_{m,l})\mathbf{x}(i) = \mathcal{K}_m S_m(i; \rho_{m,l}) H_m(\rho_{m,l}) := Y_m(i; \rho_{m,l})$ . Defining  $\tilde{\mathbf{y}}_m(i) := [Y_m(i; \rho_{m,0}) \cdots Y_m(i; \rho_{m,L+K-1})]^T$ , we deduce that

$$\tilde{\mathbf{V}}_m \mathbf{x}(i) = \tilde{\mathbf{y}}_m(i), \quad (8)$$

where  $\tilde{\mathbf{V}}_m := [\mathbf{v}(\rho_{m,0}) \cdots \mathbf{v}(\rho_{m,L+K-1})]^T$  is an  $(L+K) \times P$  Vandermonde matrix. The  $(L+K) \times 1$  vector  $\tilde{\mathbf{y}}_m(i)$  conveys complete information about  $Y_m(i; z) := \sum_{l=0}^{L+K-1} y_{m,l}(i) z^{-l} = \mathcal{K}_m S_m(i; z) H_m(z)$ , and shows how our AMOUR design converts the multiuser CDMA system into  $M$  parallel single-user systems irrespective of the multipath (see also Figure 2).

To recover  $y_{m,l}(i)$  from  $\{Y_m(i; \rho_{m,l})\}_{l=0}^{L+K-1}$ , define  $\mathbf{y}_m(i) := [y_{m,0}(i) \cdots y_{m,L+K-1}(i)]^T$ , and note that  $Y_m(i; \rho_{m,l}) = \mathbf{v}^T(\rho_{m,l})\mathbf{y}_m(i)$ , to arrive at (see also Figure 1):

$$\mathbf{y}_m(i) = \mathbf{V}_m^{-1} \tilde{\mathbf{y}}_m(i), \quad (9)$$

where the square Vandermonde matrix  $\mathbf{V}_m := [\mathbf{v}(\rho_{m,0}) \cdots \mathbf{v}(\rho_{m,L+K-1})]^T$  is full rank because it is built from distinct  $\rho_{m,l}$ 's. Matrix  $\mathbf{V}_m^{-1}$  can be combined with  $\tilde{\mathbf{V}}_m$  and  $\mathbf{V}_m^{-1} \tilde{\mathbf{V}}_m$  applied on  $\mathbf{x}(i)$  will separate blindly user  $m$ . Depending on complexity vs. performance tradeoffs, our channel-independent MUI-free receiver can be followed by any single user equalizer of linear (e.g., ZF, MMSE) or nonlinear (e.g., DF or ML) form in order to recover the block signal estimates  $\hat{\mathbf{s}}_m(i)$  from  $\mathbf{y}_m(i)$ .

To maintain an overall blind and computationally simple demodulator we recommend the filterbank approach of [8] that capitalizes on input redundancy which is also present in our transmitter design in the form of  $L$  trailing zeros. Briefly, user  $m$  collects  $I$  blocks  $\mathbf{y}_m(i) = \mathbf{H}_m \mathbf{s}_m(i)$  in a  $(L+K) \times I$  matrix  $\mathbf{Y}_m := [\mathbf{y}_m^H(0) \cdots \mathbf{y}_m^H(I-1)]$  and forms  $\mathbf{Y}_m \mathbf{Y}_m^H = \mathbf{H}_m \mathbf{S}_m \mathbf{S}_m^H \mathbf{H}_m^H$  where  $\mathcal{H}$  stands for Hermitian,  $\mathbf{S}_m := [\mathbf{s}_m(0) \cdots \mathbf{s}_m(I-1)]_{K \times I}$ , and  $\mathbf{H}_m$  is an  $(L+K) \times K$  convolution (Toeplitz) matrix. Minimal persistence of excitation guarantees that  $\mathbf{S}_m$  is full rank, and a subspace approach yields unique (within a scale) estimates of the channel coefficient vector from which  $\mathbf{H}_m$

and subsequently a ZF equalizing matrix  $\mathbf{\Gamma}_m$  can be found using the pseudo-inverse:  $\mathbf{H}_m^\dagger = \mathbf{\Gamma}_m$ . Direct, adaptive, and MMSE (if SNR is known) variants are also possible (see [8] for details).

AMOUR has low complexity if  $\rho_{m,l}$ 's are chosen regularly around the unit circle; e.g., with  $l \in [0, L+K-1]$  and  $m$ th user's signature roots

$$\rho_{m,l} = e^{j \frac{2\pi(m+lM)}{M(L+K)}}, \quad m \in [0, M-1], \quad (10)$$

matrix multiplications and inversion at the receiver can be replaced by FFTs. We expect that the "user-balanced" root selection in (9) possesses additional optimality in terms of SINR improvement, and results will be reported elsewhere.

**Remark 1:** AMOUR resembles MC CDMA although the latter does not guarantee channel-independent demodulation, and resorts to hopping in order to ameliorate performance in deep fades [1]. It also generalizes the LV/VL-CDMA systems of [4, 7, 9], which correspond to no input blocking ( $K=1$ ) and one signature root per user (as opposed to  $L+K$  roots used herein).

**Remark 2:** If channel estimates are available, MUI elimination is possible even with  $\tilde{L}+K < L+K$  signature roots which decreases the spreading in (1) and allows usage of smaller  $K$ 's to reduce decoding delays. In this case, the block and code lengths in d1), d2) are  $M(\tilde{L}+K)+L+K-1$  and  $M(\tilde{L}+K)$ , respectively. In fact, we show in Section 4 that considerable gains in Bit Error Rate (BER) are achieved with  $\tilde{L}$  as small as 1 or 2. However, to guarantee FIR ZF channel-independent blind equalization, we need  $\tilde{L} = L$  (at least  $L+K$  values of  $Y_m(i; z)$  are needed in (9)).

#### 4. PERFORMANCE AND SIMULATIONS

Because  $v(n)$  is AGN and our receiver can be ZF, theoretical BER evaluation is possible for a given constellation. For simplicity, we focus on BPSK  $s_m(n)$ 's. The  $m$ th user's ZF receiver can be described by the matrix  $\mathbf{G}_m := (\mathbf{V}_m \mathbf{H}_m)^\dagger \tilde{\mathbf{V}}_m$  whose  $k$ th row is denoted as  $\mathbf{g}_{mk}^H$ . Our figure of merit is the average BER  $\bar{P}_e := (MK)^{-1} \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} P_{e,mk}$ , where  $P_{e,mk}$  denotes BER for the  $k$ th symbol of user  $m$ . Because  $\hat{s}_m(iK+k) = \mathcal{K}_m s_m(iK+k) + \mathbf{g}_{mk}^H \mathbf{v}(i)$ , our SNR will be  $\mathcal{K}_m^2 / (N_0 \mathbf{g}_{mk}^H \mathbf{g}_{mk} / 2)$  and with  $2E_b/N_0$  denoting bit SNR, we have  $\mathcal{K}_m^2 = E_b/E_{c,m}$  where  $E_{c,m} := \sum_{p=0}^{M(L+K)-1} |c_m(p)|^2$  is the energy of the  $m$ th user's code (same  $\forall m$  for  $\rho_{m,l}$ 's as in (10)); hence,

$$P_{e,mk} = \mathcal{Q} \left( \sqrt{\frac{1}{\mathbf{g}_{mk}^H \mathbf{g}_{mk} E_{c,m}} \sqrt{\frac{2E_b}{N_0}}} \right), \quad (11)$$

where  $\mathcal{Q}(\cdot)$  denotes the  $\mathcal{Q}$ -function. In comparison,  $M$ -user OFDMA will exhibit  $m$ th equalizer output  $SNR = |H(\frac{2\pi m}{M})|^2 E_b / (N_0/2)$ , and thus average BER:

$$\bar{P}_e = \frac{1}{M} \sum_{m=0}^{M-1} \mathcal{Q} \left( \left| H\left(\frac{2\pi m}{M}\right) \right| \sqrt{\frac{2E_b}{N_0}} \right), \quad (12)$$

where  $H(z)$  is for OFDMA, a channel common to all users (downlink setup). We compared (11) with (12) on a system with  $M = 16$  users sharing a common multipath channel of order  $L = 1$  having its single root located at  $\rho = 0, 0.5, 0.7, 1$ . Because OFDMA's spreading gain is  $(M + L)/M$ , for a fair comparison with AMOUR, we chose  $K = M = 16$  (c.f. (1)). Figure 3 shows BER gains of AMOUR over OFDMA by 2-3 orders of magnitude as  $\rho$  approaches the unit circle. To avoid channel dependent performance, we averaged (11) over 100 Monte Carlo realizations of 6th order Rayleigh faded channels (simulated with complex Gaussian coefficients) and obtained AMOUR's  $\bar{P}_e$  vs  $E_b/N_0$  curves parameterized by the number of signature roots  $\tilde{L}$  assigned to each the  $M = 16$  users (Figure 4).  $K$  values were chosen according to (1) to maintain the same rate. Notwithstanding, even small values of the diversity factor  $\tilde{L}$  offer considerable BER gains over OFDMA ( $\tilde{L} = 0$ ). Further BER improvement is possible by precoding  $\mathbf{s}_m(i)$  blocks as  $\mathbf{u}_m(i) = \mathbf{F}_m \mathbf{s}_m(i)$ . It turns out that with  $\mathbf{u}_m(i)$  as input to Fig. 1 and optimal (with respect to system capacity)  $\mathbf{F}_m$  designs, it is possible to convert AMOUR to  $MK$  parallel independent flat fading channels.

To test AMOUR's ability for channel independent blind demodulation in uplink systems, we simulated  $M = 16$  users each transmitting blocks of  $K = 16$  QPSK symbols with  $L = 1$  signature root, through a two-ray channel ( $L = 1$ ). Relying only on  $I = L + K = 17$  blocks received in AWGN (SNR= 7dB), we recovered each user's constellation using the FIR-ZF direct blind equalizer of [8] (successfully equalized scatter diagrams of the first four users are depicted in Figure 5). Decision feedback (DF) schemes (see e.g., [11]) can improve performance further. We stress however, that our basic result does not rely on finite alphabet assumptions; thus, it applies to general deterministic blind separation and equalization of convolutive mixtures involving even continuous amplitude precoded (e.g., radar or speech) sources.

**Acknowledgment:** Work in this paper was supported by NSF CCR grant no. 98-05350.

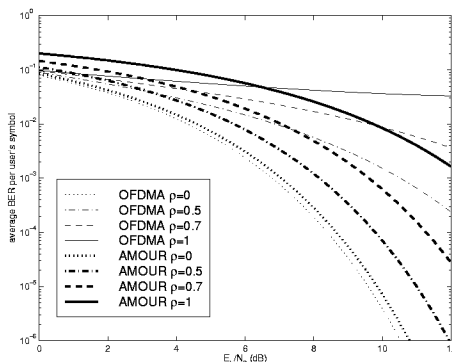


Figure 3: AMOUR vs. OFDMA, 16 users,  $L = 1$

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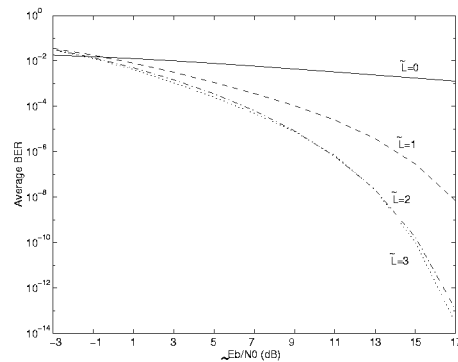


Figure 4: AMOUR with  $\tilde{L} < L = 6$  (Rayleigh fading)

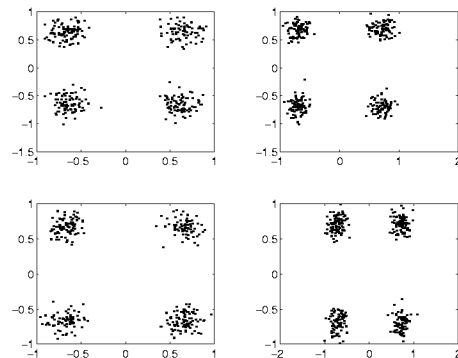


Figure 5: Blindly equalized constellations (SNR= 7dB)