JOINT SOURCE CHANNEL CODING OF IMAGES OVER FREQUENCY SELECTIVE CHANNELS USING DCT AND MULTICARRIER BPAM

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ABSTRACT

A novel approach to joint source and channel coding for frequency selective channels is presented. Multicarrier modulation is used to obtain an equivalent vector channel to the frequency selective channel and utilize the linear coding procedure of Lee and Petersen. The use of the block pulse amplitude transmission results in graceful degradation of the decoded signal for low channel SNR. The new procedure shows very good performance for image transmission over frequency selective channels with deep nulls in the frequency response, in the very low channel SNR region. Both encoder and decoder are computationally very inexpensive in terms of design and implementation, compared to the digital transmission with channel optimized vector quantization. Results for transmission of Gauss-Markov source and "Lena" image on several typical channels are presented.

1. INTRODUCTION

Recently, there has been an increased interest in joint source channel coding for both channels with Gaussian noise and fading channels. There are two basic approaches to joint source channel coding. The first approach is channel optimized source coding, including the channel optimized vector quantization (COVQ) [1]. In the so called source optimized channel coding, unequal error protection (UEP) of the bits at the output of the source coder is provided by using, for instance, the rate compatible punctured convolutional (RCPC) codes. In the context of image coding, RCPC codes have been used in [2], [3].

COVQ for vector Gaussian channels, which includes an optimization of the modulation signal set, was presented in [4] and [5]. When the source is a correlated or uncorrelated Gauss-Markov process and the channel is a vector Gaussian channel, the block pulse amplitude modulation (BPAM) provides performance gains compared to a digital system using COVQ if both systems employ a linear mapping from the source to the signal space and estimator based linear decoding [4]. Having this in mind, some authors [6, 7] advocate the use of hybrid analog and digital modulation, where some of the source information is more efficiently transmitted in an analog fashion.

So far, joint source channel coding for Gaussian channels with intersymbol interference, encountered in wireless transmission, has not been addressed adequately. In our previous work we developed COVQ for Gaussian channels with intersymbol interference based on the use of MAP equalizer as a soft decision decoder [8, 9] and, in a different approach, on the multicarrier modulation [10]. Both approaches suffer from performance loss in the high SNR region, because of the fixed rate used for the vector quantizers design.

Here, we propose BPAM for transmission of a multidimensional Gaussian source on a channel with ISI. To obtain an equivalent vector channel to the channel with ISI we use a FFT [10]. This procedure is usually referred to as discrete multitone. We show that the obtained vector channel is equivalent to the vector channel used by Lee and Petersen [11]. Then, the approach in [11] to implement linear coding that minimizes total end-to-end mean square error (MSE) is readily applicable.

We consider two examples to illustrate our novel procedure; KLT coefficients obtained from a Gauss-Markov source, and DCT coefficients, obtained by DCT of blocks of a given image. Since the distribution of the DCT coefficients can be closely approximated by a Gaussian distribution, the described procedure can be used for the transmission of images on channels with ISI.

We present the simulation results of a transmission of a Gauss-Markov source and "Lena" image on several typical frequency selective channels.

2. COMMUNICATION SYSTEM MODEL

We consider the transmission of a source signal over a frequency selective channel. The channel is modeled as FIR filter with $\nu+1$ complex coefficients $[h_0, \ldots, h_\nu]$ and complex additive white Gaussian noise w with variance σ_w^2. The channel input-output relationship is

$$z_i = \sum_{m=0}^{\nu} h_m v_{i-m} + w_i \tag{1}$$

We use boldface notation to denote vectors obtained by concatenating L complex symbols. In [10] we showed that such a channel with block serial transmission is equivalent to the following channel with block parallel transmission

$$\mathbf{y} = \mathrm{diag}(H_1, \ldots, H_L)\mathbf{u} + \mathbf{W} \tag{2}$$

where $[H_1, \ldots, H_L]$ is the DFT of $[h_0, \ldots h_\nu, 0, \ldots, 0]$, and $\mathbf{W}$ is the DFT of $\mathbf{w}$. Since $\mathbf{W}$ has the same statistics as $\mathbf{w}$, in the following we use the notation $\mathbf{w}$ instead of $\mathbf{W}$.

The vector channel is implemented by an inverse FFT at the transmitter ($\mathbf{u} \rightarrow \mathbf{v}$) and FFT at the receiver ($\mathbf{z} \rightarrow \mathbf{y}$). Assuming that we have perfect knowledge of the phases of $H_i = |H_i|e^{j\theta_i}$, we multiply the received components y_i by $e^{-j\theta_i}$ to annihilate the phase changes. We effectively get

$$y_i = |H_i|u_i + w_i \tag{3}$$

for $i = 1, \dots, L$. The same equation holds for both the real and imaginary parts. To implement the encoder we need to reorder the equations in (3) to obtain $|H_i|$ in decreasing fashion. By taking the real and imaginary parts in (3) and using real vectors, we arrive at the real system representation, presented in Figure 1, where $c_{2i-1} = c_{2i} = |H_i|$ for $i = 1, \dots, L$. The superscripts r denote real vectors. We group two by two outputs of the encoder u^r_{2i-1}, u^r_{2i}, with $i = 1, \dots, L$, to obtain L complex samples u_i, for $i = 1, \dots, L$ which form the vector $\mathbf{u}$. By taking FFT of $\mathbf{u}$ we obtain $\mathbf{v}$ and send v_i for $i = 1, \dots, L$ serially over the channel. The blocks G and Γ denote the linear encoder and decoder. By transforming the system in Figure 1 we

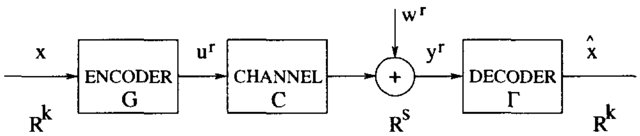


Figure 1: BPAM system with vector channel

obtain the equivalent system in Figure 2. The variances of

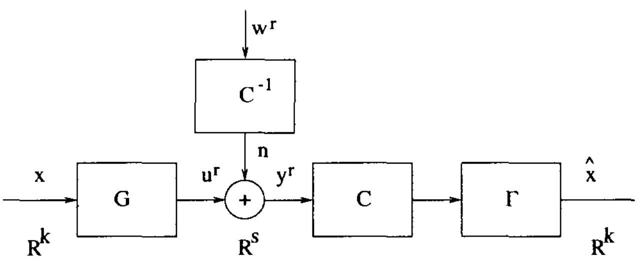


Figure 2: Equivalent BPAM system with vector channel

the entries $n_i = w^r_i/c_i$, $i = 1, \dots, 2L$ in the noise vector $\mathbf{n}$ are $N_i = \sigma^2_{w^r}/c_i^2$. Notice that $\sigma^2_{w^r} = \sigma^2_w/2$. The received complex samples are converted into real samples by taking their real and imaginary parts, $y^r_{2i-1} = Re\{y_i\}$ and $y^r_{2i} = Im\{y_i\}$, for $i = 1, \dots, L$.

3. ENCODER AND DECODER DESIGN

We assume that the vector $\mathbf{x}$ is a source vector already decorrelated with the KLT, or the suboptimal DCT in the case when we transmit an image. Let the dimension of the source space be k, and the dimension of the channel space be s. Note that $s = 2L$. The correlation matrix of the decorrelated source is the diagonal matrix $\Lambda = E[\mathbf{x}\mathbf{x}'] = \mathrm{diag}[\lambda_1, \dots \lambda_k]$. The encoder and decoder design follows [11]. Proper mapping of the components of the channel input vector $\mathbf{u}^r$ to the subchannels has to be performed, since the design described in [11] is for noise variances N_i ordered in a non-decreasing order. As previously mentioned, we achieve this by ordering the $|H_i|$'s in a decreasing order. Under a total channel input power constraint P, optimal power allocation to the components of the channel input vector is performed to obtain minimum total end-to-end mean square error (MSE) $D = \mathrm{tr}E[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})']$. The total power constraint on the signal $\mathbf{u}$ input to the channel is

$$P = \sum_{i=1}^{s} E|u^r_i|^2 = \mathrm{tr}[G\Lambda G'] \tag{4}$$

where s is the dimension of the channel space. The optimum encoder G is obtained as [11]

$$\begin{aligned} g_{ii} &= \left(\sqrt{\tfrac{\lambda_i N_i}{\theta}} - N_i\right)^{1/2} / \sqrt{\lambda_i}, && \text{for } i \leq l \\ g_{ij} &= 0, && \text{when } i \neq j \end{aligned} \tag{5}$$

For simplicity, we write g_i instead of g_{ii}. Since g_i must be real,

$$\frac{\lambda_i}{N_i} > \theta, \qquad \text{for } 1 \leq i \leq l \tag{6}$$

The optimum integer l is thus the maximum integer which satisfies (6). Once we determine the optimum encoder matrix G, i.e. its nonzero elements g_i from (5), we determine the nonzero elements of the decoder matrix Γ

$$\gamma_i = \frac{1}{c_i} \frac{\lambda_i g_i}{\lambda_i g_i^2 + N_i} \tag{7}$$

for $i = 1, \dots, l$. Note the extra factor $1/c_i$ in (7) compared to [11], due to the presence of C in front of the decoder Γ.

Given value of the parameter θ, we obtain parametric equations for the power P

$$P = \sum_{i=1}^{l} \left(\frac{\sqrt{\lambda_i N_i}}{\sqrt{\theta}} - N_i\right) \tag{8}$$

and the MSE

$$D = \sum_{i=l+1}^{k} \lambda_i + \sqrt{\theta} \sum_{i=1}^{l} \sqrt{\lambda_i N_i} \tag{9}$$

in terms of θ.

4. IMAGE TRANSMISSION ON A FIXED FREQUENCY SELECTIVE CHANNEL USING BPAM

We use the DCT to perform decorrelation and energy compaction of image blocks of $M \times M$ pixels, resulting in a dimension of the source vector of $k = M^2$. We compute the variances in the DCT coefficients by averaging over all the blocks contained in the image. We also assume that all the DCT coefficients except the 0-th are zero mean Gaussian distributed. The 0-th coefficient has a nonzero mean, and to obtain a zero mean distribution necessary for the design, we remove its mean before encoding. The number of complex subchannels is $L = s/2$. When transmitting all M^2 DCT coefficients (no compression), no information is lost due to encoding, and the end-to-end distortion (mean square error) is due to the channel noise only. In this case, we transmit one block of the image (M^2 pixels) on a block of L complex symbols. When performing signal compression, we transmit only the significant DCT coefficients. This results in a reduction of the transmitted symbol rate relative to the source rate. For example, to achieve a compression ratio of 2, we order the DCT coefficients in terms of decreasing variances. Denote the ordered variances by σ_i^2 for $i = 1, \dots, M^2$. We then retain the first $M^2/2$ most significant coefficients. We perform the encoder and decoder design by setting $\lambda_{2i-1} = \lambda_{2i} = \sigma_i^2$ for $i = 1, \dots, M^2/2$ to obtain $k = M^2$ source dimensions. In addition to the channel mapping, we now have mapping of the DCT coefficients, due to the ordering of their variances in decreasing fashion. Thus, we can transmit two image blocks ($2M^2$ pixels) on one block of L channel symbols.

5. SIMULATION RESULTS

We first consider a transmission of a Gauss-Markov (first order autoregressive) source with correlation coefficient $\rho = 0.9$. We use KLT to obtain a vector Gaussian source. For this example the channel is a FIR filter with three complex coefficients ($\nu = 2$). The dimension of the source and channel space is equal, $k = s = 32$. We use $L = 16$ complex subchannels. The channel SNR is computed as $SNR = P/(s\sigma_w^2)$. The signal to distortion ratio is computed as $SDR = (k\sigma^2)/D$ where σ^2 is the variance of the Gauss-Markov source. The simulation results for the BPAM system with an ISI channel are shown in Figure 3, together with the OPTA (optimum performance theoretically attainable) bound. The OPTA bound is obtained by computing the distortion at a rate equal to the channel capacity. The

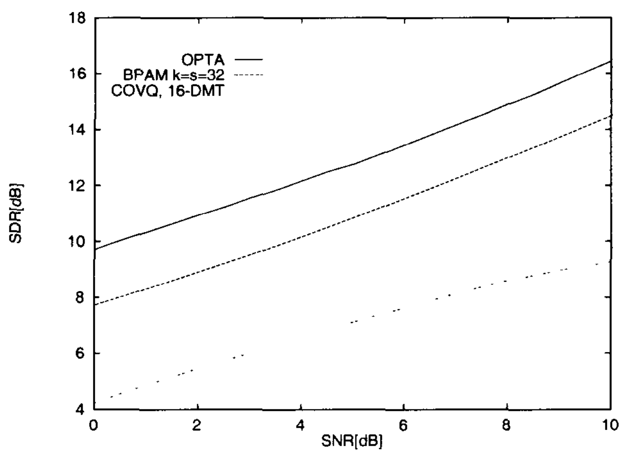


Figure 3: Performance comparison of joint source channel coding with digital and analog modulation format

BPAM performance is about 2 dB below the OPTA bound. Compared to the approach with COVQ in [10], this is an improvement of about 3-4 dB. There is always saturation of the signal to distortion ratio in the high SNR region with the COVQ approach, since the design is for a given rate of the VQ. This can be alleviated by using a high design rate, resulting in very complex COVQ. In essence this is approximating the analog by a high rate digital system.

As a second example we consider the transmission of the 512 × 512 image Lena on a frequency selective channel. We transmit DCT coefficients of 8x8 blocks. With images it is common to use the peak signal to distortion ratio PSDR. In Fig. 4 we show simulation results for 4 different channels with no compression. We consider a flat channel, the channel considered in the Gauss-Markov source example, and two channels from [12], i e. the channel (b) with a deep null in the magnitude response at the end of the bandwidth (Fig. 6.4.8b in [12]) and the channel (c) with a deep null inside the bandwidth (Fig. 6.4.8c in [12]). The

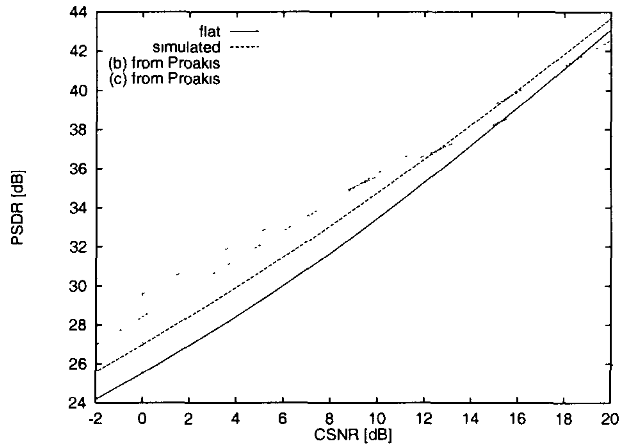
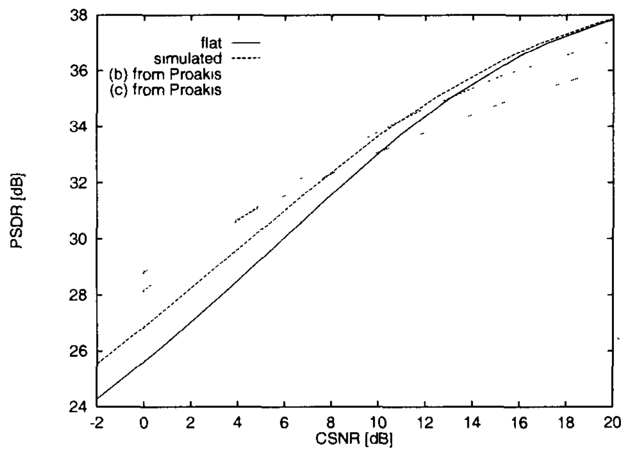


Figure 4: PSDR in terms of channel SNR; no compression

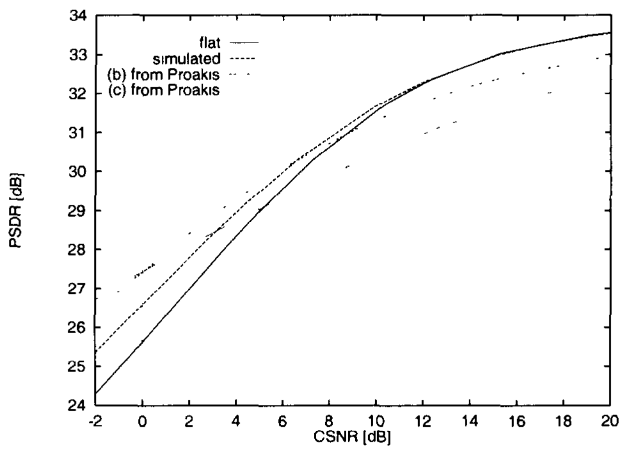
results that correspond to compression ratios of 2 and 4 are presented in Fig. 5. For low CSNR, channels with more frequency selectivity result in better performance in terms of SDR, due to the energy compaction of KLT and DCT. Our results are favorable to most of the published results in literature ([13] gives comparison of different approaches for the BSC channel, converted to Gaussian channel), in spite of the use of sophisticated source coders by other authors. Finally, we present the reconstructed "Lena" image transmitted on channel (c) from [12] with compression ratio of 2 and CSNR=-2 dB. The image has good quality in these very low CSNR conditions using a channel with high frequency distortion. In [12] it is shown that equalizers for this channel perform poorly.

6. CONCLUSION

A novel approach to joint source and channel coding for frequency selective channels based on multicarrier modulation and the linear coding procedure of [11], is presented. This analog approach is inherently robust to low signal to noise ratios, providing graceful degradation. The encoder and decoder are computationally inexpensive for design and implementation compared to COVQ. Possible drawback could be the high peak powers on some of the subchannels. Also, linear coding is not optimal for arbitrary variances of the components of the decorrelated vector source and arbitrary channel gains. With high compression ratios, the use of COVQ might be desirable. The new procedure is especially



compression ratio of 2



compression ratio of 4

Figure 5: PSDR in terms of channel SNR

efficient for channels with deep nulls in the frequency response, for which equalization is most difficult.

7. REFERENCES

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Figure 6: Reconstructed "Lena" image with compression ratio of 2 on the channel (c); CSNR=-2dB.

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