A RECURSIVE PREDICTION ERROR ALGORITHM FOR IDENTIFICATION OF CERTAIN TIME-VARYING NONLINEAR SYSTEMS

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ABSTRACT

The tracking problem in identification of certain classes of time-varying nonlinear systems is addressed. In particular. a Hammerstein type system which consists of a nonlinear part, given on a state space description, followed by a time-varying linear part is considered. A Recursive Prediction Error Method. RPEM combined with a method for on-line adjustment of the forgetting factor is proposed. This algorithm does not require estimation of the statistics of the noise and the dynamics of the true system. It is shown how the proposed scheme can be used for identification of certain nonlinear time varying acoustic echo paths. Thus. the suggested algorithm is applicable to for instance, conference telephony and mobile telephone handsfree.

1. INTRODUCTION

Certain classes of nonlinear systems can be successfully identified using models that consist of various combinations of a dynamic linear block and a static nonlinearity.

In this paper a class of Hammerstein type systems which consists of a nonlinear part given on a state space model in cascade with a linear part represented by a time-varying finite-impulse-response filter, FIR is considered. It is often convenient to use this model as many nonlinear systems can be described in state space form by a set of nonlinear ordinary differential equations.

The problem of tracking time-varying *linear* systems has been widely studied, see e.g. [1] and the references therein. Among the different methods that have been suggested, it is possible to distinguish three main approaches, namely the LMS-, RLS- and Kalman algorithms.

The problem of tracking time-varying *nonlinear* systems has been treated in e.g. [2,3,4]. In these papers the *output error method* was adopted. In [2] a recursive algorithm based on the maximum likelihood function was proposed. However, it was assumed that there were *no output mesurement noise*, (as well as slowly changing parameters). In [3] a more general formulation of a recursive prediction error algorithm was given. Several different criterion functions were considered. A recursive prediction error algorithm for identification of certain time-varying nonlinear systems given on a state space form was suggested in [4]. However, there was no correspondence to a variable forgetting factor to enhance the tracking capability in this algorithm. The problem of identifying time-varying Hammerstein systems using a subspace-based technique was addressed in [5]. The static nonlinearity was represented with a fixed polynomial and the linear block was assumed to consist of a time-varying filter.

Although, an extended Kalman filter type algorithm can be expected to yield good performance for the problem treated in the present paper, cf. [6], the associated computational burden may be considered too high. In particular, as this algorithm requires knowledge or estimates of the statistics of the noise and the dynamics of the true system. In practice these quantities are often unknown and hence need to be estimated. Furthermore, it is not possible to directly apply equation error based RLS to nonlinear models, since the regressions are assumed to contain the input and output measurements of a presumably linear system. However, the Recursive Prediction Error Method, RPEM applied to an output error model may be considered. In the following this method will be *tailored* to the specific class of nonlinear systems considered here.

2. SYSTEM AND MODEL DESCRIPTION

Thus, consider Hammerstein type systems where the nonlinear part can be described by a discrete state space model

$$\underline{x}(t+T,\underline{\theta}_n) = \underline{x}(t,\underline{\theta}_n) + T\underline{f}\left[\underline{x}(t,\underline{\theta}_n), u(t), \underline{\theta}_n\right] \quad (1)$$

$$y_n(t) = \underline{c}^T\, \underline{x}(t) + bu(t), \quad (2)$$

where T is used to denote the sampling period assuming that the discrete time formulation (1,2) has been obtained from sampling a continuous time system. Here $u(t)$ and $y_n(t)$ denote the input and output of the nonlinearity respectively. [1] $\underline{\theta}_n$ represents the time-invariant parametrization of the nonlinear part and $\underline{x}(t)$, with dimension $M \times 1$, is used to denote the state vector. Furthermore, $\underline{c}$ is a fixed real valued vector and a direct term with the coefficient b has been included. It is assumed that the nonlinear function $\underline{f}$, with dimension $M \times 1$, is continuously differentiable with respect to $\underline{x}(t)$ and $\underline{\theta}_n$.

[1] It may be noted that the commonly adopted definition of Hammerstein systems assume a *static* nonlinearity. However, here the nonlinear part exhibits a memory, (via $\underline{x}$). Thus, the considered systems can be viewed as (slightly) generalized forms of the Hammerstein type systems.

Furthermore, the linear part consists of a time-varying FIR filter and hence, the system output can be written as

$$y_l(t|\underline{\theta}) = \sum_{a=0}^{k-1} \theta_{la}(t-1)\; y_n(t-a) + e(t) = \underline{\theta}_l^T(t-1)\; \underline{y}_n(t) + e(t), \tag{3}$$

where

$$\underline{y}_n(t) = [y_n(t)\; y_n(t-1) \ldots y_n(t-k+1)]^T \tag{4}$$

and $e(t)$ is additive zero-mean white Gaussian measurement noise with variance r_2. It will be assumed that the time-varying linear parameters of the true system, $\underline{\theta}_l(t-1) = [\theta_{l0}(t-1) \ldots \theta_{l(k-1)}(t-1)]^T$, can be modeled as a random walk process. Thus,

$$\underline{\theta}_l(t) = \underline{\theta}_l(t-1) + \underline{w}_l(t), \tag{5}$$

where $\underline{w}_l(t)$ is a sequence of zero mean random vectors with covariance matrix $R_w(t)$. This assumption may be justified when considering a scenario where no information on the dynamics is available. In order to avoid ambiguity it will be assumed that the FIR filter is monic.

The following model description will be used:

$$\underline{\hat{x}}(t+T, \underline{\hat{\theta}}_n) = \underline{\hat{x}}(t, \underline{\hat{\theta}}_n) + T\underline{\hat{f}}\,[\underline{\hat{x}}(t, \underline{\hat{\theta}}_n), u(t), \underline{\hat{\theta}}_n] \tag{6}$$

$$\hat{y}_n(t) = \underline{c}^T\; \underline{\hat{x}}(t) + bu(t). \tag{7}$$

$$\hat{y}_l(t|\underline{\hat{\theta}}) = \sum_{a=0}^{k-1} \hat{\theta}_{la}(t-1)\; \hat{y}_n(t-a) = \underline{\hat{\theta}}_l^T(t-1)\; \underline{\hat{y}}_n(t), \tag{8}$$

Thus, it is assumed that the system can be completely described by the chosen model structure. If the coefficient for the direct term is unknown $\hat{b}$ should be substituted for b in (7). In addition, $\underline{\hat{y}}_n(t)$ is an estimate of (4). It is assumed that $\underline{\hat{f}}$ has the same continuity properties as $\underline{f}$.

3. A RECURSIVE PREDICTION ERROR ALGORITHM

Recalling the description of the system and the model in section 2, it follows that the prediction error becomes

$$\varepsilon_l(t) = y_l(t|\underline{\theta}) - \hat{y}_l(t|\underline{\hat{\theta}}). \tag{9}$$

A Recursive Prediction Error Method, RPEM can be obtained as a result of minimizing the quadratic criterion function $V(t) = \frac{1}{2}E[\varepsilon_l^2(t)]$, [7, pp. 88-92] using the stochastic Gauss Newton method. In the following, the RPEM will be applied to the class of nonlinear Hammerstein type systems treated here. In order to avoid matrix inversion, a 'P-matrix formulation' of the algorithm will be considered. Thus, calculate

$$P(t) = \left(I - \frac{P(t-1)\underline{\psi}(t)\underline{\psi}^T(t)}{\underline{\psi}^T(t)P(t-1)\underline{\psi}(t) + \lambda(t)}\right)\frac{1}{\lambda(t)}P(t-1) \tag{10}$$

and update the parameter estimates according to

$$\underline{\hat{\theta}}(t) = \underline{\hat{\theta}}(t-1) + P(t)\underline{\psi}(t)\varepsilon_l(t), \tag{11}$$

where $\underline{\hat{\theta}} = [\underline{\hat{\theta}}_l^T\; \underline{\hat{\theta}}_n^T\; \hat{b}]^T$ and the gradient vector, i.e. the derivative of the prediction error w.r.t. the parameter estimates, can be evaluated as

$$\underline{\psi}(t) = \begin{pmatrix} (-\frac{\partial \varepsilon_l}{\partial \underline{\hat{\theta}}_l(t-1)})^T \\ (-\frac{\partial \varepsilon_l}{\partial \underline{\hat{\theta}}_n})^T \\ (-\frac{\partial \varepsilon_l}{\partial \hat{b}})^T \end{pmatrix} = \begin{pmatrix} (\frac{\partial \hat{y}_l(t)}{\partial \underline{\hat{\theta}}_l(t-1)})^T \\ (\frac{\partial \hat{y}_l(t)}{\partial \underline{\hat{y}}_n(t)} \frac{\partial \underline{\hat{y}}_n(t)}{\partial \underline{\hat{\theta}}_n})^T \\ (\frac{\partial \hat{y}_l(t)}{\partial \hat{b}})^T \end{pmatrix} =$$

$$\begin{pmatrix} \underline{\hat{y}}_n(t) \\ \left(\frac{\partial}{\partial \underline{\hat{\theta}}_n}\underline{\hat{x}}^T(t, \underline{\hat{\theta}}_n)\; \underline{c} \ldots \frac{\partial}{\partial \underline{\hat{\theta}}_n}\underline{\hat{x}}^T(t-k+1, \underline{\hat{\theta}}_n)\; \underline{c}\right)\; \underline{\hat{\theta}}_l(t-1) \\ \underline{\hat{\theta}}_l^T(t-1)\; [u(t) \ldots u(t-k+1)]^T \end{pmatrix}. \tag{12}$$

In case the coefficient b for the direct term in (2) is known, the last row of the expression for the gradient above vanishes. In order to obtain $\frac{\partial \underline{\hat{x}}(t, \underline{\hat{\theta}}_n)}{\partial \underline{\hat{\theta}}_n}$ differentiate (6):

$$\frac{\partial \underline{\hat{x}}(t+T, \underline{\hat{\theta}}_n)}{\partial \underline{\hat{\theta}}_n} =$$

$$= \left(I + T\frac{\partial \underline{\hat{f}}(t, \underline{\hat{\theta}}_n)}{\partial \underline{\hat{x}}(t)}\right)\frac{\partial \underline{\hat{x}}(t, \underline{\hat{\theta}}_n)}{\partial \underline{\hat{\theta}}_n} + T\frac{\partial \underline{\hat{f}}(t, \underline{\hat{\theta}}_n)}{\partial \underline{\hat{\theta}}_n} \tag{13}$$

where

$$\frac{\partial \underline{\hat{f}}(t, \underline{\hat{\theta}}_n)}{\partial \underline{\hat{x}}} = \begin{pmatrix} \frac{\partial \hat{f}_1(t)}{\partial \hat{x}_1} & \frac{\partial \hat{f}_1(t)}{\partial \hat{x}_2} \ldots \frac{\partial \hat{f}_1(t)}{\partial \hat{x}_M} \\ \vdots & \vdots \\ \frac{\partial \hat{f}_M(t)}{\partial \hat{x}_1} & \frac{\partial \hat{f}_M(t)}{\partial \hat{x}_2} \ldots \frac{\partial \hat{f}_M(t)}{\partial \hat{x}_M} \end{pmatrix} \tag{14}$$

and

$$\frac{\partial \underline{\hat{f}}(t, \underline{\hat{\theta}}_n)}{\partial \underline{\hat{\theta}}_n} = \begin{pmatrix} \frac{\partial \hat{f}_1(t)}{\partial \hat{\theta}_{n1}} & \frac{\partial \hat{f}_1(t)}{\partial \hat{\theta}_{n2}} \ldots \frac{\partial \hat{f}_1(t)}{\partial \hat{\theta}_{np}} \\ \vdots & \vdots \\ \frac{\partial \hat{f}_M(t)}{\partial \hat{\theta}_{n1}} & \frac{\partial \hat{f}_M(t)}{\partial \hat{\theta}_{n2}} \ldots \frac{\partial \hat{f}_M(t)}{\partial \hat{\theta}_{np}} \end{pmatrix}. \tag{15}$$

Here M is used to denote the dimension of the vectors $\underline{\hat{f}}$ and $\underline{\hat{x}}$. Furthermore, p is the number of nonlinear parameter estimates. In order to evaluate (13), the initial estimates $\underline{\hat{\theta}}_n(0)$ and $\frac{\partial \underline{\hat{x}}(0, \underline{\hat{\theta}}_n(0))}{\partial \underline{\hat{\theta}}_n(0)}$ are required.

As a *time − varying* scenario is considered, the forgetting factor $\lambda(t)$ should be allowed to assume values less than 1.0 in order to be able to discount old information and thus respond more quickly to changes. It is proposed that a method devised in [8] for on-line adjustment of $\lambda(t)$ is adopted. Hence, calculate

$$\lambda_1(t) = \rho + (1-\rho)[1 - exp(-\frac{t}{\tau})], \quad 0.9 \leq \rho \leq 1.0 \tag{16}$$

$$\lambda_2(t) = 1 - \frac{\varepsilon_l^2(t-1)}{\tau s(t)}, \tag{17}$$

$$\lambda(t) = \lambda_1(t)\; \lambda_2(t), \tag{18}$$

Here τ is the desired memory or equivalently the time constant with which the influence of old information vanishes. In (17) $s(t)$ is a weighted sum of over τ samples of the squared prediction error, calculated as

$$s(t) = \frac{\tau - 1}{\tau} s(t-1) + \frac{\varepsilon_l^2(t-1)}{\tau}. \quad (19)$$

Inclusion of the factor $\lambda_1(t)$ guarantees that the forgetting factor $\lambda(t)$ attains a relatively low value initially. Furthermore, $\lambda_2(t)$, and thus also $\lambda(t)$, decreases when the squared residual $\varepsilon_l^2(t)$ increases and vice versa. It is noted that this method does not require an estimate of (or knowledge of) neither the noise variance nor the statistics of the time dynamics.

In conclusion, the proposed algorithm with a variable forgetting factor, hereafter named the VRPEM, based on a Hammerstein model with a nonlinear part on a state space description in cascade with a time-varying FIR-filter, is given by eqs. (6–19).

4. NUMERICAL EXAMPLE - ACOUSTIC ECHO PATH

In telephony systems, such as conference telephony and mobile telephone handsfree, where the loudspeaker and the microphone are separated, the remote end talker's speech may be echoed via acoustic coupling. Typically the echo propagates via a direct path between the loudspeaker and the microphone as well as via one or several reflections in the room. It is usually possible to model the echo path in the acoustic cavity using a linear filter. The duration of an acoustic echo in a teleconferencing system may be several hundred milliseconds. In addition, the system may be time-varying. While the propagation can be considered as essentially linear, it is usually necessary to account for nonlinearities in some of the electrical components. Typically, the loudspeaker is the main source of nonlinear distortion. This effect is mainly due to nonlinearity in the flexible suspensions to which the diaphragm, (cone) is mounted and inhomogeneity in the magnetic flux density in the air gap of the permanent magnet, [9, ch.7]. In particular for high input levels, these sources of distortion may severely degrade the sound quality. These nonlinearities can be well approximated by respectively third and second order polynomials in the diaphragm displacement, [10]. The following loudspeaker model, has been suggested in [10]:

$$\left[\frac{\partial x_1(t)}{\partial t}\ \frac{\partial x_2(t)}{\partial t}\ \frac{\partial x_3(t)}{\partial t}\right]^T =$$

$$\begin{pmatrix} \theta_{n1}u + \theta_{n2}x_1 \\ 0 \\ \theta_{n6}x_1 + \theta_{n7}x_1x_2 + \theta_{n8}x_1x_2^2 + \theta_{n9}x_2 + \theta_{n10}x_2^2 \end{pmatrix} + \begin{pmatrix} \theta_{n3}x_3 + \theta_{n4}x_2x_3 + \theta_{n5}x_2^2x_3 \\ x_3 \\ \theta_{n11}x_2^3 + \theta_{n12}x_3 \end{pmatrix}. \quad (20)$$

For an interpretation of the state $\underline{x}(t) = [x_1(t)\ x_2(t)\ x_3(t)]^T$ and the nonlinear parameters $\underline{\theta}_n = [\theta_{n1}, \theta_{n2}, \ldots, \theta_{n12}]^T$ in terms of actual physical loudspeaker parameters cf. [10,6].

Assuming that the derivative is approximated with a forward difference and that the sampling period equals unity, the corresponding discrete form is obtained as

$$\underline{x}(t+1) = \underline{x}(t) + \underline{f}\left[\underline{x}(t,\underline{\theta}_n), u(t), \underline{\theta}_n\right] =$$

$$\underline{x}(t) + \begin{pmatrix} -1.1 & 0 & -0.2 \\ 0 & 0 & 1 \\ 0.6 & -0.5 & -1.15 \end{pmatrix} \underline{x}(t) + \begin{pmatrix} 0.4 \\ 0 \\ 0 \end{pmatrix} u(t)$$

$$+ \begin{pmatrix} -0.04x_2(t)x_3(t) \\ 0 \\ -0.04x_2^2(t) - 0.32x_2^3(t) \end{pmatrix}$$

$$+ \begin{pmatrix} -0.05x_2^2(t)x_3(t) \\ 0 \\ 0.01x_1(t)x_2(t) + 0.02x_1(t)x_2^2(t) \end{pmatrix} \quad (21)$$

$$y_n(t) = [0\ 1\ 0]\ \underline{x}(t) \quad (22)$$

which corresponds to the associated nonlinear parameters

$$\underline{\theta}_n = [\theta_{n1}, \theta_{n2} \ldots \theta_{n12}]^T$$

$$[\theta_{n1} \ldots \theta_{n6}] = [0.4, -1.1, -0.2, -0.04, -0.05, 0.6],$$

$$[\theta_{n7} \ldots \theta_{n12}] = [0.01, 0.02, -0.5, -0.04, -0.32, -1.15], \quad (23)$$

(cf.(20)). This choice of parameters corresponds to a loudspeaker with rather low quality. The level of nonlinear distortion is quite high. In particular, with a 0 dB sinusoidal input, the first odd order harmonic is only 24 dB below the fundamental tone.

Next, it is noted that for the considered problem the gradients corresponding to (15,14) become

$$\frac{\partial \hat{\underline{f}}(t, \hat{\underline{\theta}}_n)}{\partial \hat{\underline{\theta}}_n} =$$

$$\begin{pmatrix} u & \hat{x}_1 & \hat{x}_3 & \hat{x}_2\hat{x}_3 & \hat{x}_2^2\hat{x}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \hat{x}_1 & \hat{x}_1\hat{x}_2 & \hat{x}_1\hat{x}_2^2 & \hat{x}_2 & \hat{x}_2^2 & \hat{x}_2^3 & \hat{x}_3 \end{pmatrix}, \quad (24)$$

$$\frac{\partial \hat{\underline{f}}(t, \hat{\underline{\theta}}_n)}{\partial \hat{\underline{x}}} =$$

$$\begin{pmatrix} \hat{\theta}_{n2} & \hat{\theta}_{n4}\hat{x}_3 + 2\hat{\theta}_{n5}\hat{x}_2\hat{x}_3 & \hat{\theta}_{n3} \\ 0 & 0 & 1 \\ \hat{\theta}_{n6} + \hat{\theta}_{n7}\hat{x}_2 & \hat{\theta}_{n7}\hat{x}_1 + 2\hat{\theta}_{n8}\hat{x}_1\hat{x}_2 + \hat{\theta}_{n9} & \hat{\theta}_{n12} \end{pmatrix}.$$

$$+ \begin{pmatrix} 0 & 0 & \hat{\theta}_{n4}\hat{x}_2 + \hat{\theta}_{n5}\hat{x}_2^2 \\ 0 & 0 & 0 \\ \hat{\theta}_{n8}\hat{x}_2^2 & 2\hat{\theta}_{n10}\hat{x}_2 + 3\hat{\theta}_{n11}\hat{x}_2^2 & 0 \end{pmatrix}. \quad (25)$$

To summarize, a 'typical' acoustic echo path in a conference telephony or telephone handsfree system can be modeled with a Hammerstein system composed of a nonlinear part as described above in series with a time varying FIR filter. Consequently, the algorithm derived in section 3 can be used for acoustic echo cancelation. [2] In this example

[2] In practice the problem is further complicated since the echo is typically contaminated by various kinds of disturbances that are picked up by the microphone. Also during periods with double-talk, i.e. simultaneous two-way speech, the near-end speaker acts as an additive disturbance.

the initial values of the 12'th order linear time varying FIR filter were chosen as

$$\underline{\theta}_l^T = (1.0\ 0.1578\ 0.3548\ 0.3820\ 0.2872\ 0.1388\ -0.0018\ -0.0954\ -0.1300\ -0.1147\ -0.0706\ -0.0201\ 0.0197). \quad (26)$$

The estimation problem becomes very difficult in case all the nonlinear parameters are unknown. In practice, the odd harmonics often dominate over the even harmonics. Consequently, it might be interesting to study a scenario where all the nonlinear parameters are assumed known, except for those corresponding to the nonlinear relationship between the magnetic force and the diaphragm, i.e. $[\ \theta_{n9}\ \theta_{n10}\ \theta_{n11}\ ]$. The variance of the white Gaussian measurement noise was $r_2 = 10^{-5}$ and the covariance matrix of the random walk parameters was fixed $R_w = 10^{-5} \times I$. The input to the system was zero mean white Gaussian noise with variance 1.0 The variable forgetting factor, used in the VRPEM, was calculated according to (16–19) with $\rho = 0.94$ and $\tau = 8$. This value of τ was found empirically to be a good choice for the considered scenario. As a comparison, the RPEM, with a fixed forgetting factor $\lambda = 0.99$ was also simulated.

As evident from Fig.1 the tracking performance is clearly improved by inclusion of a variable forgetting factor. The VRPEM exhibits a small tracking lag, but follows the linear parameter variations very good. In addition, it was noted that the VRPEM yields better performance, in terms of the expectation of the squared prediction error, $E[\varepsilon_l^2(t)]$, (estimated with data from 20 independent trials), as compared to the RPEM with a fixed forgetting factor. The squared prediction error corresponds to the level of the residual echo that remains after subtraction of the echo replica from the actual echo. In this experiment, the average attenuation in the system was 6 dB. The peaks of $E[\varepsilon_l^2(t)]$, were approximately 37 dB below the input level, when running the VRPEM, hence corresponding to an echo suppression in excess of 30 dB. This figure is quite good for this application, considering the time variations and the high level of nonlinear distortion. The echo suppression provided by the RPEM in this case was approximately 24 dB.

5. CONCLUSIONS

The problem of identifying a class of Hammerstein systems, composed of a nonlinear block given on a state space description followed by a time-varying FIR filter, was addressed. A Recursive Prediction Error Method with a variable forgetting factor was derived. It was demonstrated that the suggested algorithm can be successfully applied to the problem of acoustic echo cancelation.

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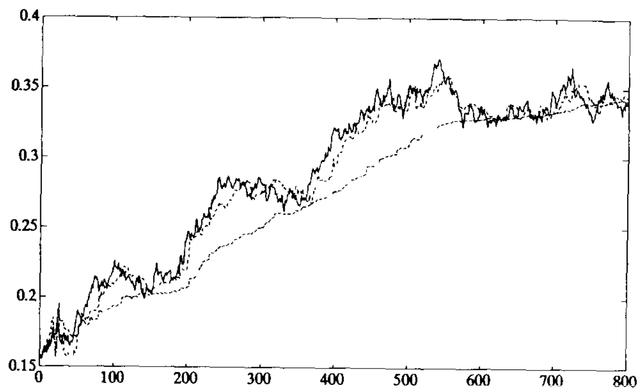

Fig 1 The true value of the second linear parameter, (*solid*), the VRPEM-estimate, (*dashdotted*) and the RPEM-estimate, (*dashed*) $\hat{\underline{\theta}}_l(0) = \underline{\theta}_l(0)$, $\hat{\theta}_{n9}(0) = 0\,9 \times \theta_{n9}$, $\hat{\theta}_{n10}(0) = 0\,9 \times \theta_{n10}$, $\hat{\theta}_{n11}(0) = 0\,9 \times \theta_{n11}$