A Velocity Control of Frictional Servosystems Using an Adaptive Fuzzy Control

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Abstract --- Based on the system relative degree, the frictional servosystem is transformed into external and internal parts. By using a feedback linearizing control, the external part becomes a linear dynamic system with uncertainties. A reference model with the desired amplitude and phase properties is given to obtain an error dynamics in the presence of uncertainties. The unmatched uncertainty is also examined. To improve the system performance, an on-line fuzzy-model is employed to model these uncertainties in a compact subset. An updating law with *e*-modification for the weight of fuzzy-model is designed to obtain an effective learning of the uncertainties. Then, an equivalent control using the known part of frictional servosystem and the learning fuzzy-model of uncertainties is established to achieve the desired result. The unmodeled dynamics caused by the error of approximated fuzzy-model and estimated weight are tackled by a switching control. In summary, the adaptive fuzzy control includes two parts: a feedback linearizing control with a reference model and an adaptive fuzzy variable structure control (AFVSC). The stability of the overall system is then verified by the Lyapunov theory so that the uniformly ultimately bounded tracking is accomplished. Simulations are also presented to verify the usefulness of the proposed control.

Index Terms --- Friction, Adaptive fuzzy variable structure control, Lyapunov stability.

I. INTRODUCTION

Friction is an important aspect of many control systems. Friction can lead to tracking errors, limit cycles, and undesired stick-slip motion [1,2]. The controller designs for the servosystems with friction can be adaptive control [3], fuzzy control [4] and neural-network-based adaptive control [5]. Because a friction is a natural phenomenon that is quite hard to model, and it is not yet completely understood [1,2]. Especially, the results of application with high-precision positioning and low-velocity tracking are not satisfactory.

Fuzzy (or adaptive fuzzy) control has widened its applicability to many engineering fields, it is increasing the need of theoretic analysis, e.g., stability, robustness and performance. It is generally applicable to the systems that are mathematically poorly modeled [6-10]. Although many papers discuss the adaptive fuzzy controls [6,7,10], they have made many assumptions: (i) The controller must adapt itself to every change of reference signal. (ii) The method is limited to the system where the Lie-derivative of system output is constant. (iii) To ensure the convergence of weight, fuzzy basis function must be persistently excited. (iv) The relative degree, r, must be equal to the order of system, n.

In this paper, the frictional servosystems include known part and unknown part. The known part is achieved by deriving from the physical law or by experimental modeling. Then, a coordinate transformation satisfying some conditions is employed to achieve a transformed system including an external part with the order of relative degree and an internal part with the remain degree of system [11]. The external part of transformed frictional servosystem via a feedback linearizing control becomes a linear dynamic system with uncertainties. Then a prescribed reference model with the desired amplitude and phase properties is designed to obtain an error dynamic model subject to uncertainties. Then the matched and unmatched uncertainties are discussed. For the purpose of improving the system performance, an on-line fuzzy-model is applied to model these uncertainties. Without the requirement of persistent excitation for the fuzzy basis function, the boundedness of weight is guaranteed by an updating law with e-modification [12,13]. Then, an equivalent control using the known part of frictional servosystem and the learning fuzzy-model of uncertainties is applied to achieve the desired control behavior. Because the fuzzy-model is not applied to the whole nonlinear functions, the resolution of fuzzy-model increases or good description of system uncertainties is accomplished. In addition, the unmodeled dynamics caused by the error of approximated fuzzy-model and estimated weight are tackled by a switching control. Under the circumstances, the switching gain is reduced as compared with that in convectional variable structure control.

In summary, the adaptive fuzzy control includes two parts: a feedback linearizing control with a reference model and an adaptive fuzzy variable structure control (AFVSC).The system performance is better than those of other (adaptive) fuzzy control (e.g., [6], [7], [10]) or convectional variable structure control (VSC) (e.g., [14]). The proposed control can deal with extra uncertainties to attain an excellent tracking result without the occurrence of chattering control. The simulations are given to confirm the proposed control.

II. PROBLEM FORMULATION

The motion of a mass that moves on the surface with static friction is described as follows:

$$m\ddot{p}(t) = F(t) - F_f(u, p, \dot{p}),\ u(t) = g(p, \dot{p})F(t) \qquad (1)$$

where $p(t), \dot{p}(t)$ denote the position, velocity of servosystem, $F_f(u, p, \dot{p})$ denotes the friction force of servosystem, $m \neq 0$ stands for the mass of moving part of servosystem; $g(p, \dot{p}) > 0 \forall p(t), \dot{p}(t)$ denotes the gain between control input $u(t)$ and input force of servosystem $F(t)$. Let the friction force be: $F_f(u, p, \dot{p}) = F_{slip}(\dot{p})\sigma(\dot{p}) + F_{stick}(u, p, \dot{p})[1 - \sigma(\dot{p})]$ where $\sigma(\dot{p}) = 1 \text{ as } |\dot{p}(t)| > \varepsilon_0$, and $\sigma(\dot{p}) = 0$, otherwise. The sticking force is modeled as follows:

$$F_{stick}(u, p, \dot{p}) = \begin{cases} F_s^+, & F(t) > F_s^+ > 0 \\ F(t), & F_s^- \leq F(t) \leq F_s^+ \\ F_s^-, & F(t) < F_s^- < 0 \end{cases} \qquad (2)$$

The slipping force is modeled as follows:

$$F_{slip}(\dot{p}) = \begin{cases} F_k^+(\dot{p}) = F_s^+ - \Delta F^+ \left\{1 - \exp\left[-\left| \dot{p}(t)/\dot{p}^+ \right|\right]\right\} \\ \qquad + C^+ \dot{p}(t), \ \dot{p}(t) > 0 \\ F_k^-(\dot{p}) = F_s^- - \Delta F^- \left\{1 - \exp\left[-\left| \dot{p}(t)/\dot{p}^- \right|\right]\right\} \\ \qquad + C^- \dot{p}(t), \ \dot{p}(t) < 0 \end{cases} \qquad (3)$$

Because the following facts: (i) $F_{slip}(\dot{p})$ is continuous on $(-\infty, -\varepsilon_0) \cup (\varepsilon_0, \infty)$, (ii) $F_s^+ \geq F_{slip}(0_+) \cong \lim_{\dot{p} \to 0_+} F_{slip}(\dot{p})$ and $F_s^- \leq F_{slip}(0_-) \cong \lim_{\dot{p} \to 0_-} F_{slip}(\dot{p})$, the Caratheodory solution of systems (1)-(3) exists [15]. Furthermore, because ε_0 is a small value and $F_s^- \leq F_{stick}(u, p, \dot{p}) \leq F_s^+$, based on the mean-value theorem the following result exists: $F_f(u, p, \dot{p}) = F_e(p, \dot{p})$, where $F_e(p, \dot{p})$ is a relay-like piecewise continuous function. Then the dynamics of frictional servosystem for velocity control becomes

$$\dot{x}(t) = A(x) + B(x)u(t), y(t) = C^T x(t) \qquad (4)$$

where $x(t) = [x_1(t) \ \ x_2(t)]^T = [p(t) \ \ \dot{p}(t)]^T, A(x) = [x_2(t) \ \ F_e(x)/m]^T$, $B(x) = [0 \ \ 1/mg(x)]^T, C^T = [0 \ \ 1]$. Furthermore, $A(x) = \bar{A}(x) + \Delta A(x)$, $B(x) = \bar{B}(x) + \Delta B(x)$, where $\bar{A}(x)$ and $\bar{B}(x)$ denote the nominal part of system matrices, $\Delta A(x)$ and $\Delta B(x)$ represent the unknown part of system matrices, and are bounded and piecewise continuous.

The derivative of output with respect to time is described as follows:

$$\dot{y}(t) = A_2(x) + B_2(x)u(t) \qquad (5)$$

where $A_2(x), B_2(x)$ denote the second component of $A(x), B(x)$, respectively. Because $B_2(x) \neq 0, \forall x(t)$, the frictional servosystems (1) have the relative degree one. It is assumed that the state $x(t)$ is available. The definition of diffeomorphism can allude to [11].

First, the following matched uncertainties are considered.

$$\Delta A(x) = \bar{B}(x)\Delta a(x) \quad \text{and} \quad \Delta B(x) = \bar{B}(x)\Delta b(x) . \qquad (6)$$

Lemma 1[11]: Consider the nonlinear system (1) with the following global diffeomorphism $z(t) = T(x) = [z_1(x) \ \ z_2(x)]^T$. The matching condition (6) and the following conditions: $\partial z_1(x)/\partial x \bar{A}(x) = -\beta^{-1}(x)\alpha(x)$, $\partial z_1(x)/\partial x \bar{B}(x) = \beta^{-1}(x) \neq 0$, and $\partial z_2(x)/\partial x \bar{B}(x) = 0$ with $z_2(0) = 0$ are satisfied. Then the following dynamic system is achieved

$$\dot{z}_1(x) = \beta_0^{-1}(z_1, z_2)[u(t) - \alpha_0(z_1, z_2) + \Delta a_0(z_1, z_2) + \Delta b_0(z_1, z_2)u(t)] \qquad (7)$$

$$\dot{z}_2(x) = A_0(z_1, z_2) \qquad (8)$$

where $z_1(x), z_2(x) \in \Re$, the functions $\alpha_0(z_1, z_2)$, $\beta_0(z_1, z_2)$, $\Delta a_0(z_1, z_2), \Delta b_0(z_1, z_2)$ are the functions $\beta(x) = 1/\{\partial z_1(x)/\partial x \bar{B}(x)\}$, $\alpha(x) = -\beta(x)\{\partial z_1(x)/\partial x \bar{A}(x)\}$, $\Delta a(x), \Delta b(x)$ evaluated at $x(t) = T^{-1}(z)$, respectively. $A_0(z_1, z_2)$ is the representation in the transformed coordinate $x(t) = T^{-1}(z)$ of $\partial z_2(x)/\partial x \bar{A}(x)$.

Equation (7)-(8) is said to be in the normal form. This form decomposes the system into an external part $z_1(x)$ and an internal part $z_2(x)$. The external part is linearized by the (11). The problem is to develop an adaptive fuzzy control for frictional servosystem (1) subjected to uncertainties (i.e., $\Delta A(x)$ and $\Delta B(x)$) which are not necessarily to satisfy the matching condition (6) (Fig. 1).

III. FEEDBACK LINEARIZING CONTROL

First, the following reference model is considered.

$$\dot{\bar{z}}_1(t) = g_1 \bar{z}_1(t) + g_2 r_d(t + \phi) \qquad (9)$$

where $r_d(t)$ is a bounded trajectory, constant $g_1 < 0$ is selected to obtain the desired response, g_2 is chosen to accomplish the desired steady-state amplitude between input and output, and ϕ denotes the phase lag of reference model. For example, the tracking of sinusoidal signal $A_m \sin(wt)$, the values of $g_2 = \sqrt{g_1^2 + w^2}$ and $\phi = -\tan^{-1}(w/g_1)$ are chosen. The state tracking error of system can be written as follows:

$$\dot{\tilde{z}}_1(t) = \beta_0^{-1}(z_1, z_2)[u(t) - \alpha_0(z_1, z_2) + \Delta a_0(z_1, z_2) + \Delta b_0(z_1, z_2)u(t)] - g_1 \bar{z}_1(t) - g_2 r_d(t + \phi) \qquad (10)$$

where $\tilde{z}_1(t) = z_1(x) - \bar{z}_1(t)$. The following linearizing feedback control is designed for the system (10)

$$u(t) = \alpha_0(z_1, z_2) + \beta_0(z_1, z_2)v(t) \qquad (11)$$

where $v(t)$ is an adaptive fuzzy variable structure control discussed in the next section and contains the following two parts:

$$v(t) = v_{eq}(t) + v_{sw}(t) . \qquad (12)$$

Then the external part of system by using the linearizing feedback control (11) becomes the following linear error dynamic system with the uncertainties:

$$\dot{\tilde{z}}_1(t) = [1 + \Delta b_0(z_1, z_2)]v_{sw}(t) + v_{eq}(t) - g_1 \bar{z}_1(t) - g_2 r_d(t + \phi) + f(\xi) \qquad (13)$$

where

$$\xi(t) = [v_{eq}(t) \quad z_1(x) \quad z_2(x)]^T \qquad (14)$$

$$f(\,°) = \Delta b_0(z_1,z_2)v_{eq}(t) + \;\;_0^{-1}(z_1,z_2)$$
$$\cdot\left[\Delta a_0(z_1,z_2) + \Delta b_0(z_1,z_2)\text{~}_0(z_1,z_2)\right] \quad (15)$$

The uncertainties in (13) include two parts: (i) $\Delta b_0(z_1,z_2)v_{sw}(t)$ affecting the stability of closed-loop system, (ii) $f(\,°)$ affecting the performance of closed-loop system. Therefore, the uncertainties $f(\,°)$ can't contain the signal $v_{sw}(t)$ used to deal with uncertainties. Furthermore, the following assumption about the uncertainty of control gain is made.

A1: $\left|\Delta b_0(z_1,z_2)\right| \le ¥_0 < 1,\ z(t)\in\Omega,$ where $\Omega = \left\{ z(t)\in\Re^2 \,\middle|\, \|z(t)\| < c_b \right\}$ is a compact set.

In short, (8) and (13) represent the stability of closed-loop system. If the control $v(t)$ asymptotically stabilizes the dynamics $\tilde{z}_1(t)$ in (13) and the dynamics $z_2(x)$ is input-to-state stable in (8), then the asymptotic tracking of the closed-loop system is guaranteed. Setting $z_1(x)=0$ in (8) results in

$$\dot{z}_2(x) = A_0(0,z_2) \quad (16)$$

which is called the zero dynamics. For input-to-state stability of (8), the origin of (16) must be exponentially stable and $A_0(z_1,z_2)$ is Lipschitz in (z_1,z_2), i.e., $\|A_0(z_{12},z_{22}) - A_0(z_{11},z_{21})\| \le L\left\|\left[(z_{12}-z_{11})^T \quad (z_{22}-z_{21})^T\right]\right\|,$ where L is a constant.

Consider the uncertainties with the unmatched form:

$$\Delta A(x) = \overline{B}(x)\Delta a(x) + \Delta A_u(x) \quad (17)$$

where $\Delta A_u(x)$ denotes the uncertainties those do not satisfy the matching condition. Then (8) and (13) become

$$\dot{z}_2(x) = A_0(z_1,z_2) + \Delta A_0(z_1,z_2) \quad (18)$$

$$\dot{\tilde{z}}_1(t) = \left[1+\Delta b_0(z_1,z_2)\right]v_{sw}(t) + v_{eq}(t)$$
$$- g_1\tilde{z}_1(t) - g_2 r_d(t+¤) + f_u(\,°) \quad (19)$$

where

$$\Delta A_0(z_1,z_2) = \partial z_2(x)/\partial x\,\Delta A_{u0}(z_1,z_2) \quad (20)$$

$$f_u(\,°) = f(\,°) + \partial z_1/\partial x\,\Delta A_{u0}(z_1,z_2) \quad (21)$$

and the function $\Delta A_{u0}(z_1,z_2)$ is the function $\Delta A_u(x)$ evaluated at $x(t) = T^{-1}(z)$. For input-to-state stability of (18), the origin of the following system (22) must be exponentially stable and $A_0(z_1,z_2)+\Delta A_0(z_1,z_2)$ is Lipschitz in (z_1,z_2).

$$\dot{z}_2(x) = A_0(0,z_2) + \Delta A_0(0,z_2)\,. \quad (22)$$

As compared with (13) and (19), an extra term caused by the unmatched uncertainties (i.e., $\partial z_1(x)/\partial x\,\Delta A_{u0}(z_1,z_2)$) is approximated by the fuzzy-model. Similarly, an extra term caused by the unmatched uncertainties (i.e., $\Delta A_0(0,z_2)$) is added into the (8). As compared with the matched case, the input-to-state stability margin of internal system decreases. If the uncertainty $f(\,°)$ or $f_u(\,°)$ is large, a convectional VSC for the system (13) or (19) will be poor [13].

IV. ADAPTIVE FUZZY VARIABLE STRUCTURE CONTROL

The fuzzy logic system performs a mapping from $U\in\Re^n$ to $\Re$. There are l fuzzy control rules and k is the kth rule from human experts in the following form:

IF $°_1(t)$ is $F_1^k \cdots °_n(t)$ is F_n^k, THEN $f(\,°)$ is G^k (23)

where $°(t) = [\,°_1(t) \quad °_2(t) \quad \cdots \quad °_n(t)]^T \in U\subset\Re^n$ and $f(\,°)\in V\subset\Re$ are the input and output of the fuzzy logic system, respectively, $F_i^k\ (1\le i\le n, 1\le k\le l)$ and G^k are labels of sets in U_i and V, respectively. The fuzzy inference engine performs a mapping from fuzzy sets in $U\subset\Re^n$ to fuzzy sets in $V\subset\Re$, based upon the fuzzy IF-THEN rules in the fuzzy rule base and the compositional rule of inference. Let $A_\circ$ be an arbitrary fuzzy set in U. The fuzzifier maps a crisp point $°(t)$ into a fuzzy set $A_\circ$ in U. The defuzzifier maps a fuzzy set in V to a crisp point in V.

Let $\mu_{F_i^k}(\,°_i)$ and $\mu_{G^k}(\overline{w}_k)$ are membership functions. The fuzzy logic system with center-average defuzzifier, product inference and singleton fuzzifier are in the following form [6-10]:

$$f_z(\,°) = \sum_{k=1}^{l}\overline{w}_k\left(\prod_{i=1}^{n}\mu_{F_i^k}(\,°_i)\right)\bigg/\sum_{k=1}^{l}\left(\prod_{i=1}^{n}\mu_{F_i^k}(\,°_i)\right) \quad (24)$$

where $\overline{w}_i\,(1\le i\le l)$ denotes center of the *ith* fuzzy set and is the point at which μ_{G^k} achieves its maximum value and $\mu_{G^k}(\overline{w}_k)=1$. Equation (24) can be rewritten as

$$f_z(\,°) = \overline{W}^T\Phi(\,°) \quad (25)$$

where $\overline{W} = [\overline{w}_1 \quad \overline{w}_2 \quad \cdots \quad \overline{w}_l]^T$ is a parameter vector, and $\Phi(\,°) = [¤_1(\,°) \quad ¤_2(\,°) \quad \cdots \quad ¤_l(\,°)]^T$ is a fuzzy basis function defined as follows:

$$¤_k(\,°) = \prod_{i=1}^{n}\mu_{F_i^k}(\,°_i)\bigg/\sum_{k=1}^{l}\left(\prod_{i=1}^{n}\mu_{F_i^k}(\,°_i)\right). \quad (26)$$

Theorem 1(Universal Approximation Theorem [7]): Suppose that the input universe of discourse U is a compact set in $\Re^n$. Then, for any given real continuous function $f(\,°)$ on U and arbitrary $\varepsilon>0$, there exists a fuzzy system $f_z(\,°)$ in the form of (23) such that $\sup_{°\in U}\left|f_z(\,°) - f(\,°)\right| < \varepsilon$.

Based on the approximation theory of *Theorem 1*,

$$f(\,°) = \overline{W}^T\Phi(\,°) + \varepsilon(\,°),\ |\varepsilon(\,°)| < \varepsilon_1,$$
$$\overline{W}\in\Re^l, \|\overline{W}\|_F \le w_{\max} \quad (27)$$

where $\|\cdot\|_F$ denotes the Frobenius' norm (i.e., $\|\overline{W}\|_F^2 = tr\{\overline{W}^T\overline{W}\} = tr\{\overline{W}\,\overline{W}^T\}$) and $l, w_{\max}$ are assumed to be known. The fact that the dimension of $\overline{W}$, the upper bound of $\varepsilon(\,°)$ and the fuzzy basis function $\Phi(\,°)$ is known, implies that the function $\overline{W}^T\Phi(\,°)+\varepsilon(\,°)$ can represent a class of uncertainties $f(\,°)$. In sequence, the following switching surface is defined [13,14].

$$s(t) = \tilde{z}_1(t) + d_1\int_0^t \tilde{z}_1(²)d², d_1 \ge 0. \qquad (28)$$

The following updating law for the weight is considered.

$$\dot{\hat{W}}(t) = s(t)\Lambda\Phi(\,°) - ¦\Lambda|s(t)|\hat{W}(t) \qquad (29)$$

where $\Lambda = diag\{ª_{ii}\} \in \Re^l$, $ª_{ii} > 0$ denotes constant learning rate and $¦ > 0$. Because the fuzzy basis function $\Phi(\,°)$ in (25) is small as compared with the radical basis function in neural-network control (e.g., [13]), the learning rate of the paper is chosen large enough to accomplish an effective learning of uncertainties. The following theorem discusses the adaptive fuzzy variable structure control for the system (8) and (13) or the system (18) and (19).

Theorem 2: Consider the system (8) and (13) satisfying matched uncertainties or the system (18) and (19) satisfying unmatched uncertainties. The adaptive fuzzy variable structure control (12) is designed as follows:

$$v_{eq}(t) = -d_1\tilde{z}_1(t) + g_1\bar{z}_1(t) + g_2 r_d(t+¤) - \hat{W}^T(t)\Phi(\,°) \qquad (30)$$

$$v_{sw}(t) = -\left[¥_1 s(t) + \frac{¥_2 s(t)}{|s(t)| + ¶}\right]\Big/(1 - ¥_0),$$

$$¥_1, ¥_2 > 0,\ ¶ \ge 0. \qquad (31)$$

The overall system satisfies the following conditions: (i) the assumption A1, (ii) the satisfaction of input-to-state stability for (16) or (22), and (iii) $°(t) \in U \subset \Omega$. Then $s(t), \hat{W}(t), v(t), u(t)$ and $x(t)$ are uniformly ultimately bounded [11], and the tracking performance satisfying $|s(t)| < h$ as $t \to \infty$, where

$$h_1 = \left\{¶ + \frac{¥_2}{¥_1} - \frac{¦w_{\max}^2}{4¥_1} - \frac{£_1}{¥_1}\right\}\Big/2, \quad h_2 = \frac{¦w_{\max}^2 ¶}{4¥_1} + \frac{£_1 ¶}{¥_1},$$

$$h = \sqrt{h_1^2 + h_2} - h_1. \qquad (32)$$

Proof: See Appendix.

VI. SIMULATIONS

Consider the following nominal values of system:

$$F_s^+(F_s^-) = 4.5(-4.2)Nt, \qquad ¢F^+(¢F^-) = 2.1(-1.8)Nt,$$

$$\dot{p}^+(\dot{p}^-) = 0.2(0.16)\,m/s, \quad C^+(C^-) = 0.5\,Nt\$m, \quad £_0 = 0.01\,m/s,$$

$$m = 2kg,\ g(p,\dot{p}) = 1\,vol/Nt. \qquad (33)$$

The reference input is assigned as $r(t) = 0.5\sin(2®ft)\,m/s$, where $f = 0.05Hz$. From the previous study [5], the responses of PID control are poor. Because $B(x)$ is constant, only the matched uncertainties are considered. For example, the real values of (33), which are 50% variations of the nominal values, are described.

$$F_s^+(F_s^-) = 6.75(-6.3)Nt, \qquad ¢F^+(¢F^-) = 3.15(-2.7)Nt,$$

$$\dot{p}^+(\dot{p}^-) = 0.1(0.08)\,m/s, \quad C^+(C^-) = 0.75\,Nts/m, \quad m = 3kg,$$

$$g(p,\dot{p}) = 1.5\,vol/Nt. \qquad (34)$$

Firstly, the response of the proposed control without the compensation of uncertainties is shown in Fig.2 that is poor as the trajectory nears the zero velocity. This is motivation of the paper to provide a more effective control for the servosystem with friction.

Suppose the following coordinate transformation:

$$z_1(x) = °_1 x_1(t) + °_2 x_2(t),\ z_2(x) = °_3 x_1(t). \qquad (35)$$

The values of $°_1 = 2$, $°_2 = °_3 = 1$ are selected because as $z_1(x) = 0$, $\dot{z}_2(x) = -z_2(x)\,°_1/°_2 = -2z_2(x)$ is exponentially stable. Furthermore, $\partial T(x)/\partial x$ is nonsingular for all $x(t)$. In short, the input-to-state stability is satisfied [11].

The desired velocity is set as $x_{2d}(t) = v_m \sin(2®ft + ¤)\,rad/s$, where $¤ = \tan^{-1}[2®f/g_1]$, then $x_{1d}(t) = \int x_{2d}(²)d² = -v_m\cos(2®ft+¤)/(2®f)\,rad$. The parameters of reference model are chosen as $g_1 = -0.5$ and $g_2 = \sqrt{(2®f)^2 + g_1^2}$. Hence, the reference input for (9) becomes $r_d(t) = °_1 x_{1d}(t) + °_2 x_{2d}(t)$. Three fuzzy sets $F^k(k = 1,2,3)$ have the following Gaussian membership functions:

$$«_i(z_1) = e^{-((z_1 - c_{1i})/±_1)},\ c_{1i} = [-6 \quad -3 \quad 0 \quad 3 \quad 6],$$

$$i = 1,2,...,5 \text{ and } ±_1 = 100$$

$$«_i(z_2) = e^{-((z_2 - c_{2i})/±_2)},\ c_{2i} = [-3 \quad -1.5 \quad 0 \quad 1.5 \quad 3],$$

$$i = 1,2,...,5 \text{ and } ±_2 = 100$$

$$«_i(v_{eq}) = e^{-((v_{eq} - c_{3i})/±_3)},\ c_{3i} = [-15 \quad -7.5 \quad 0 \quad 7.5 \quad 15],$$

$$i = 1,2,...,5 \text{ and } ±_3 = 200. \qquad (36)$$

The total number of fuzzy rule is $l = 125$. The control parameters are set as follows:

$$d_1 = 0.1, ¥_0 = 0.02, ¥_1 = 15, ¥_2 = 2, ¶ = 0.02, ¦ = 10^{-3},$$

$$£_1 = 0.01,\ ª_{ii} = 5\times10^{-5} \text{ for } i = 1,2,...,125.$$

The responses of the proposed control are shown in Fig. 3. The tracking performance is excellent except a little tracking error occurs in the vicinity of zero velocity. However, the result is much improved as compared with the result of Fig. 2. The control signal in Fig. 3 (b) is also smooth enough. The responses of real and learning uncertainties are quiet matched (refer to Fig. 3(c)). The proposed fuzzy logic system (24) and the updating law for weight (29) demonstrate an effective tool for the learning of uncertainties. Furthermore, the response of switching surface (Fig. 3(d)) is much better than that in Fig. 2(b). This is the main reason to obtain a better tracking performance.

VII. CONCLUSIONS

The external part of transformed frictional servosystem becomes a linear error dynamic system with uncertainties by using a feedback linearizing control and a reference model. An on-line fuzzy-model with an updating law having *e*-modification is then employed to achieve an effective learning of these uncertainties. The adaptive fuzzy variable structure control contains equivalent control and switching control. The equivalent control uses the known part of frictional servosystem and the learning fuzzy-model of uncertainties. The switching control is applied to deal with the unmodeled dynamics caused by the error of approximated fuzzy-model and estimated weight. It is not necessary to assume that the matching condition of uncertainties must be satisfied. Then the tracking performance including tracking accuracy and smoothness of control input is much better than that of PID control or convectional variable structure control or (adaptive) fuzzy control. Simulations confirm the usefulness of the proposed control. The experimental work is in progress.

REFERENCES

[1] K. C. Cheok, H. Hu, and N. K. Loh, "Modeling and identification of a class of servomechanism systems with stick-slip friction," *ASME Trans., J. of Dyna. Syst., .Meas. Contr.*, vol. 110, pp. 322-328, 1988.

[2] De. C.C. Wit, H. Dlsson, K.J. Astrom, and P. Lischinsky, "A new model for control of system with friction," *IEEE Trans. Automat. Contr.,* vol. 40, no.3, pp.419-425, 1995.

[3] S. W. Lee, and J. H. Kim, "Robust adaptive stick-slip friction compensation," *IEEE Trans. Ind. Electron.*, vol. 42, no. 5, pp. 474-479, 1995.

[4] J. T. Teeter, M. Y. Chow, and J. J. Brickley, "A novel fuzzy friction compensation approach to improve the performance of a DC motor control system," *IEEE Trans. Ind. Electron.*, vol. 43, no. 1, pp. 113-120, 1996.

[5] C. L. Hwang, "Fourier-series neural-network-based adaptive variable structure control for servosystems with friction," *IEE Proc. D, Contr. Theory Applicat.*, vol. 144, no.6, pp. 559-565, 1997.

[6] J. T. Spooner, and K. M. Passino, "Stable adaptive control using fuzzy systems and neural networks," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 3, pp. 339-359, 1996.

[7] L. X. Wang, *A Course in Fuzzy Systems and Control.* Prentice-Hall Int. Inc., 1997.

[8] S. Y. Yi, and M. J. Chung, "Robustness of fuzzy logic control for an uncertain dynamic system," *IEEE Trans. Fuzzy Syst.*, vol. 6, no. 2, pp. 216-224, 1998.

[9] H. J. Kang, C. Kwon, H. Lee, and M. Park, "Robust stability analysis and design method for the fuzzy feedback linearization regulator," *IEEE Trans. Fuzzy Syst.*, vol. 6, no. 4, pp. 464-472, 1998.

[10] K. Fischle, and D. Schroder, "An improved stable adaptive fuzzy control method," *IEEE Trans. Fuzzy Syst.*, vol. 7, no. 1, pp. 27-40, 1999.

[11] A.Isidori, *Nonlinear Control Systems.* Springer-Verlag, London Limited, 3rd Ed., 1995.

[12] K. S. Narendra, and A. M. Annaswamy, "A new adaptation law for robust adaptation without persistent excitation," *IEEE Trans. Automat. Contr.*, vol. 32, no. 2, pp. 134 -145, 1987.

[13] C. L. Hwang, "Neural-network-based variable structure control of electrohydraulic servosystems subject to huge uncertainties without persistent excitation," *IEEE/ASME Trans. Mechatron.*, vol. 4, no. 1, pp. 50-59, 1999.

[14] Y. Hung, W. Gao, and J. C. Hung, "Variable structure control: A survey," *IEEE Trans. Ind. Electron.*, vol. 40, no. 1, pp.2-22, 1993.

[15] S. J. Kim, and I. J. Ha, "On the existence of Caratheodory solutions in mechanical systems with friction," *IEEE Trans. Automat. Contr.*, vol. 44, no. 11, pp. 2086-2089, 1999.

APPENDIX

Appendix (the proof of Theorem 2):

First, the matched uncertainties are considered. Define the following Lyapunov function candidate:

$$V = s^2/2 + \tilde{W}^T \Lambda^{-1} \tilde{W}/2 = \pm^T P \pm > 0, \text{as } s \neq 0, \tilde{W} \neq 0 \quad \text{(A1)}$$

where $\tilde{W} = \overline{W} - \hat{W}, \pm = [s \quad \tilde{W}^T]^T$, $P = daig\{1 \quad a_{11}^{-1} \quad \dots \quad a_{ll}^{-1}\}/2$.

The derivative $\dot{V}$ is given

$$\dot{V} = s\dot{s} + \tilde{W}^T \Lambda^{-1} \dot{\tilde{W}}. \quad \text{(A2)}$$

Similarly, the derivative of s using (13), (15), (28) and (30) is given as follows:

$$\begin{aligned}\dot{s} &= (1+\Delta b_0) v_{sw} + v_{eq} - g_1 \bar{z}_1 - g_2 r_d + f + d_1 \tilde{z}_1 \\ &= (1+\Delta b_0) v_{sw} - \hat{W}^T \Theta + f = (1+\Delta b_0) v_{sw} + \tilde{W}^T \Theta + \pounds\end{aligned} \quad \text{(A3)}$$

Substituting (A3), (29) and (31) into (A2), and using the inequality $-(1+\Delta b_0)/(1-\text{¥}_0) \le -1$ yields

$$\begin{aligned}\dot{V} &= s\{(1+\Delta b_0) v_{sw} + \tilde{W}^T \Theta + \pounds\} - s\tilde{W}^T \Theta + \text{¦}|s| \tilde{W}^T \hat{W} \\ &\le |s| \left\{ \pounds_1 - \left[\text{¥}_1 |s| + \frac{\text{¥}_2 |s|}{|s| + \P} \right] + \text{¦}\, \tilde{W}^T (\overline{W} - \tilde{W}) \right\} = \frac{-\text{¥}_1 |s|}{|s| + \P} \\ &\quad \cdot \left\{ -\frac{|s| + \P}{\text{¥}_1} \pounds_1 + |s|(|s| + \P) + \frac{\text{¥}_2}{\text{¥}_1} |s| + \frac{\text{¦}(|s| + \P)}{\text{¥}_1} \tilde{W}^T (\tilde{W} - \overline{W}) \right\} \\ &\le \frac{-\text{¥}_1 |s|}{|s| + \P} \left\{ H(|s|) + K(\|\tilde{W}\|) \right\}\end{aligned} \quad \text{(A4)}$$

where

$$H(|s|) = |s|^2 + 2h_1 |s| - h_2 \quad \text{(A5)}$$

$$K(\|\tilde{W}\|) = \text{¦}(|s| + \P)\left[\|\tilde{W}\| - w_{\max}/2\right]^2 / \text{¥}_1 . \quad \text{(A6)}$$

Because $h_2 > 0, h > 0$ is achieved. Hence, if $|s| > h$, then $\dot{V} \le -\text{¸}_1(|s|, \|\tilde{W}\|)$, where $\text{¸}_1(|s|, \|\tilde{W}\|) > 0$. Similarly, if $\|\tilde{W}\| > w_{\max}/2 + \sqrt{\frac{w_{\max}^2}{4} + \frac{\pounds_0}{\text{¦}}} = w_{in}$, then $\dot{V} \le -\text{¸}_2(|s|, \|\tilde{W}\|)$, where $\text{¸}_2(|s|, \|\tilde{W}\|) > 0$. Hence, if $|s| > h$ and $\|\tilde{W}\| > w_{in}$, then

$$\dot{V} \le -\min\left\{ \text{¸}_1(|s|, \|\tilde{W}\|), \text{¸}_2(|s|, \|\tilde{W}\|) \right\}. \quad \text{(A7)}$$

Then outside of the domain $\overline{\Omega}$ making (A7) is described as follows:

$$\overline{\Omega} = \left\{ \pm \in \Re^{l+1} \,\middle|\, 0 \le \|\tilde{W}\| \le w_{in}, 0 \le |s| \le h \right\} \quad \text{(A8)}$$

Finally, from (30)-(31), v is UUB. Because the dynamics (8) is input-to-state stable, z_1, z_2 or x are UUB. From Lemma 1, are also UUB. Then u is UUB.

Similarly, the system with unmatched uncertainties can be achieved. For simplicity, those are omitted.

Q.E.D.

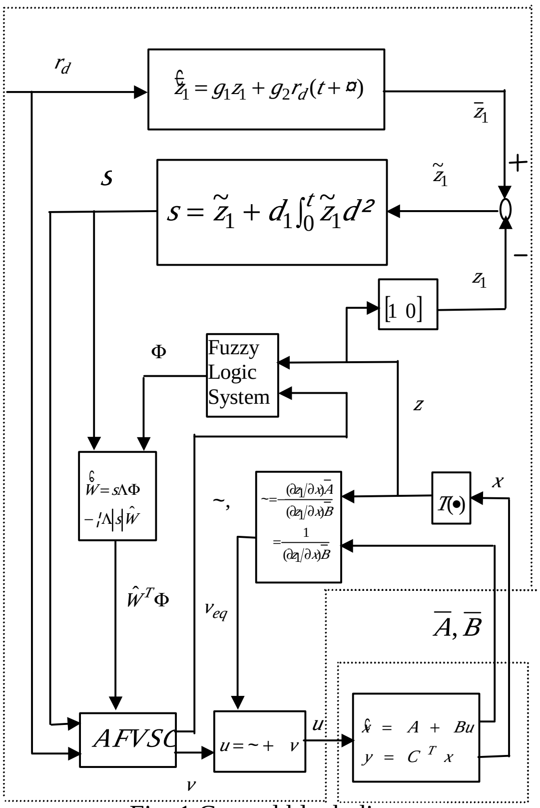
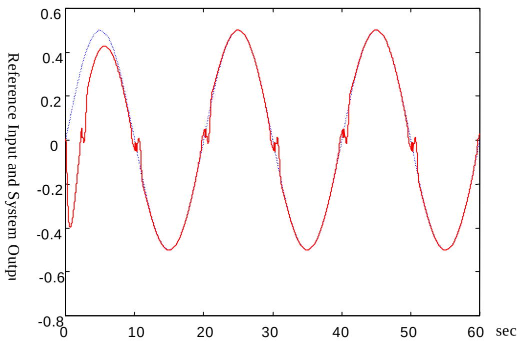


Fig. 1 Control block diagram.



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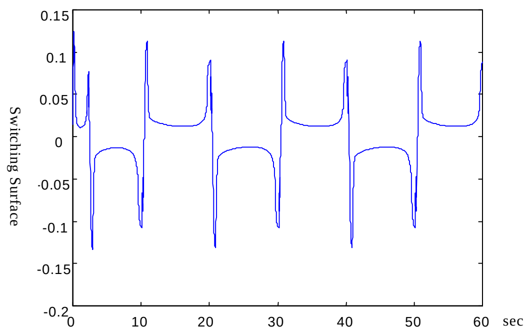
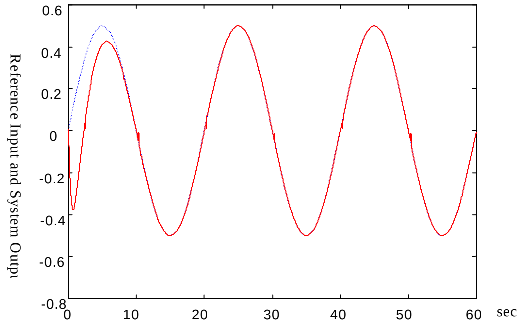


(b)

Fig. 2 The response of the proposed control without the compensation of uncertainties.

(a) Reference input $r(t)$ and system output $y(t)$.

(b) Switching surface $s(t)$.



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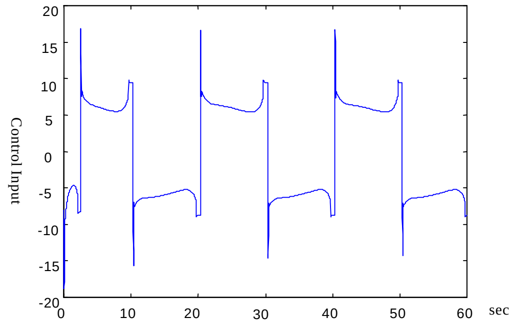

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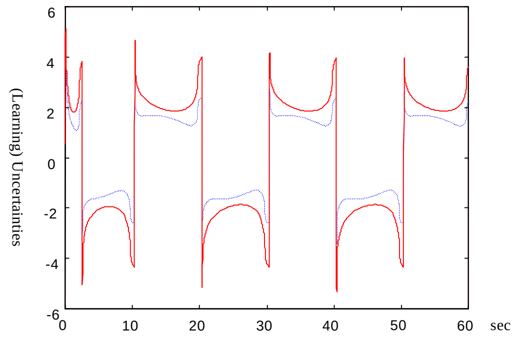
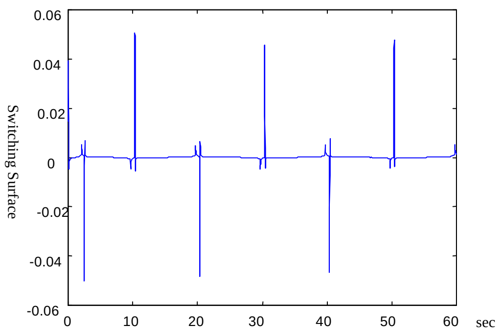


(c)



(d)

Fig. 3 The responses of the proposed control.

(a) Reference input $r(t)$ and system output $y(t)$.

(b) Control input $u(t)$.

(c) $f(\circ)$ and Uncertainties learning uncertainties $\hat{W}^T(t)\Phi(\circ)$.

(d) Switching surface $s(t)$.