Robust Pole Placement with Widely Adjustable Parameter

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Abstract

This paper develops robust pole placement that satisfies mixed sensitivity specification and places pole of the closed-loop system in pre-assigned half plane simultaneously. The feature of the proposed method is that the pre-assigned half plane is broad in comparison with that of affine transformation of standard H^∞ control and multipurpose H^∞ synthesis in LMI. Theoretical background and illustrative numerical example are presented

Key Words : Robust Pole Placement, H^∞ control, Disturbance Rejection

1. Introduction

In design of the control systems, low sensitivity and low complementary one are required for good control performances under the model error and the perturbations of the plants. Since there is trade-off between them, the weighting functions are introduced to be low sensitivity for low frequency and low complementary one for high frequency. The mixed sensitivity problem is to find a controller that satisfies the specification, and it is solved via the generalized plant by H^∞ control [1][2] and LMI [3][4].

However, If the plant and/or the weighting function for the sensitivity has any poles on the imaginary axis, then the generalized plant has to be modified such that the transfer function from the external signal to the measured output consists of all poles on the imaginary axis. The solution without such a modification for the weighting functions with poles on the imaginary and right half plane is proposed as extended H^∞ control by Mita et.al.[8]. If the plant has poles near the imaginary axis, then the responses to the input disturbances is sluggish. Kimura [5]and Saeki[6] proposed the robust stability degree assignment which was based on affine transfomation [7]. In its application to mixed sensitivity problem and multipurpose H^∞ synthesis in LMI, the adjustable region, however, is too narrow to improve the responses.

The purpose of this paper is to break through these problems. It is clarified that the problems cause to the structure of the H^∞ control for mixed sensitivity problems. The restrict structural defect is removed by combination of modification of the second Riccati equation and affine transformation. The proposed method can deal with plants with poles on the imaginary axis and the weighting functions without any modification of the generalized plant. The poles can be shifted widely within the mixed sensitivity specification.

2. **H**¥ Control for Mixed Sensitivity Problem

Consider the controlled system

$$G_p(s) = [I + \Delta(s)]G(s) \tag{1}$$

where $G(s) = C(sI - A)^{-1}B \in R^{m\times m}(s)$ is the nominal plant, $A \in R^{nn}$ 、 $B \in R^{n}$ 、 $C \in R^{m\times n}$, (A, B) is controllable and (C, A) is observable. It is assumed that the number of unstable poles of $G_p(s)$ is equal to that of $G(s)$, $G(s)$ has no zeroes on the imaginary axis. The $\Delta(s)$ is the model error satisfying

$$\bar{\sigma}[|\Delta(j\omega)|] < |W_T(j\omega)|, \quad \forall\omega \ge 0 \tag{2}$$

$\bar{\sigma}$ denotes the maximum singular value and $W_T(s) \in R^{m\times m}(s)$.

The feedback control system is shown in Fig.1, where $G_c(s)$ is the controller.

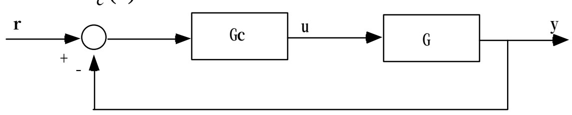

Fig.1 Feedback System

The sensitivity $S(s)$ and the complementary one $T(s)$ at the output are given by

$$S(s) = \{I + G(s)G_c(s)\}^{-1} \tag{3}$$

$$T(s) = \{I + G(s)G_c(s)\}^{-1}G(s)G_c(s) \tag{4}$$

For good control performance, the weighting function $W_s(s) \in R(s)$ with poles on the imaginary axis is introduced. The 1 disk mixed sensitivity problem is to design the control system satisfying

$$\left\| \begin{matrix} W_s(s)S(s) \\ W_T(s)T(s) \end{matrix} \right\|_\infty < 1 \tag{5}$$

This is equivalent to that the H^{∞} norm of the transfer matrix from the disturbance d to the output $\begin{bmatrix} z_s^T & z_T^T \end{bmatrix}^T$ in Fig.2 is less than 1 .

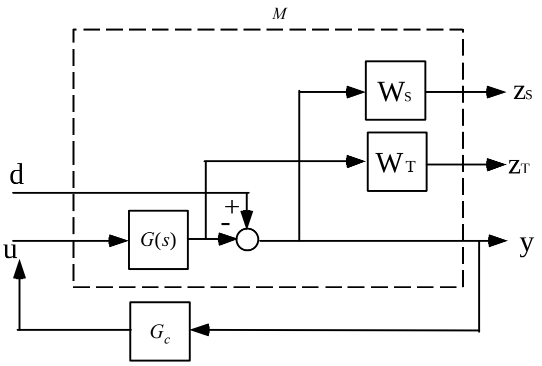

Fig.2 generalized plant

The generalized plant is represented by

$$\begin{bmatrix} z_s \\ z_T \\ y \end{bmatrix} = M \begin{bmatrix} d \\ u \end{bmatrix} \tag{6}$$

$$u = G_c y$$

where

$$M = \begin{bmatrix} W_S & -W_S G \\ 0 & W_T G \\ I & -G \end{bmatrix}$$

Denote $W_T G$ and $W_S(s)$ by

$$\begin{bmatrix} W_T G(s) \\ G \end{bmatrix} = \left[\begin{array}{c|c} A_T & B_T \\ \hline C_{T1} & D_T \\ C_{T2} & 0 \end{array}\right], \; W_S(s) = \left[\begin{array}{c|c} A_s & B_s \\ \hline C & 0 \end{array}\right] \tag{7}$$

where $A_s \in R^{n_s \times n_s}$, $B_s \in R^{n_s \times m}$, $C_s \in R^{m \times n_s}$, the generalized plant is represented by

$$M := \left[\begin{array}{c|c|c} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ \hline C_2 & D_{21} & 0 \end{array}\right]$$

$$= \left[\begin{array}{cc|c|c} A_S & -B_S C_{T2} & B_s & 0 \\ 0 & A_T & 0 & B \\ \hline C_s & 0 & 0 & 0 \\ 0 & C_{T1} & 0 & D_T \\ \hline 0 & -C_{T2} & I & 0 \end{array}\right] \tag{8}$$

where

$A \in R^{(n+n_s)\times(n+n_s)}$, $B_1 \in R^{(n+n_s)\times m}$, $B_2 \in R^{(n+n_s)\times m}$

, $C_1 \in R^{2m\times(n+n_s)}$, $C_2 \in R^{m\times(n+n_s)}$,

$D_{12} \in R^{2m\times m}$, $D_{21} \in R^{m\times m}$.

The state equation of the generalized plant is denoted by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\ z(t) &= C_1 x(t) \qquad + D_{12} u(t) \\ y(t) &= C_2 x(t) + D_{21} w(t) \end{aligned} \tag{9}$$

Theorem 1[1][2]: It is assumed that

（1） (A, B_2) is stabilizable

（2） $rank D_{12} = m$

（3） $rank \begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + m, \forall \omega \geq 0$

（4） (C_2, A) is detectable

（5） $rank D_{21} = r$

（6） $rank \begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + r, \forall \omega \geq 0$

The necessary and sufficient condition for the controller satisfying (3) is that there exist the stabilizing solution $X = X^T \geq 0$ of the Riccati equation

$$\begin{aligned} & X(A - B_2 D_{12}^{+} C_1) + (A - B_2 D_{12}^{+} C_1)^T X \\ & + X(B_1 B_1^T - B_2 E_{12}^{-1} B_2^T)X \\ & + C_1^T C_1 - C_1^T D_{12} D_{12}^{+} C_1 = 0 \end{aligned} \tag{10}$$

$$E_{12} = D_{12}^T D_{12}, \quad D_{12}^{+} = (D_{12}^T D_{12})^{-1} D_{12}{}^T$$

and the stabilizing solution $Y = Y^T \geq 0$ of the Riccati equation

$$\begin{aligned} & Y(A - B_1 D_{21}^{+} C_2)^T + (A - B_1 D_{21}^{+} C_2)Y \\ & + Y(C_1^T C_1 - C_2^T E_{21}^{-1} C_2)Y \\ & + B_1^T B_1 - B_1^T D_{21}^{+} D_{21} B_1 = 0 \end{aligned} \tag{11}$$

$$E_{12} = D_{21} D_{21}^T, \qquad D_{21}^{+} = D_{21}{}^T (D_{21} D_{21}^T)^{-1}$$

and they satisfy the inequality

$$\rho(XY) < 1 \; . \tag{12}$$

If the condition is met, then a controller satisfying (3) is given by

$$G_c(s) = \left[\begin{array}{c|c} A_c & K \\ \hline -F & 0 \end{array}\right] \tag{13}$$

$$A_c = A + B_1 B_1^T X - B_2 F - K(C_2 + D_{21} B_1^T X)$$

$$F = D_{12}^{+} C_1 + E_{12}^{-1} B_2^T X$$

$$K = (I - YX)^{-1}(B_1 D_{21}^{+} + Y C_2^T E_{21}^{-1})$$

or equivalently

$$\begin{aligned} G_c(s) &= \left[\begin{array}{c|c} (I - YX) A_c (I - YX)^{-1} & (I - YX)K \\ \hline -F(I - YX)^{-1} & 0 \end{array}\right] \\ &= \left[\begin{array}{c|c} \overline{A}_c & \overline{K} \\ \hline -\overline{F} & 0 \end{array}\right] \end{aligned} \tag{14}$$

$$\overline{A}_c = A + Y C_1^T C_1 - (B_2 + Y C_1^T D_{12})\overline{F} - \overline{K} C_2$$

$$\overline{F} = (D_{12}^{+} C_1 + E_{12}^{-1} B_2^T X)(I - YX)^{-1}$$

$$\overline{K} = B_1 D_{21}^{+} + Y C_2^T E_{21}^{-1}$$

3. Structural Problems

The conventional H^{∞} control approach to the mixed sensitivity problem suffers from three serious problems in mixed sensitivity problem. One of them is the assumption (4). It requires that the weighting function $W_s(s)$ is asymptotically stable, because of

$$(C_2, A) = \left(\begin{bmatrix} 0 & -C_{T2} \end{bmatrix}, \begin{bmatrix} A_S & -B_S C_{T2} \\ 0 & A_T \end{bmatrix} \right).$$

The second problem is the assumption (6), which requires that $C_2(sI-\hat{A})^{-1}B_1 + D_{21}$ does not contains of any zeroes on the imaginary axis. This is equivalent to that $\hat{A} - B_1 D_{21}^{+} C_2$ does not have any eigenvalues on the imaginary axis. It follows from (8) that

$$D_{21} = I \ , \quad E_{12} = D_{21} D_{21}^T = I \ ,$$

$$D_{21}^{+} = D_{21}{}^{T} (D_{21} D_{21}^T)^{-1} = I$$

and we have

$$A - B_1 D_{21}^{+} C_2 = \begin{bmatrix} A_s & 0 \\ 0 & A \end{bmatrix} \tag{15}$$

If the weighting function $W_s(s)$ and/or the plant $G(s)$ contains of poles on the imaginary axis, then the assumption (6) is not satisfied. In order to satisfy the condition, some modifications are required for the generalized plant and it complicates the design of the multi-input and multi-output plants..

The other problem is fatal for the plant with stable poles near the imaginary axis. Suppose that $G(s)$ and $W_s(s)$ do not contain any poles on the imaginary axis. Since $D_{21} = I$, $D_{21}^{+} = I$ 、 we have

$$B_1 B_1^T - B_1 D_{21}^{+} D_{21} B_1^T = 0 \tag{16}$$

and (11) becomes

$$Y(A - B_1 D_{21}^{+} C_2 + Y(C_1^T C_1 - C_2^T E_{21}^{-1} C_2))^T + (A - B_1 D_{21}^{+} C_2) Y = 0 \tag{17}$$

This yields

$$\lambda(A + YC_1^T C_1 - \overline{K} C_2) = \begin{cases} \lambda(A) & \mathrm{Re}\,\lambda(A) < 0 \\ -\lambda(A) & \mathrm{Re}\,\lambda(A) > 0 \end{cases} \tag{18}$$

where $\lambda(A)$ denotes eigenvalue of A. The controller is denoted by

$$G_c = \left[\begin{array}{c|cc} A + YC_1^T C_1 - \overline{K} C_2 & B_2 + YC_1^T D_{12} & \overline{K} \\ \hline -\overline{F} & 0 & 0 \end{array} \right] \tag{19}$$

It should be noted that the controller contains the same stable poles of the plant, which are connected by parallel to the input as shown in Fig.3. Therefore, there exist uncontrollable modes. If they are derived by input disturbances, then they affect the output. If the plant contains poles near the imaginary axis, then the responses to input disturbances is very sluggish or oscillate for long time.

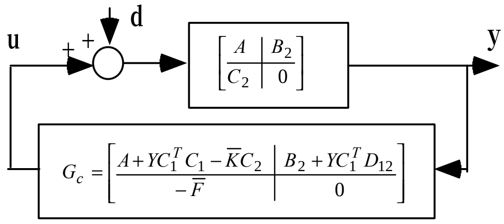

Fig.3 Control system

For example, consider the plant

$$G(s) = \frac{s+5}{(s+0.1+j)(s+0.1-j)(s-1)} \tag{20}$$

and the weighting functions

$$W_s(s) = \frac{1}{s+0.01} \quad , \qquad W_T(s) = \frac{(s+1)^2}{100} \ .$$

(21)

The standard H^{∞} control can yield the solution as shown in Fig.4. The response to a step disturbance, however, oscillates for long time as shown in Fig. 5.

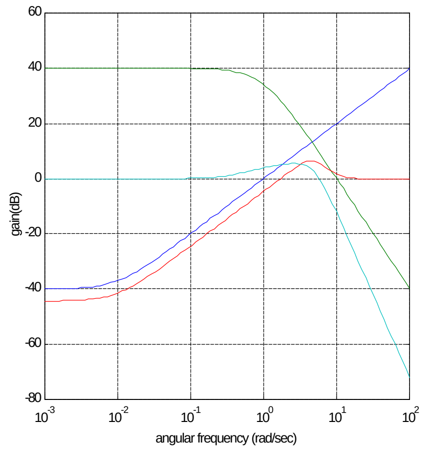

Fig.4 Bode diagrams of $|S(j\omega)|$ and $|T(j\omega)|$

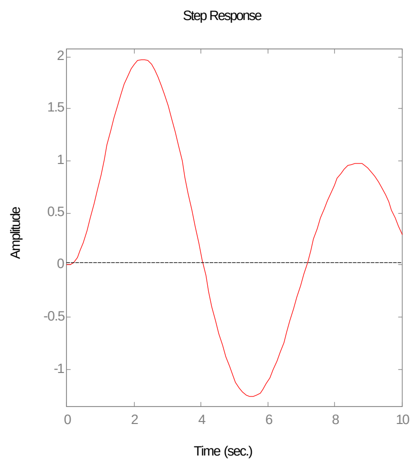

Fig.5 Disturbance response

In order to improve the disturbance response, the standard H^∞ control is modified by affine transformation such that

$$X(A+\alpha I-B_2D_{12}^{+}C_1)+(A+\alpha I-B_2D_{12}^{+}C_1)^T X$$
$$+X(B_1B_1^T-B_2E_{12}^{-1}B_2^T)Xb$$
$$+C_1^TC_1-C_1^TD_{12}D_{12}^{+}C_1=0$$

and

$$Y(A+\alpha I-B_1D_{21}^{+}C_2)^T+(A+\alpha I-B_1D_{21}^{+}C_2)Y$$
$$+Y(C_1^TC_1-C_2^TE_{21}^{-1}C_2)Y=0 \quad (22)$$

In this approarch, however, adjustable parameter α is too small to improve the response. For above example, we have solution from $\alpha=0.0$ to $a=0.009$. Multi-purpose H^∞ synthesis hinfmix in LMI yields the solution to $\alpha=0.009$. The response to disturbance is not improved. These are caused from the assumption (4) and (6).

4. Robust Pole Placement

In order to overcome above problems, we consider the robust pole placement problem to find a controller such that (3) is satisfied and poles of the closed-loop systems are located in pre-assigned half plane (real parts of poles are less than $-\alpha\ (\alpha\geq 0)$).

The key of the break-through is that the eigenvalues of A_s must be reserved in $G_c(s)$ to satisfy the sensitivity specification. It follows from (13) that

$$A_c=A+B_1B_1^TX-B_2F-K(C_2+D_{21}B_1^TX)$$
$$=\begin{bmatrix}A_s & 0\\ 0 & A_T\end{bmatrix}-\begin{bmatrix}0\\ B_t\end{bmatrix}F+\begin{bmatrix}B_s\\ 0\end{bmatrix}(C_2+B_1^TX) \quad (23)$$
$$-K(C_2+B_1^TX)$$

In order to keep A_s, K should be

$$K=\begin{bmatrix}B_s\\ *\end{bmatrix} \quad (24)$$

It follows from (24) and

$$K=(I-YX)^{-1}(B_1D_{21}^{+}+YC_2^TE_{21}^{-1})$$

that Y has to be of the form

$$Y=\begin{bmatrix}0 & 0\\ Y_q & Y_p\end{bmatrix} \quad (25)$$

The Riccati equation (22) becomes

$$Y_p(A_T^T+\alpha I)+(A_T+\alpha I)Y_p$$
$$+Y_p(C_{T1}^TC_{T1}-C_{T2}^TC_{T2})Y_p=0 \quad (26)$$

and

$$Y_q(A_s^T+\alpha I)+(A_T+\alpha I)Y_q$$
$$+Y_p(C_{T1}^TC_{T1}-C_{T2}^TC_{T2})Y_q=0 \quad (27)$$

If the Y_q is not zero, then it is restrictive rather than the case of $Y_q=0$. Therefore, the case of $Y_q=0$ is general..

This implies that the condition that the restrictive assumption(4) is replaced by general condition that (C_{T2},A_T) is detectable, and that the assumption(6) can be removed. They allow the solution for lager $\alpha\geq 0$.

Theorem2: Suppose that the assumptions (1)~(3) and (5) in Theorem 1 and the assumption that (C_{T2},A_T) is detectable are met. If there exists the stabilizing solution $X=X^T\geq 0$ of the Riccati equation

$$X(A+\alpha I-B_2D_{12}^{+}C_1)+(A+\alpha I-B_2D_{12}^{+}C_1)^TX$$
$$+X(B_1B_1^T-B_2E_{12}^{-1}B_2^T)X \quad (28)$$
$$+C_1^TC_1-C_1^TD_{12}D_{12}^{+}C_1=0$$

and there exists the stabilizing solution $Y_p=Y_p^T\geq 0$ of the Riccati equation

$$Y_p(A_T^T+\alpha I)+(A_T+\alpha I)Y_p$$
$$+Y_p(C_{T1}^TC_{T1}-C_{T2}^TC_{T2})Y_p=0 \quad (29)$$

and the inequality

$$\rho(XY)<1\ . \quad (30)$$

$$Y=\begin{bmatrix}0 & 0\\ 0 & Y_p\end{bmatrix} \quad (31)$$

is satisfied, then there exists a solution satisfying (3). The controller are given by (13) or (14).

The poles of the closed-loop system are located in the half plane with real part less than $-\alpha$.

(Proof) Eqs. (16),(29) and (31) yield

$$Y(A+\alpha I-B_1D_{21}^{+}C_2)^T+(A+\alpha I-B_1D_{21}^{+}C_2)Y$$
$$+Y(C_1^TC_1-C_2^TE_{21}^{-1}C_2)Y \quad (32)$$
$$+B_1^TB_1-B_1^TD_{21}^{+}D_{21}B_1=0$$

Y is the stabilizing solution of Riccati equation (32) except eigenvalues of $A_s+\alpha I$.

Define

$$\hat{M}(v)=\left[\begin{array}{c|c|c}A+\alpha I & B_1 & B_2\\ \hline C_1 & 0 & D_{12}\\ \hline C_2 & D_{21} & 0\end{array}\right] \quad (33)$$

$$\hat{G}_c(v)=\left[\begin{array}{c|c}A_c+\alpha I & K\\ \hline -F & 0\end{array}\right] \quad (34)$$

then we have

$$M(v-\alpha)=\hat{M}(v),\quad M(s)=\hat{M}(s+\alpha) \quad (35)$$
$$G_c(v-\alpha)=\hat{G}_c(v),\quad G_c(s)=\hat{G}_c(s+\alpha)\ .$$

Let

$$\hat{M}(v)=\begin{bmatrix}\hat{M}_{11}(v) & \hat{M}_{12}(v)\\ \hat{M}_{21}(v) & \hat{M}_{22}(v)\end{bmatrix} \quad (36)$$

and define LFT transform as

$$LFT\{\hat{M}(v),\hat{G}_c(v)\} = \hat{M}_{11}(v) + \hat{M}_{12}(v)(I - \hat{G}_c(v)\hat{M}_{22}(v))^{-1}\hat{G}_c(v)\hat{M}_{21}(v) \quad (37)$$

In the similar manner to standard H^∞ control, we have

$$LFT\{\hat{M}(v),\hat{G}_c(v)\} = LFT\{\hat{M}^a(v), LFT\{\hat{M}^b(v),\hat{G}_c(v)\}\} \quad (38)$$

where

$$\hat{M}^a(v) = \left[\begin{array}{c|cc} A+\alpha I - B_2F & B_1 & B_2E_{12}^{-1/2} \\ \hline C_1 - D_{12}F & 0 & D_{12}E_{12}^{-1/2} \\ -B_1^TX & I & 0 \end{array}\right] \quad (39)$$

is inner and

$$\hat{M}^b(v) = \left[\begin{array}{c|cc} A+\alpha I + B_1B_1^TX & B_1 & B_2 \\ \hline E_{12}^{1/2}F & 0 & E_{12}^{1/2} \\ C_2 + D_{21}B_1^TX & D_{21} & 0 \end{array}\right]. \quad (40)$$

Transforming

$$LFT\{\hat{M}^b(v),\hat{G}_c(v)\} = \left[\begin{array}{cc|c} A+\alpha I + B_1B_1^TX & -B_2F & B_1 \\ K(C_2 + D_{21}B_1^TX) & A+\alpha I + B_1B_1^TX - B_2F - K(C_2+D_{21}B_1^TX) & KD_{21} \\ \hline E_{12}^{1/2}F & -E_{12}^{1/2}F & 0 \end{array}\right]$$

by

$$\begin{bmatrix} I & I \\ 0 & I \end{bmatrix}$$

yields

$$LFT\{\hat{M}^b(v),\hat{G}_c(v)\} = \left[\begin{array}{c|c} A+\alpha I + B_1B_1^TX - K(C_2+D_{21}B_1^TX) & B_1 - KD_{21} \\ \hline E_{12}^{1/2}F & 0 \end{array}\right]. \quad (41)$$

It follows that

$$\sigma_{\max\limits_{\forall\omega\ge 0}}[LFT\{\hat{M}^b(j\omega),\hat{G}_c(j\omega)\}] < 1 \quad (42)$$

in the same manner to standard H^∞ control. Furthermore, we have

$$A+\alpha I + B_1B_1^TX - K(C_2+D_{21}B_1^TX) = \begin{bmatrix} A_s & 0 \\ * & * \end{bmatrix} \quad (43)$$

and

$$B_1 - KD_{21} = \begin{bmatrix} 0 \\ * \end{bmatrix} \quad (44)$$

These imply that the eigenvalues of A_s are not controllable in (41) and they are eliminated in the transfer function. Then (41) is stable and

$$\left\|LFT\{\hat{M}^b(v),\hat{G}_c(v)\}\right\|_\infty < 1 \quad (45)$$

Denote

$$\hat{M}^a(v) = \begin{bmatrix} \hat{M}_{11}^a(v) & \hat{M}_{12}^a(v) \\ \hat{M}_{21}^a(v) & \hat{M}_{22}^a(v) \end{bmatrix} \quad (46)$$

Since $\hat{M}^a(v)$ is inner, $\left\|\hat{M}_{22}^a(v)\right\|_\infty \le 1$. From the small gain theorem, it follows that

$$\begin{aligned} &LFT\{\hat{M}^a(v), LFT\{\hat{M}^b(v),\hat{G}_c(v)\}\} \\ &= \hat{M}_{11}^a(v) + \hat{M}_{12}^a(v)(I - LFT\{\hat{M}^b(v),\hat{G}_c(v)\}\hat{M}_{22}^a(v))^{-1} \\ &\quad LFT\{\hat{M}^b(v),\hat{G}_c(v)\}M_{21}^a(v) \end{aligned} \quad (47)$$

is stable. Furthermore, we have

$$\sigma_{\max\limits_{\forall\omega\ge 0}} LFT\{\hat{M}^a(j\omega), LFT\{\hat{M}^b(j\omega),\hat{G}_c(j\omega)\}\} < 1 \quad (48)$$

It follows from (47) and (48) that

$$\left\|LFT\{\hat{M}(v),\hat{G}_c(v)\}\right\|_\infty < 1 \quad (49)$$

This and the definition of infinity norm on Hardy space imply that

$$\begin{aligned} &\|LFT\{M(s),G_c(s)\}\|_\infty \\ &\le \left\|LFT\{\hat{M}(s+\alpha),\hat{G}_c(s+\alpha)\}\right\|_\infty \\ &\le \left\|LFT\{\hat{M}(v),\hat{G}_c(v)\}\right\|_\infty < 1 \end{aligned} \quad (50)$$

$LFT\{M(s),G_c(s)\}$ has poles, real parts of which are less than $-\alpha$. □

Let $\bar{F} = [F_1 \quad F_2]$, then the controller is denoted by

$$G_C = \left[\begin{array}{cc|c|c} A_s & 0 & 0 & B_s \\ 0 & A_T + Y_p(C_{T1}^TC_{T1} - C_{T2}^TC_{T2}) & B_T + Y_pC_{T1}^TD_T & -Y_pC_{T2}^T \\ \hline -F_1 & -F_2 & 0 & 0 \end{array}\right]. \quad (51)$$

The control system is shown in Fig. 5. The weighting unction $W_S(s)$ is introduced as internal model and thus $\|W_S(s)S(s)\|_\infty < 1$ is satisfied broadly. Compared with the proposed method, conventional H^∞ control loses this structure and thus α is restricted to be small.

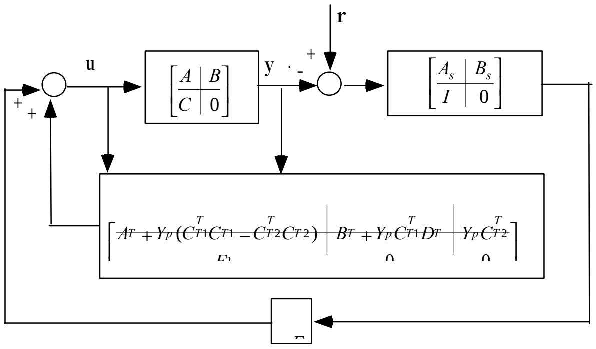

Fig.6 propose control system

For above example, the weighting function

$W_S(s)$ is changed to

$$W_s(s) = \frac{1}{s} \qquad (52)$$

The proposed method yields the solution from $\alpha = 0$ to $\alpha = 1.08$. Bode diagrams of $|S(j\omega)|$ and $|T(j\omega)|$, responses to step reference input and step disturbances for $\alpha = 0$ are similar to those of the standard H^∞ control. Bode diagrams of $|S(j\omega)|$ and $|T(j\omega)|$ for $\alpha = 1.08$. are shown in Fig.7. Responses to a step reference input and step disturbance are shown in Fig.8 and9 respectively. It should be noted that responses to disturbance is improved considerably.

5. Conclusions

This paper proposed the broad robust pole placement that the mixed sensitivity specification is satisfied and poles are located in pre-assigned half plane. The restrict assumption of the conventional H^∞ control is removed and the second Riccati equation is modified. The weighting function to sensitivity is directly introduced as inner model and the sensitivity specification is met broadly.

The proposed method is directly applied to the plants and the weighting function with poles on the imaginary axis. The response to disturbance can be easily adjusted. This makes the design of control system more practical.

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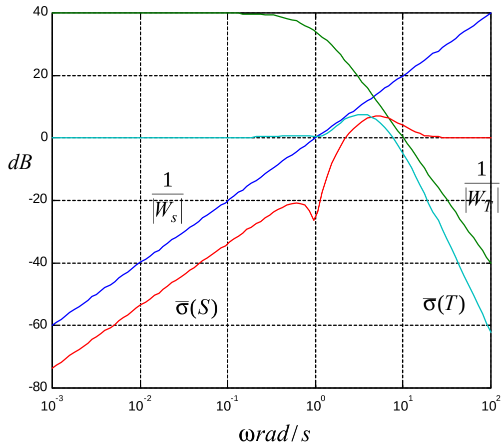

Fig.7 Bode diagrams of $|S(j\omega)|$ and $|T(j\omega)|$

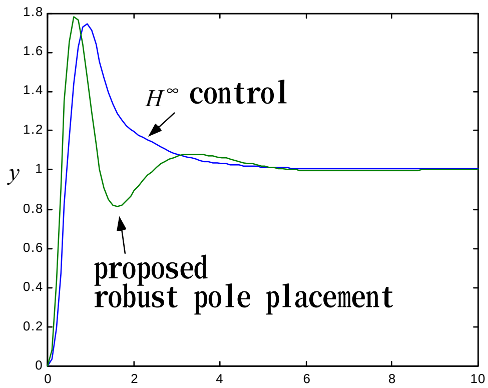

Fig.8 Step reference

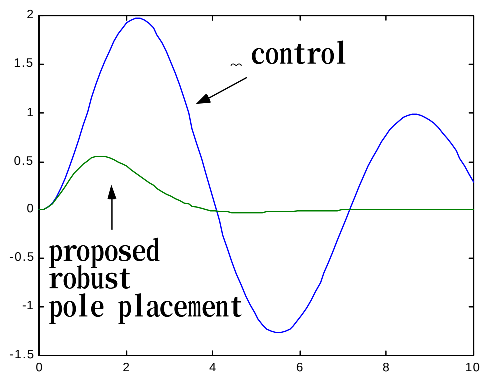

Fig.9 Step disturbance