A new robust observer for the perspective system

Xinkai Chen*, Hiroyuki Kano* and Toshio Fukuda**
*Department of Information Sciences, Tokyo Denki University, Hiki-gun, Saitama 350-0394, Japan
E-mail: chen@j.dendai.ac.jp; kano@j.dendai.ac.jp; Fax: +81-492-96-6403
**Dept. Mechano-Informatics and Systems, Nagoya University, Chikusa-ku, Nagoya 464-8603, Japan
E-mail: fukuda@mein.nagoya-u.ac.jp; Fax: +81-52-789-3115

Abstract: In this paper, we consider the problem of estimating the position of an object moving in the space by observing its image with the aid of a CCD camera. The problem can be converted into the observation of a dynamical system with nonlinearities. A new method, which is inspired by the sliding mode techniques, is proposed to identify the obtained dynamical system. The attraction of the new method is that the identification can be finished in a very short time, the algorithm is very simple and easy to be implemented, and it is robust to measurement noises. Further, minor *a priori* knowledge of the system is required in the new formulation. Simulation results show the superiority of the new method to the traditional ones.
***Keywords*:** Observer; Perspective system; Machine vision

1. Introduction

Observing the position of a moving object in the space by the image data with the aid of a CCD camera has been studied in the past years [1,3-6,8]. A very typical method is the application of the extended Kalman filter (EKF) [6,8]. It is well known that the convergence cannot be guaranteed theoretically and it takes a long time to finish the identification. Further, the *a priori* knowledge about the noise is required. A fatal shortcoming of EKF is that the algorithm is very complicated and can hardly be implemented practically with real image data. To overcome these difficulties, Jankovic and Ghosh [4] proposed a new recursive formulation called "identifier based observer" (IBO) based on a parameter identifier considered in model reference adaptive control (see [7]). The proposed IBO is guaranteed to converge in an arbitrary large (but bounded) set of initial conditions, and since the convergence is exponential it is believed that the performance of IBO is reliable, robust and would quickly compute the position on real data [4]. For the EKF, no such convergence guarantees are available. It should be noted that the performance of IBO is similar to that of the EKF. However, in the formulation of IBO, the *a priori* information about the upper bound of the state is required, which may not be easy to be obtained in practice.

In this paper, we consider the position identification problem of the perspective system, where the motion parameters are assumed known. The formulated problem can be converted into the observation of a dynamical system with nonlinearities. To identify this class of nonlinear system, the method proposed by the authors in [2] may be considerable. In this paper, a new identification method, which is inspired by the sliding mode techniques, is proposed to identify the space position of a moving object. The information about the upper bound of the state is not required, and it is adaptively updated online. The new observer converges very fast, and the estimation process can be monitored online. The attraction of the new method lies in that the algorithm is very simple, easy to be implemented practically, and robust to measurement noises. The proposed algorithm can be extended to the observation problem for the general *n*-dimensional perspective systems (Remark 13). Further, simulation results show that the new method is superior to the traditional ones.

The organization of the paper is as follows: Section 2 formulates the problem. In section 3, the robust observer is proposed for the perspective system. In section 4, an example is given to illustrate the new algorithm, and to compare the performance with the traditional methods.

2. Problem statement

Consider the movement of the object described by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad (1)$$

where $x(t) = [x_1 \quad x_2 \quad x_3]^T$ is the position in the space, $a_{ij}\,(i,j=1,2,3)$ and $b_i\,(i=1,2,3)$ are the motion parameters (see [5]).

Also, we suppose the observed position in the image plane is defined by

$$y(t) = [y_1(t), \quad y_2(t)] = \left[\frac{x_1}{x_3}, \quad \frac{x_2}{x_3}\right] \qquad (2)$$

Thus, the perspective system is composed of equations (1) and (2).

In this paper, we make the following assumptions.

(A1). The parameters $a_{ij}\,(i,j=1,2,3)$ and b_i

$(i = 1,2,3)$ are known.

(A2). $x_3(t)$ meets the condition $x_3(t) > \alpha > 0$, where α is a positive constant.

(A3). $x(t)$ is bounded in finite time.

Remark 1: It is easy to see that assumptions (A2) and (A3) are reasonable by referring to the practical systems.

The purpose of this paper is to estimate the position $x(t)$ by using the observed information $y_1(t)$ and $y_2(t)$ in the image plane.

3. Formulation of the observer

Define

$$y_3(t) = \frac{1}{x_3(t)} \tag{3}$$

Then, equation (1) can be rewritten as

$$\begin{cases} \dot{y}_1(t) = a_{13} + (a_{11} - a_{33})y_1 + a_{12}y_2 - a_{31}y_1^2 \\ \qquad - a_{32}y_1y_2 + (b_1 - b_3y_1)y_3 \\ \dot{y}_2(t) = a_{23} + a_{21}y_1 + (a_{22} - a_{33})y_2 - a_{31}y_1y_2 \\ \qquad - a_{32}y_2^2 + (b_2 - b_3y_2)y_3 \\ \dot{y}_3(t) = -(a_{31}y_1 + a_{32}y_2 + a_{33})y_3 - b_3y_3^2 \end{cases} \tag{4}$$

It is obvious that the position of the object in the space can be calculated as

$$x_1(t) = \frac{y_1(t)}{y_3(t)}, \quad x_2(t) = \frac{y_2(t)}{y_3(t)}, \quad x_3(t) = \frac{1}{y_3(t)} \tag{5}$$

if $y_3(t)$ is available. So, the remaining task is to estimate $y_3(t)$.

Lemma 1: If $(b_1 - b_3y_1)^2 + (b_2 - b_3y_2)^2 \equiv 0$, then the position cannot be observed by using the image data.

Proof: In this case, equation (4) reduces to

$$\begin{cases} \dot{y}_1(t) = a_{13} + (a_{11} - a_{33})y_1 + a_{12}y_2 - a_{31}y_1^2 - a_{32}y_1y_2 \\ \dot{y}_2(t) = a_{23} + a_{21}y_1 + (a_{22} - a_{33})y_2 - a_{31}y_1y_2 - a_{32}y_2^2 \\ \dot{y}_3(t) = -(a_{31}y_1 + a_{32}y_2 + a_{33})y_3 - b_3y_3^2 \end{cases} \tag{6}$$

It can be seen that the first two equations has no relation with $y_3(t)$. Based solely on the third equation in (6), it is impossible to identify $y_3(t)$.

Remark 2: If $b_3 \neq 0$, Lemma 1 means that the position in the space cannot be identified by using a definite point $(\frac{b_1}{b_3}, \frac{b_2}{b_3})$ in the image plane. This fact is very intuitive and well known. The point $(\frac{b_1}{b_3}, \frac{b_2}{b_3})$ is the so-called *focus of expansion* (FOE) [4]. If $b_3 = 0$, then $(b_1 - b_3y_1)^2 + (b_2 - b_3y_2)^2 \equiv 0$ means that $b_1 = b_2 = b_3 = 0$. In this case, the position in the space cannot be identified due to the observed data is not "rich".

In the following, the observer of system (4) is formulated. We consider the system described by

$$\begin{cases} \dot{\hat{y}}_1(t) = a_{13} + (a_{11} - a_{33})y_1 + a_{12}y_2 - a_{31}y_1^2 \\ \qquad - a_{32}y_1y_2 + (b_1 - b_3y_1)\hat{y}_3 + \hat{\lambda}_1(t)\frac{e_1}{|e_1| + \delta_1} \\ \dot{\hat{y}}_2(t) = a_{23} + a_{21}y_1 + (a_{22} - a_{33})y_2 - a_{31}y_1y_2 \\ \qquad - a_{32}y_2^2 + (b_2 - b_3y_2)\hat{y}_3 + \hat{\lambda}_2(t)\frac{e_2}{|e_2| + \delta_2} \\ \dot{\hat{y}}_3(t) = -(a_{31}y_1 + a_{32}y_2 + a_{33})\hat{y}_3 - b_3\hat{y}_3^2 \end{cases} \tag{7}$$

where the initial condition is determined as

$$\hat{y}_1(0) = y_1(0), \quad \hat{y}_2(0) = y_2(0), \quad \hat{y}_3(0) = \hat{y}_{30}, \tag{8}$$

with $\hat{y}_{30} > 0$; $e_1(t)$ and $e_2(t)$ are respectively defined as

$$e_1 = y_1 - \hat{y}_1, \quad e_2 = y_2 - \hat{y}_2; \tag{9}$$

$\delta_i\ (i = 1,2)$ are positive constants; $\hat{\lambda}_1(t)$ and $\hat{\lambda}_2(t)$ are respectively defined as

$$\dot{\hat{\lambda}}_1(t) = \begin{cases} 2\alpha_1|e_1| & if\ |e_1| > \delta_1 \\ 0 & otherwise \end{cases}, \tag{10}$$

$$\dot{\hat{\lambda}}_2(t) = \begin{cases} 2\alpha_2|e_2| & if\ |e_2| > \delta_2 \\ 0 & otherwise \end{cases}, \tag{11}$$

α_1 and α_2 are positive constants, $\hat{\lambda}_1(0)$ and $\hat{\lambda}_2(0)$ can be any positive constants.

Remark 3: By observing the third equation in (7), it differs from the third equation in (4) with only the initial condition. Since $\hat{y}_3(0)$ is finite and $y_3(t)$ is bounded in finite time, it can be seen that $\hat{y}_3(t)$ is bounded at least when t is not so large.

Now, combining (4) and (7) yields

$$\begin{cases} \dot{e}_1(t) = (b_1 - b_3y_1)e_3 - \hat{\lambda}_1(t)\frac{e_1}{|e_1| + \delta_1}, \quad e_1(0) = 0 \\ \dot{e}_2(t) = (b_2 - b_3y_2)e_3 - \hat{\lambda}_2(t)\frac{e_2}{|e_2| + \delta_2}, \quad e_2(0) = 0 \\ \dot{e}_3(t) = -(a_{31}y_1 + a_{32}y_2 + a_{33})e_3 - b_3(y_3 + \hat{y}_3)e_3 \end{cases} \tag{12}$$

where $e_3(t)$ is defined as

$$e_3 = y_3 - \hat{y}_3. \tag{13}$$

Define

$$e_3' = \frac{1}{(b_1 - b_3y_1)^2 + (b_2 - b_3y_2)^2}\left\{(b_1 - b_3y_1)\hat{\lambda}_1(t)\frac{e_1}{|e_1| + \delta_1}\right.$$

$$+(b_2-b_3y_2)\hat{\lambda}_2(t)\frac{e_2}{|e_2|+\delta_2}\Bigg\} \quad (14)$$

and

$$\hat{\hat{y}}(t)=e_3'(t)+\hat{y}_3(t). \quad (15)$$

We have the next theorem to illustrate our new observer.

Theorem 1. Suppose $(b_1-b_3y_1(t))^2+(b_2-b_3y_2(t))^2\neq 0$ for all $t\geq 0$. Then, e_3' can be regarded as the approximate estimate of $e_3(t)$ after some instant. Thus, $\hat{\hat{y}}_3(t)$ can be thought of the approximate estimate of $y_3(t)$.

Proof: The proof is composed of the following three steps.

Step 1 The proof of that $|e_1(t)|$ and $|e_2(t)|$ are uniformly bounded, and there exist $t_1, t_2>0$ such that

$$|e_1(t)|\leq\delta_1 \quad (16)$$

for $t>t_1$, and

$$|e_2(t)|\leq\delta_2 \quad (17)$$

for $t>t_2$.

By remark 3 and the assumptions, it can be seen that $|(b_1-b_3y_1)e_3(t)|$ is bounded. Suppose its upper bound is λ_1, i.e.

$$|(b_1-b_3y_1)e_3(t)|<\lambda_1. \quad (18)$$

Now, consider the Lyapunov candidate

$$V_1(t)=(e_1(t))^2+\frac{1}{\alpha_1}\left(\frac{1}{2}\hat{\lambda}_1(t)-\lambda_1\right)^2 \quad (19)$$

If $|e_1(t)|>\delta_1$, then differentiating $V_1(t)$ yields

$$\frac{d}{dt}\left\{(e_1(t))^2+\frac{1}{\alpha_1}\left(\frac{1}{2}\hat{\lambda}_1(t)-\lambda_1\right)^2\right\}$$

$$=2e_1(t)(b_1-b_3y_1)e_3-2\hat{\lambda}_1(t)\frac{e_1^2}{|e_1|+\delta_1}+2\left(0.5\hat{\lambda}_1(t)-\lambda_1\right)|e_1|$$

$$=2\left(e_1(t)(b_1-b_3y_1)e_3-\lambda_1|e_1|\right)+2\hat{\lambda}_1(t)\frac{|e_1|\delta_1}{|e_1|+\delta_1}-\hat{\lambda}_1(t)|e_1|$$

$$\leq-\hat{\lambda}_1(t)|e_1|\left(1-\frac{2\delta_1}{|e_1|+\delta_1}\right)<0 \quad (20)$$

Thus, $V_1(t)$ decreases monotonically. Further, from (20), it can be seen that the condition $|e_1(t)|\leq\delta_1$ can be satisfied in finite time. Then, there exists $t_1>0$ such that

$$|e_1(t)|\leq\delta_1 \quad (21)$$

for $t>t_1$, and $V(t)$ is uniformly bounded for $0\leq t\leq t_1$. Thus, $|e_1(t)|$ and $\hat{\lambda}_1(t)$ are also uniformly bounded for $0\leq t\leq t_1$. By the definition of $\hat{\lambda}_1(t)$ and (21), it can be seen that $\hat{\lambda}_1(t)=\hat{\lambda}_1(t_1)$ for $t>t_1$. Thus, it can be concluded that $\hat{\lambda}_1(t)$ is uniformly bounded for all $t\geq 0$. Therefore, $|e_1(t)|$ is uniformly bounded and (16) is valid.

Similarly, it can be proved that $|e_1(t)|$ and $\hat{\lambda}_2(t)$ are uniformly bounded, and there exists $t_2>0$ such that (17) is valid for all $t>t_2$.

Step 2 The proof of that $|\dot{e}_1(t)|$ and $|\dot{e}_2(t)|$ are very small after some instant.

For $t>t_1$, differentiating the first equation in (12) yields

$$\ddot{e}_1(t)=\frac{d}{dt}\left((b_1-b_3y_1)e_3\right)-\hat{\lambda}_1(t_1)\frac{\dot{e}_1\delta_1}{\left(|e_1|+\delta_1\right)^2} \quad (22)$$

Differentiating $(\dot{e}_1(t))^2$ yields

$$\frac{d}{dt}(\dot{e}_1(t))^2=2\dot{e}_1(t)\cdot\frac{d}{dt}\left((b_1-b_3y_1)e_3\right)-2\hat{\lambda}_1(t_1)\frac{\dot{e}_1^2\delta_1}{\left(|e_1|+\delta_1\right)^2}$$

$$\leq 2\dot{e}_1(t)\cdot\frac{d}{dt}\left((b_1-b_3y_1)e_3\right)-2\hat{\lambda}_1(t_1)\frac{\dot{e}_1^2}{4\delta_1} \quad (23)$$

where the fact $|e_1(t)|\leq\delta_1$ is employed. If $|\dot{e}_1(t)|>\frac{4\delta_1}{\hat{\lambda}_1(t_1)}\max_{t\geq t_1}\left(\left|\frac{d}{dt}\left((b_1-b_3y_1)e_3\right)\right|\right)$, then $\frac{d}{dt}(\dot{e}_1(t))^2<0$, i.e. $|\dot{e}_1(t)|$ decreases until $|\dot{e}_1(t)|\leq\frac{4\delta_1}{\hat{\lambda}_1(t_1)}\max_{t\geq t_1}\left(\left|\frac{d}{dt}\left((b_1-b_3y_1)e_3\right)\right|\right)$ is satisfied. Therefore, there exists an instant $t_3\geq t_1$ such that

$$|\dot{e}_1(t)|\leq\frac{4\delta_1}{\hat{\lambda}_1(t_1)}\max_{t\geq t_1}\left(\left|\frac{d}{dt}\left((b_1-b_3y_1)e_3\right)\right|\right) \quad (24)$$

for all $t>t_3$.

Since $\frac{1}{\hat{\lambda}_1(t_1)}\max_{t\geq t_1}\left(\left|\frac{d}{dt}\left((b_1-b_3y_1)e_3\right)\right|\right)$ could be considered to be bounded, it can be concluded that $|\dot{e}_1(t)|$ is very small as $t>t_3$ by choosing very small δ_1. Further, from (23), it can be seen that the decreasing speed of $|\dot{e}_1(t)|$ can be increased by choosing very small δ_1.

Similarly, it can be proved that there exists an instant $t_4\geq t_2$ such that $|\dot{e}_2(t)|$ is very small as $t>t_4$ by choosing very small δ_2.

Step 3 The proof of that $e_3'(t)$ is an approximate estimate of $e_3(t)$.

From the results in the first and the second steps in this proof, by observing the first and the second equations in (12), it can be seen that $(b_1 - b_3 y_1)e_3 - \hat{\lambda}_1(t)\frac{e_1}{|e_1| + \delta_1}$ and $(b_2 - b_3 y_2)e_3 - \hat{\lambda}_2(t)\frac{e_2}{|e_2| + \delta_2}$ are very small as $t > t_3$ and as $t > t_4$, respectively. Thus, by least square method, $e_3'(t)$ defined in (14) can be considered as an approximate estimate of $e_3(t)$, i.e. there exists $\varepsilon(\delta_1, \delta_2) > 0$ such that

$$|e_3'(t) - e_3(t)| \le \varepsilon(\delta_1, \delta_2) \tag{25}$$

as $t > \max(t_3, t_4)$, where $\varepsilon(\delta_1, \delta_2)$ is very small depending on δ_1 and δ_2.

Therefore, the theorem is proved.

Remark 4: As the bounds of $|(b_1 - b_3 y_1)e_3|$ and $|(b_2 - b_3 y_2)e_3|$ are unknown, we adaptively update them by (10) and (11).

Remark 5: By the proof of Theorem 1, it can be seen that the observation process can be stopped after confirming that both $e_1(t)$ and $e_2(t)$ converge for some seconds. These convergences can be monitored online.

Remark 6: The design parameters $\alpha_i > 0$ and $\delta_i > 0$ ($i = 1,2$) determine the estimating speed and the estimating precision. The parameters $\alpha_i > 0$ ($i = 1,2$) should be chosen large enough to rapidly adjust $\hat{\lambda}_i(t)$.

Remark 7: Because the converging speed can be increased by choosing appropriate parameters, it can be seen that the identification can be finished in a short time by observing Theorem 1. Thus, only the condition that $(b_1 - b_3 y_1)^2 + (b_2 - b_3 y_2)^2$ is not zero in some interval is needed. Therefore, the assumption $(b_1 - b_3 y_1)^2 + (b_2 - b_3 y_2)^2 \neq 0$ can be weakened. This condition can be monitored online.

By combining Theorem 1 and Remark 7, we have the following theorem.

Theorem 2. The sufficient condition for the system (4) to be observable by using the image data (y_1, y_2) is that $(b_1 - b_3 y_1)^2 + (b_2 - b_3 y_2)^2$ is not zero during some necessary long interval. If the system is observable, then $y_3(t)$ can be estimated as in Theorem 1.

Remark 8: By solving the linear differential equation (1), the initial value $x(0)$ can be expressed by using $y_i(t)$ $(i = 1,2,3)$. Further, by replacing $y_3(t)$ by its estimate $\hat{y}_3(t)$, the calculated initial value, say $x(t,0) = [x_1(t,0) \quad x_2(t,0) \quad x_3(t,0)]^T$, converges when $t > \max\{t_3, t_4\}$. The convergence can be tested online by monitoring that whether the differences $\left|\frac{x_1(t,0)}{x_3(t,0)} - y_1(0)\right|$ and $\left|\frac{x_2(t,0)}{x_3(t,0)} - y_2(0)\right|$ converge to very small values and whether $x_3(t,0)$ converges. After the convergence is confirmed, the identification process can be stopped, and the position in the space at arbitrary instant t can be approximately calculated out by using the estimated initial condition. Certainly, the identified position at the stopped instant, say t_0, can be treated as the initial condition to compute the position at instant t $(t \ge t_0)$ based on the dynamical system (1). Of course, if $(b_1 - b_3 y_1)^2 + (b_2 - b_3 y_2)^2$ is not zero at any instant, the position in the space at instant t can also be approximately calculated online as $x(t) = \begin{bmatrix} \frac{y_1(t)}{\hat{y}_3(t)} & \frac{y_2(t)}{\hat{y}_3(t)} & \frac{1}{\hat{y}_3(t)} \end{bmatrix}^T$.

Remark 9: For $b_3 \neq 0$, in the presence of measurement noise, the position in the space cannot be identified reliably if the observed data (y_1, y_2) is very close to $(\frac{b_1}{b_3}, \frac{b_2}{b_3})$. Thus, for $b_3 \neq 0$, in the sense of practical application, the necessary and sufficient condition for the system to be observable should be $(b_1 - b_3 y_1)^2 + (b_2 - b_3 y_2)^2 > \gamma$ in some necessary long interval, where $\gamma > 0$ is very small.

Remark 10: Consider the object is moving in the Ricatti dynamics [3] described by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + \begin{bmatrix} c_1 & c_2 & c_3 & 0 & 0 & 0 \\ 0 & c_1 & 0 & c_2 & c_3 & 0 \\ 0 & 0 & c_1 & 0 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} x_1^2 \\ x_1 x_2 \\ x_1 x_3 \\ x_2^2 \\ x_2 x_3 \\ x_3^2 \end{bmatrix} \tag{26}$$

where c_i $(i = 1,2,3)$ are known constants. The problem is to estimate the state $x = [x_1 \quad x_2 \quad x_3]^T$ by using the observed information $y_1 = \frac{x_1}{x_3}$ and $y_2 = \frac{x_2}{x_3}$. By defining $y_3 = \frac{1}{x_3}$, system (26) gives

$$\begin{cases} \dot{y}_1(t) = a_{13} + (a_{11} - a_{33})y_1 + a_{12}y_2 - a_{31}y_1^2 \\ \quad - a_{32}y_1y_2 + (b_1 - b_3y_1)y_3 \\ \dot{y}_2(t) = a_{23} + a_{21}y_1 + (a_{22} - a_{33})y_2 - a_{31}y_1y_2 \\ \quad - a_{32}y_2^2 + (b_2 - b_3y_2)y_3 \\ \dot{y}_3(t) = -(c_1y_1 + c_2y_2 + c_3) \\ \quad - (a_{31}y_1 + a_{32}y_2 + a_{33})y_3 - b_3y_3^2 \end{cases} \quad (27)$$

Since $(c_1y_1 + c_2y_2 + c_3)$ is an available signal, the observer can be similarly constructed.

Remark 12: The proposed method can be extended to the general n-dimensional perspective systems. Consider the system described by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}\begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} \quad (28)$$

with $x_n(t) > 0$, and $x = [x_1 \ \cdots \ x_n]^T$ is bounded in finite time. The problem is to estimate the state $x(t)$ by using the available signals $y_1 = \frac{x_1}{x_n}$, …, $y_{n-1} = \frac{x_{n-1}}{x_n}$.

By defining $y_n = \frac{1}{x_n}$, systems (28) can be rewritten as

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ \vdots \\ y_{n-1} \\ y_n \end{bmatrix} = \begin{bmatrix} \omega_1(t) \\ \vdots \\ \omega_{n-1}(t) \\ 0 \end{bmatrix} + \begin{bmatrix} b_1 - b_ny_1 \\ \vdots \\ b_{n-1} - b_ny_{n-1} \\ 0 \end{bmatrix}y_n - \begin{bmatrix} 0 \\ \vdots \\ 0 \\ a_{n1}y_1 + \cdots + a_{n,n-1}y_{n-1} + a_{nn} + b_ny_n \end{bmatrix}y_n \quad (29)$$

where $\omega_i(t)$ are available signals defined by

$$\omega_i(t) = (a_{i1} - a_{n1}y_i)y_1 + \cdots + (a_{i,n-1} - a_{n,n-1}y_i)y_{n-1} + (a_{in} - a_{nn}y_i). \quad (30)$$

Thus, the observer of (29) can be constructed as

$$\frac{d}{dt}\begin{bmatrix} \hat{y}_1 \\ \vdots \\ \hat{y}_{n-1} \\ \hat{y}_n \end{bmatrix} = \begin{bmatrix} \omega_1(t) \\ \vdots \\ \omega_{n-1}(t) \\ 0 \end{bmatrix} + \begin{bmatrix} b_1 - b_ny_1 \\ \vdots \\ b_{n-1} - b_ny_{n-1} \\ 0 \end{bmatrix}\hat{y}_n - \begin{bmatrix} 0 \\ \vdots \\ 0 \\ a_{n1}y_1 + \cdots + a_{n,n-1}y_{n-1} + a_{nn} + b_n\hat{y}_n \end{bmatrix}\hat{y}_n + \begin{bmatrix} \hat{\lambda}_1 \frac{e_1}{|e_1| + \delta_1} \\ \vdots \\ \hat{\lambda}_{n-1} \frac{e_{n-1}}{|e_{n-1}| + \delta_{n-1}} \\ 0 \end{bmatrix} \quad (31)$$

where the initial condition is determined as

$$\hat{y}_i(0) = y_i(0) \quad \hat{y}_n(0) = \hat{y}_{n0}, \ (\hat{y}_{n0} > 0), \quad (32)$$

for $i = 1, \cdots, n-1$; δ_i ($i = 1, \cdots, n-1$) are positive constants; $e_i(t)$ and $\hat{\lambda}_i(t)$ are respectively defined as

$$e_i = y_i - \hat{y}_i, \quad \dot{\hat{\lambda}}_i(t) = \begin{cases} 2\alpha_i|e_i| & if\ |e_i| > \delta_i \\ 0 & otherwise \end{cases}, \quad (33)$$

α_i are positive constants, $\hat{\lambda}_i(0)$ can be any positive constants. Similarly, it can be concluded that if $\sum_{i=1}^{n-1}(b_i - b_ny_i)^2 \neq 0$ is satisfied in some necessary long time interval, then

$$\left(\sum_{i=1}^{n-1}(b_i - b_ny_i)^2\right)^{-1}\sum_{i=1}^{n-1}\left((b_i - b_ny_i)\frac{\hat{\lambda}_i(t)\cdot e_i}{|e_i| + \delta_i}\right) + \hat{y}_n(t) \quad (34)$$

can be thought of the estimate of $y_n(t)$. Thus, the state $x(t)$ can be approximately calculated out.

Remark 13: By the analysis in this section, it can be seen that the parameters a_{ij} $(i, j = 1, \cdots, n)$, b_i $(i = 1, \cdots, n)$ and c_i $(i = 1, \cdots, n)$ can be expanded to any known bounded piecewise differentiable functions of time t.

4. Example and simulation results

Consider the movement of the object described by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} -0.2 & 0.4 & -0.6 \\ 0.1 & -0.2 & 0.3 \\ 0.3 & -0.4 & 0.4 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.25 \\ 0.3 \end{bmatrix}, \quad (35)$$

with $[x_1(0) \quad x_2(0) \quad x_3(0)] = [1 \quad 1.5 \quad 2.5]$.

The observer is constructed as

$$\begin{cases} \dot{\hat{y}}_1(t) = -0.6 - 0.6y_1 + 0.4y_2 - 0.3y_1^2 + 0.4y_1y_2 \\ \quad + (0.5 - 0.3y_1)\hat{y}_3 + \frac{\hat{\lambda}_1(t)(y_1 - \hat{y}_1)}{|y_1 - \hat{y}_1| + \delta_1}, \ \hat{y}_1(0) = 0.4 \\ \dot{\hat{y}}_2(t) = 0.3 + 0.1y_1 - 0.6y_2 - 0.3y_1y_2 + 0.4y_2^2 \\ \quad + (0.25 - 0.3y_2)\hat{y}_3 + \frac{\hat{\lambda}_2(t)(y_2 - \hat{y}_2)}{|y_2 - \hat{y}_2| + \delta_2}, \ \hat{y}_2(0) = 0.6 \\ \dot{\hat{y}}_3(t) = -(0.3y_1 - 0.4y_2 + 0.4)\hat{y}_3 - 0.3\hat{y}_3^2, \ \hat{y}_3(0) = 1 \end{cases} \quad (36)$$

where $y_1 = \frac{x_1}{x_3}$ and $y_2 = \frac{x_2}{x_3}$ are the available image data; $\hat{\lambda}_1(t)$ and $\hat{\lambda}_2(t)$ are respectively defined as

$$\dot{\hat{\lambda}}_1(t) = \begin{cases} 2\alpha_1|y_1 - \hat{y}_1| & if\ |y_1 - \hat{y}_1| > \delta_1 \\ 0 & otherwise \end{cases}, \quad \hat{\lambda}_1(0) = 2, \quad (37)$$

$$\dot{\hat{\lambda}}_2(t) = \begin{cases} 2\alpha_2|y_2 - \hat{y}_2| & if\ |y_2 - \hat{y}_2| > \delta_2 \\ 0 & otherwise \end{cases}, \quad \hat{\lambda}_2(0) = 2. \quad (38)$$

Thus, $y_3 = \frac{1}{x_3}$ can be estimated by $\hat{y}(t) = \hat{y}_3(t) + e_3'(t)$, where e_3' is defined as

$$e_3' = \frac{1}{(0.5-0.3y_1)^2+(0.25-0.3y_2)^2}\left((0.5-0.3y_1)\frac{\hat{\lambda}_1(t)(y_1-\hat{y}_1)}{|y_1-\hat{y}_1|+\delta_1} + (0.25-0.3y_2)\frac{\hat{\lambda}_2(t)(y_2-\hat{y}_2)}{|y_2-\hat{y}_2|+\delta_2}\right) \quad (39)$$

The sampling period is chosen as 0.05. The simulation is done by Matlab. The parameters are chosen as $\alpha_1 = \alpha_2 = 5$, $\delta_1 = \delta_2 = 0.3$.

Suppose the measured image data is corrupted by 1% with random noise. Comparison between the proposed new observer and the IBO in [4] is performed. Because of the trade-off of the estimating error and the converging speed, the estimating error is compared based on the same converging speed. Figure 1 shows the difference between y_3 and its estimate $\hat{\hat{y}}_3$ by using the new observer. Figure 2 shows the difference between y_3 and its estimate $\hat{\hat{y}}_3$ by using the IBO in [4]. It can be seen that the tracking error in Figure 1 is much smaller than that in Figure 2, i.e. the performance of the new observer is better than that of the IBO. Since the performance of the extended Kalman filter (EKF) is similar to that of IBO (see [4]), it can be concluded that the performance of the new observer is also better than that of the EKF. Further, it can be seen that the formulation of the new observer is much simpler than that of the IBO, not to say EKF. And the *a priori* knowledge about the noise and the bound of the state is not needed in the new method. Therefore, it can be concluded that the new observer is superior to the traditional IBO and EKF.

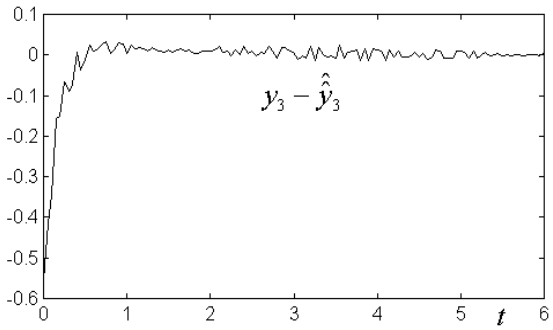

Fig. 1 The difference between y_3 and its estimate $\hat{\hat{y}}_3$ by using the new observer.

5. Conclusions

In this paper, we consider the position identification problem of the perspective system, where the motion parameters are assumed known. The formulated problem can be converted into the observation of a dynamical system with nonlinearties. A new observer, which is inspired by the sliding mode techniques, is proposed to identify the space position of a moving object. The information about the upper bound of the state is not required, and it is adaptively updated online. The new observer converges very fast, and the estimation process can be monitored online. The attraction of the new method lies in that the algorithm is very simple and easy to be implemented. Further, simulation results show the robustness to measurement noises and the superiority to the traditional ones. The new method is also extended to the observation problem for the general n-dimensional perspective systems.

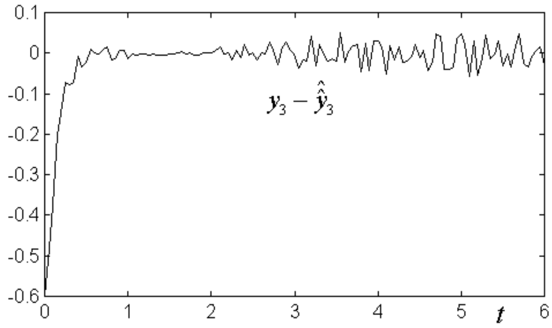

Fig. 2 The difference between y_3 and its estimate $\hat{\hat{y}}_3$ by using the IBO in [4].

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