

# Redundancy Controllable System and Control of Snake Robots Based on Kinematic Model

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## Abstract

We define the redundancy controllable system of hyper-redundant mechanical systems. We derive the condition that the hyper-redundant snake robots become redundancy controllable, and the control law with considering the redundancy. We also propose a concept of a unit and the system design strategy of the snake robots. Simulation results are shown.

## 1 Introduction

Unique and interesting gait of the snakes makes them able to crawl, climb a hill, climb a tree by winding and move on very slippery floor [1]. It is useful to consider and understand the mechanism of the gait of the snakes for mechanical design and control law of snake robots.

Hirose has long investigated snake robots and produced several snake robots, and he models the snake by a wheeled link mechanism with no side slip [2]. Some other snake-like mechanisms are developed in [3] and [4]. Burdick and Chirikjian discuss the sidewinding locomotion of the snake robots based on the kinematic model [5]. Ostrowski and Burdick analyze the controllability of a class of nonholonomic systems that the snake robots are included on the basis of the geometric approach [6]. The feedback control law for the snake head's position using Lyapunov method has been developed by Prautesch et al. on the basis of the wheeled link model [7]. They point out the controller can stabilize the head position of the snake robot to its desired value, but the configuration of it converges to a singular configuration. From the model we find that the snake robot does not have the redundant degrees of freedom, and this leads to the difficulty in the control objective of the singular configuration avoidance.

In this paper we define the redundancy controllable system and propose control law and structure design methodology of redundant snake robots based on the wheeled link model. We find that introduction of links without wheels and shape controllable points in the snake robot's body makes the system redundancy controllable. In this case the head's velocity of the snake

robot does not determine all joint velocities of the robot uniquely. We introduce the cost function related to the measure for the singularity and the manipulability of the system, and construct a controller with considering the redundancy. Using redundancy, it becomes possible to accomplish both the main objective of controlling the position and the posture of the snake robot head and the shape of the snake robot, and the sub-objective of the singular configuration avoidance. From simulation results we find that the crawling motion of the snake robot is natural.

## 2 Redundancy Controllable System

Let  $\mathbf{q} \in R^{\bar{n}}$  be the state vector,  $\mathbf{u} \in R^{\bar{p}}$  be the input vector,  $\mathbf{w} \equiv S\mathbf{q} \in R^{\bar{q}}$  be the state vector to be controlled,  $S$  be a selection matrix whose row vectors are independent unit vectors related to generalized coordinates,  $A(\mathbf{q}) \in R^{\bar{m} \times \bar{q}}$ ,  $B(\mathbf{q}) \in R^{\bar{m} \times \bar{p}}$ . We define that the system

$$A(\mathbf{q})\dot{\mathbf{w}} = B(\mathbf{q})\mathbf{u}, \quad \mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2 \quad (1)$$

is redundancy controllable if  $\bar{p} > \bar{q}$  (redundancy I),  $\bar{p} > \bar{m}$  (redundancy II),<sup>1</sup> the matrix  $A$  is full column rank,  $B$  is full row rank, and there exists an input  $\mathbf{u}_1$  which accomplishes the main objective of the convergence of the vector  $\mathbf{w}$  to the desired state  $\mathbf{w}_d$  ( $\mathbf{w} \rightarrow \mathbf{w}_d$ ,  $\dot{\mathbf{w}} \rightarrow \dot{\mathbf{w}}_d$ ) and an input  $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$  which accomplishes the increase (or decrease) of a cost function  $V(\mathbf{q})$ , which is related to the sub-objective, compared to the input  $\mathbf{u}_1$  and does not disturb the main objective. For a snake robot based on the wheeled link model we discuss a condition that the system is redundancy controllable.

## 3 Kinematic Model of Snake Robots

We consider a redundant  $n$ -link snake robot. Let  $n$  be the number of links,  $m$  be the number of wheeled

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<sup>1</sup> In the case of  $\bar{m} = \bar{p}$ , if the state vector to be controlled  $\dot{\mathbf{w}}$  in (1) is given, the input  $\mathbf{u}$  is determined uniquely. In this sense the system is not redundant, so we introduce the redundancy II.





where  $B^*$  is a pseudo-inverse matrix of  $B$ ,  $\boldsymbol{\eta} = \nabla_{\boldsymbol{\theta}} V(\boldsymbol{q}) = [\partial V/\partial \theta_1 \cdots \partial V/\partial \theta_{n-1-s}]$  is the gradient of the cost function  $V(\boldsymbol{q})$  with respect to the vector  $\boldsymbol{\theta}$  related to the input vector  $\boldsymbol{u}$ , and  $\alpha \geq 0$ ,  $K > 0$ . The first term of the right side of (13) is the control input term to accomplish the main objective of the convergence of the state vector  $\boldsymbol{w}$  to the desired value  $\boldsymbol{w}_d$ . As the second term  $(I - B^*B)\alpha\boldsymbol{\eta}$  belongs to the null space of the matrix  $B$ , we obtain

$$B\boldsymbol{u} = A\{\dot{\boldsymbol{w}}_d - K(\boldsymbol{w} - \boldsymbol{w}_d)\}. \quad (14)$$

As the vector  $B\boldsymbol{u}$  can be expressed as a linear combination of column vectors of the matrix  $A$ , the condition of the existence of the solution (10) is satisfied. The second term in (13) does not disturb the dynamics of the controlled vector  $\boldsymbol{w}$ . As there is no interaction between  $\boldsymbol{w}$  and  $\boldsymbol{\theta}$ , we find that the control law (13) accomplishes the sub-objective. Actually we can derive

$$\begin{aligned} \dot{V}(\boldsymbol{q}) &= (\partial V/\partial \boldsymbol{w})\dot{\boldsymbol{w}} + (\partial V/\partial \boldsymbol{\theta})\dot{\boldsymbol{\theta}} \\ &= (\partial V/\partial \boldsymbol{w})\dot{\boldsymbol{w}} + \boldsymbol{\eta}^T B^* A\{\dot{\boldsymbol{w}}_d - K(\boldsymbol{w} - \boldsymbol{w}_d)\} \\ &\quad + \boldsymbol{\eta}^T (I - B^*B)\alpha\boldsymbol{\eta}. \end{aligned} \quad (15)$$

As  $I - B^*B \geq 0$  [8], we find that the second term of the input (13) accomplishes the increase of the cost function  $V$ .

The closed-loop system is expressed as

$$A\{(\dot{\boldsymbol{w}} - \dot{\boldsymbol{w}}_d) + K(\boldsymbol{w} - \boldsymbol{w}_d)\} = 0. \quad (16)$$

If the matrix  $A$  is full column rank, the uniqueness of the solution is guaranteed. The solution of (16) is given as

$$\dot{\boldsymbol{w}} - \dot{\boldsymbol{w}}_d + K(\boldsymbol{w} - \boldsymbol{w}_d) = 0$$

and we find that the controller ensures the convergence of the controlled state vector to the desired value ( $\boldsymbol{w} \rightarrow \boldsymbol{w}_d$ ). A set of joint angles which satisfies  $\text{rank} A < q$  ( $A$  is not full column rank) means the singular configuration, for example a straight line ( $\phi_i = 0, i = 1, \dots, n-1$ ).

## 6 Simulation

To demonstrate the validity of the proposed control law simulations have been carried out. In this simulation we set  $B^* = B^T(BB^T)^{-1}$  and

$$V = a'(\det(A^T A)) + b'(\det(BB^T)) \quad (17)$$

where  $a', b' > 0$ . The first term of the right side of (17) implies the measure of the singular configuration. The second term of the right side of (17) is related to the manipulability of the system.

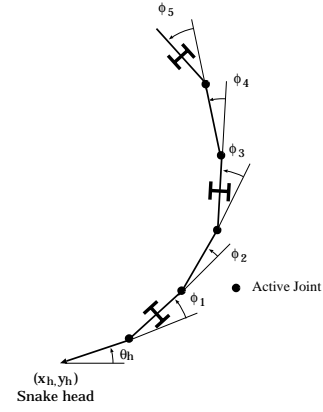


Fig. 4 A 6-link snake robot

For simplicity we consider a 6-link snake robot with 3 wheeled links and does not introduce the shape controlled points ( $s = 0$ ) (Fig. 4). In this case the matrix  $A$  is square. We set the initial condition  $\boldsymbol{w}(0), \boldsymbol{\theta}(0)$  and the desired condition  $\boldsymbol{w}_d(t)$  as  $\boldsymbol{w}(0) = [0 \ 0.1 \ \frac{3\pi}{4}]^T$ ,  $\boldsymbol{\theta}(0) = [\frac{\pi}{100} \ \frac{\pi}{90} \ \frac{\pi}{80} \ \frac{\pi}{70} \ \frac{\pi}{60}]^T$ ,  $\boldsymbol{w}_d(t) = [t \ 0 \ \pi]^T$ ,  $\dot{\boldsymbol{w}}_d(t) = [1 \ 0 \ 0]^T$ , and  $l = 0.05[\text{m}]$ ,  $K = \text{diag}(3, 3, 3)$ . We set coefficients of the cost function  $V$  as  $a' = a/l^2, b' = b/l^6$  in order to normalize with respect to the link length  $l$ . Figs. 5-7 show the transient responses. The left column in each figure shows transient responses for  $x_h - x_{h,d}[\text{m}], y_h - y_{h,d}[\text{m}], \theta_h - \theta_{h,d}[\text{rad}], \det A/l, \sqrt{\det(BB^T)}/l^3$  and the right column shows transient responses for  $u_1, \dots, u_5$ .

Fig. 5 shows the responses for  $\alpha = 0$  (case 1). In this case the controller does not use the redundancy. From the figure we find that the snake head tracks the desired state, but  $\det A$  converges to zero. In case 2 and 3 we set  $\alpha = 9$ . In these cases the controller uses the redundancy so as to attain the increase of the cost function  $V$ . We set  $a = 5, b = 1$  (case 2) and  $a = 2, b = 1$  (case 3). In Figs. 6 and 7 the snake head converges to the desired state. From Fig. 6 (case 2) the avoidance of the singular point is accomplished, but the amplitude of the input is very big and the frequency of the input is too high. It is impossible to realize the obtained input. From Fig. 7 (case 3) the snake robot becomes singular about 15 times per second, but the amplitude and the frequency of the input are acceptable. Fig. 8 shows the movement of the snake robot for each case. We find the crawling of the snake in the case 3. In the case 1 the snake robot converges to a singular configuration of the straight line. In the case 2 the movement of the snake robot is quite far from that of natural snakes.

We find that the crawling movement of the snake robot around the singular configuration of the straight line is natural in view of the restriction of the input power and that the redundancy is important for the movement.

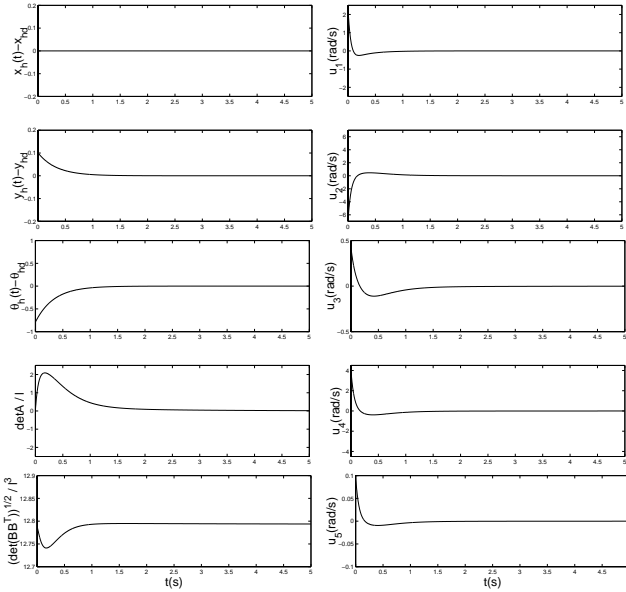


Fig. 5 Transient responses for the controller without considering the redundancy (case 1 :  $\alpha = 0$ )

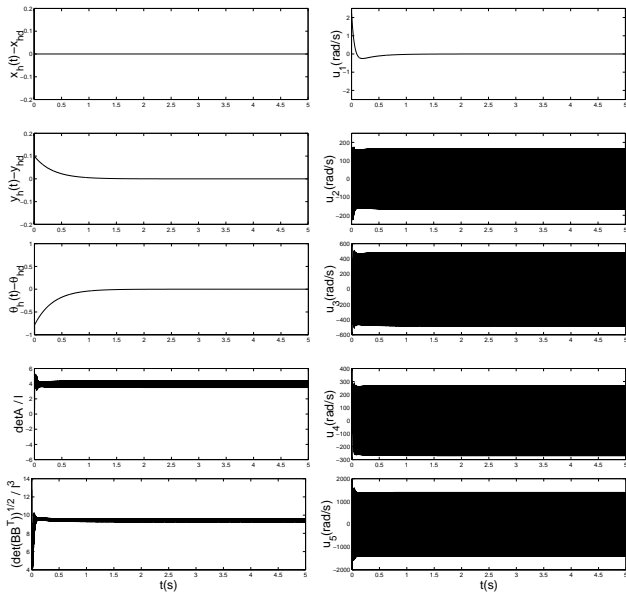


Fig. 6 Transient responses with considering the redundancy and a big weight for the singularity avoidance (case 2 :  $\alpha = 9, a = 5, b = 1$ )

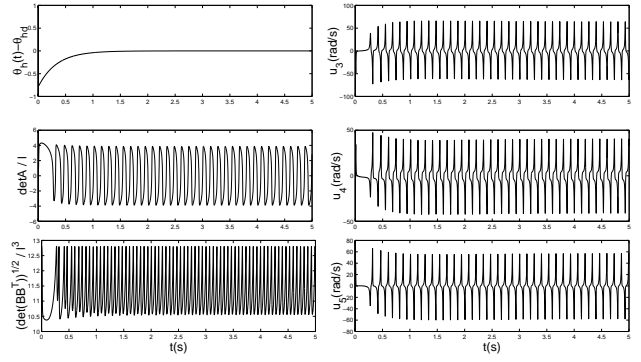
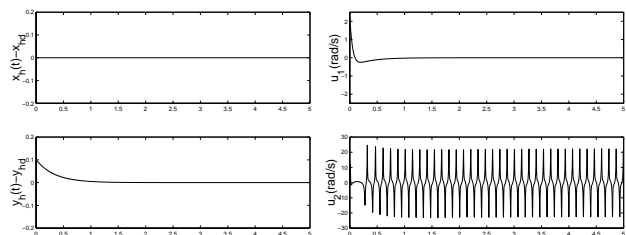


Fig. 7 Transient responses with considering the redundancy and a small weight for the singularity avoidance (case 3 :  $\alpha = 9, a = 2, b = 1$ )

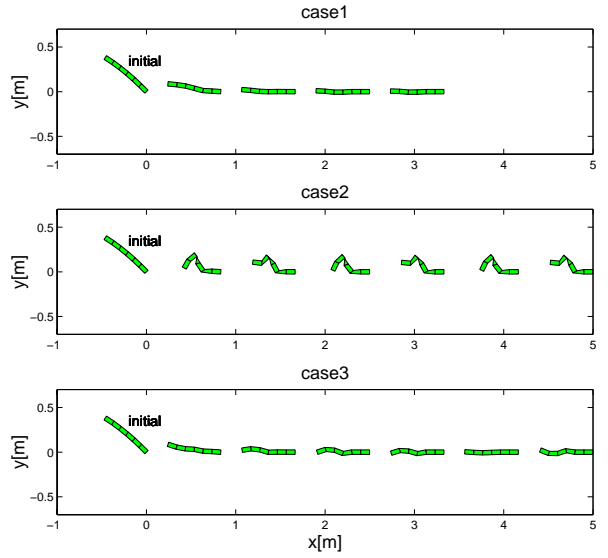


Fig. 8 Movement of the snake robot

Fig. 9 A 8-link snake robot that is constructed by connecting two Type(4,2) units

Next, we consider an 8-link snake robot that is constructed by connecting two Type(4,2) units [9] as

shown in Fig. 9. The Type(4,2) unit has four links, two wheeled links and no shape controllable points. The connected point is the shape controllable point. In this case  $\mathbf{w} = [x_h \ y_h \ \theta_h \ \phi_4]^T$  and the matrix  $A$  is square. We set the initial condition  $\mathbf{w}(0), \boldsymbol{\theta}(0)$  and the desired condition  $\mathbf{w}_d(t)$  as  $\mathbf{w}(0) = [0 \ 0 \ \frac{3\pi}{4} \ \frac{\pi}{90}]^T$ ,  $\boldsymbol{\theta}(0) = [\frac{\pi}{120} \ \frac{\pi}{110} \ \frac{\pi}{100} \ \frac{\pi}{80} \ \frac{\pi}{70} \ \frac{\pi}{60}]^T$ ,  $\mathbf{w}_d = [t \ 0 \ \pi \ \phi_{4_d}]^T$ ,  $\dot{\mathbf{w}}_d = [1 \ 0 \ 0 \ \dot{\phi}_{4_d}]^T$ , and  $l = 0.05[\text{m}]$ ,  $K = \text{diag}(3, 3, 3, 3)$ . We set coefficients of the cost function  $V$  as  $a' = a/l^4, b' = b/l^8$  in order to normalize with respect to the link length  $l$ .

In the case of  $\alpha = 0, \phi_{4_d} = 0$  (case 4) the controller does not use the redundancy and the desired value for the shape controllable point is zero. From the simulation result we find that the snake head tracks the desired trajectory, but  $\det A$  converges to zero. In this case we find that the snake robot converges to a singular configuration of a straight line [10].

In the case of  $\alpha = 0, \phi_{4_d} = \frac{\pi}{10} \cos(9t)$  (case 5). the controller does not use the redundancy but the desired value of the shape controllable point is not zero. From the simulation result we find that the snake head tracks the desired trajectory without converging to the singular configuration and the movement of the snake robot is like the *side winding motion* [2] of snakes (Fig. 10).

In the case of  $\alpha = 1, a = 1, b = 0.85, \phi_{4_d} = 0$  (case 6). the controller uses the redundancy but the desired value of the shape controllable point is zero. From the simulation result we find that the snake head tracks the desired trajectory and the snake robot crawls without converging to the singular configuration (Fig. 10).

From simulation results we find that the second term of the control law (13) can ensure the singularity avoidance and the vibratory motion of the shape controllable point can avoid convergence of the singular configuration.

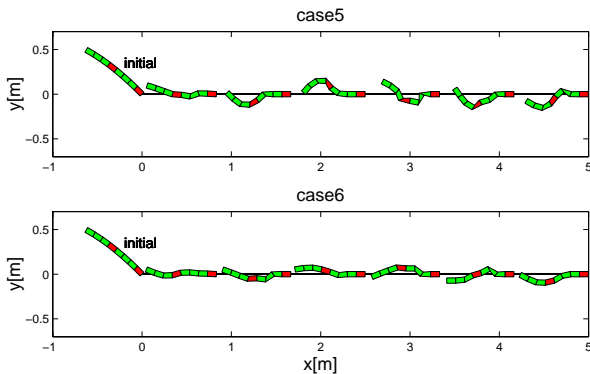


Fig. 10 Movements of the snake robot (case 5, 6)

## 7 Conclusion

We derive the condition that the system for the snake robot becomes redundancy controllable, and we introduce the wheel free links in the snake robot body so

as to satisfy the condition. Using the redundancy, it becomes possible to accomplish both the main objective of controlling the position and the posture of the robot head and the shape controlled points, and also the sub-objective of the singular configuration avoidance. We find that the crawling movement of the snake around the singular configuration of the straight line is natural in view of the restriction of the input power. Though the desired values for the shape controllable points should determine so as to accomplish the given tasks, the relation between them is not clear. We should discuss how to determine the desired values for the shape controllable points related to the various given tasks. As the future works, we should expand the obtained results to the dynamic model.

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