Output Tracking Controller Design for MIMO Nonlinear Systems with Higher-Order and Unmatched Uncertainties

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Abstract

Based on the combination of the sliding mode control with fuzzy control and the simple adaptation laws, this paper presents a new and feasible design algorithm to synthesize a robust fuzzy sliding mode controller which can easily tackle the output tracking control problem of MIMO nonlinear systems in the presence of higher-order and unmatched uncertainties without the knowledge of the upper bounds on the norm of the uncertainties. A series of computer simulations are included to verify the effectiveness of the proposed design algorithm.

Keywords: sliding mode control, fuzzy control, output tracking, unmatched uncertainties, adaptation law.

1. Introduction

A considerable number of researchers have studied the output tracking control problem of nonlinear systems from differential perspectives. An appealing approach is based on elementary differential geometric methods which are summarized in Isidori's outstanding book [1]. In particular, the theory [1,2] that involves the exact linearization of the input-output response has received considerable attention. Generally speaking, the feedback linearization technique requires the accurate mathematical model of the plant to achieve exact linearization of the closed-loop system. This presents a limitation of the theory: in the presence of modeling errors or uncertainties exact cancellations are not possible. By the technique of input-output linearization, generalized uncertainties include the matched uncertainties and the unmatched (mismatched) uncertainties [3-5] in the feedback linearization. For many practical systems, unmatched uncertainties are common in control practice. Therefore, the design of a robust controller that deals with a nonlinear system with significant unmatched uncertainties is an important subject for the design of a good and efficient control system in the recent years. So far, the systematic design of feedback linearizable systems with the unmatched uncertainties has been conducted, and three main extensions have been proposed by [7, 8, 9, 11].

In this paper, we study the output tracking problem of multi-input multi-output nonlinear systems with higher-order and unmatched uncertainties when the upper bounds on the norm of the uncertainties may not be easily known. For a robust control strategy to treat this class of systems, the sliding mode control has attracted a lot of attention, and significant progress has been made in recent years. However, the conventional sliding mode control produces the discontinuous control input and causes the chattering phenomenon, which is particularly undesirable, such that the actuator mechanism may be damaged by the rapid switching. Some remedies for this problem have been proposed in literature [12-13]. Kim and Lee [12] design a fuzzy controller with fuzzy sliding surface to reduce the high-frequency chattering; however, their results are valid while the controlled nonlinear systems do not include the uncertainties. Chiang and Tzeng [13] provide a dynamic sliding mode controller, which can alleviate the chattering phenomenon, for the output tracking problem of SISO nonlinear sytsems with higher-order and unmatched uncertainties.

In the light of the combination of the sliding mode control with fuzzy control and the simple adaptation laws, this paper presents a systematic and feasible design algorithm to synthesize a robust fuzzy sliding mode controller which can easily tackle the output tracking control problem of MIMO nonlinear systems in the presence of higher-order and unmatched uncertainties without the knowledge of the upper bounds on the norm of the uncertainties. The fuzzy controller output is used to replace the sign function which is usually used in the conventional sliding mode control. Thus, the chattering phenomenon inherent to a sliding mode control can be avoided. Moreover, due to the complexity of the structure of the uncertainties, we propose the simple adaptation laws for the upper bounds on the norm of the uncertainties, which may not be easily obtained.

2. Preliminaries and Problem Formulation

Consider a class of MIMO nonlinear systems in the presence of the unmatched uncertainties of the following form:

$$
\begin{aligned}
\dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x)\,u_i + \Theta(x) && (1)\\
&= f(x) + G(x)\,u + \Theta(x) \qquad x(t_0) = x_0 &&\\
y_i &= h_i(x) \qquad\qquad i = 1, \cdots, m &&
\end{aligned}
$$

where $x(\cdot): R_+ \to R^n$ is the state vector, $u(\cdot), y(\cdot): R_+ \to R^m$ are the system input vector and output vector, respectively. $f(\cdot), g_i(\cdot): R^n \to R^n,\ i = 1, \cdots, m$ are sufficiently smooth vector field, $h_i(\cdot): R^n \to R,\ i = 1, \cdots, m$ are sufficiently smooth scalar functions, and $\Theta(\cdot): R_+ \to R^n$ represent uncertainties continuously differentiable with respect to x. Apparently, $G(x) = [\ g_1(x), \cdots, g_m(x)\ ]^T$ is the nominal matrix.

We are interested in the problem of finding a control vector u that would result in asymptotic convergence of the output y to the desired trajectory y_d.

Definition 1: A multivariable nonlinear system of the form (1) is said to have a (vector) relative degree $r \equiv (r_1, \cdots, r_m)$ at a point x_0 if

(i) $L_{g_j} L_f^k h_i = 0$ for all $1 \le j \le m$, for all $1 \le i \le m$, for all $k < r_i - 1$, and for all x in a neighborhood of x_0.

(ii) The $m \times m$ matrix

$$
A(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix} \tag{2}
$$

is nonsingular at $x = x_0$

Proposition 1[1]**:** Suppose a system has a (vector) relative degree $r \equiv (r_1, \cdots, r_m)$ at x_0, then $r_1 + \cdots + r_m \le n$. Let

$$
\begin{aligned}
z_{i1} &= h_i(x)\\
&\ \vdots && (3)\\
z_{ir_i} &= L_f^{r_i-1} h_i(x) \qquad i = 1, \cdots, m
\end{aligned}
$$

if $r = r_1 + \cdots + r_m$ is strictly less than n, it is always to find $n - r$ smooth functions $\eta_{r+1}, \cdots, \eta_n$ such that the mapping

$$
\bar{z} = [\ z_{11}, \cdots, z_{1r_1}, \cdots, z_{m1}, \cdots, z_{mr_m}, \eta_{r+1}, \cdots, \eta_n\ ]^T \tag{4}
$$

has a Jacobean matrix which is nonsingular at x_0. The $\eta_{r+1}, \cdots, \eta_n$ are chosen to satisfy

$$
L_{g_j} \eta_k = 0 \tag{5}
$$

for all $1 \le j \le m,\ r+1 \le k \le n$, and all x around x_0. Now, we set

$$
\begin{aligned}
z &= [\ z_{11}, \cdots, z_{1r_1}, \cdots, z_{m1}, \cdots, z_{mr_m}\ ]^T\\
&= [\ h_1, L_f h_1, \cdots, L_f^{r_1-1} h_1, \cdots, h_m, L_f h_m, \cdots, L_f^{r_m-1} h_m\ ]^T\\
\eta &= [\ \eta_{r+1}, \cdots, \eta_n\ ]^T && (6)
\end{aligned}
$$

According to Proposition 1, there exists a diffeomorphic coordinate transformation $(z, \eta) = T(x)$ which transforms the nominal system of (1) into the normal form:

$$
\begin{aligned}
\dot{z}_{i1} &= z_{i2}\\
\dot{z}_{i2} &= z_{i3} && (7a)\\
&\ \vdots\\
\dot{z}_{i(r_i-1)} &= z_{ir_i}
\end{aligned}
$$

$$\dot{z}_{ir_i} = b_i + \sum_{j=1}^{m} a_{ij} u_j\,,\ \ i=1,\cdots,m,\ \ j=1,\cdots,m \tag{7b}$$

$$\xi = Q(z,\xi)\,, \tag{7c}$$

$$y_i = z_{i1}\,,\ \ i=1,\cdots,m \tag{7d}$$

Since $A(x)$ is nonsingular in the domain of definition, then the feedback control law $u \in R^m$ can be represented as

$$u = A^{-1}(z,\xi)[\,w - b(z,\xi)] \tag{8}$$

where

$b(z,\xi) = \left[\, L_f^{r_1} h_1, \cdots, L_f^{r_m} h_m \,\right]^T = [\, b_1, \cdots, b_m]^T$ and $w = [\,w_1, \cdots, w_m]^T$. From (2) and (7b), it is obvious that

$$\dot{z}_{ir_i} = w_i\,,\ \ i=1,\cdots,m. \tag{9}$$

Now the question is how to synthesize a feedback controller (8) by means of the auxiliary control input $w = [\,w_1, \cdots, w_m]^T$ such that the robust stability of the MIMO nonlinear system with the uncertainties $\Theta(x)$ as shown in (1) can be guaranteed.

Assumption 1: The zero dynamics $\xi = Q(0,\xi)$ is exponentially stable in the domain of definition, the function $Q(z,\xi)$ is Lipschitz in z, and uniformly in ξ.

If the coordinate transformation $(z,\xi) = T(x)$ is performed to the system (1) subjected to the unmatched uncertainties, we obtain the following form:

$$\dot{z}_{i1} = z_{i2} + \Delta\varphi_{i1}(z,\xi)\,,$$

$$\dot{z}_{i2} = z_{i3} + \Delta\varphi_{i2}(z,\xi)$$

$$\vdots$$

$$\dot{z}_{i(r_i-1)} = z_{ir_i} + \Delta\varphi_{i(r_i-1)}(z,\xi) \tag{10}$$

$$\begin{aligned}\dot{z}_{ir_i} &= b_i + \sum_{j=1}^{m} a_{ij} u_j + \Delta\varphi_{ir_i}(z,\xi)\\ &= w_i + \Delta\varphi_{ir_i}(z,\xi)\end{aligned} \tag{11}$$

$$\xi = Q(z,\xi) + \Delta\Omega(z,\xi) \tag{12}$$

$$y_i = h_i = z_{i1}\,,\ \ i=1,\cdots,m \tag{13}$$

$$\begin{aligned}A(x) &= \begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mm} \end{bmatrix}\\ &= \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix}\end{aligned} \tag{14}$$

$$u = [\, u_1 \ \ \cdots \ \ u_m]^T = A^{-1}[w - b] \tag{15}$$

$$\Delta\varphi_{ij} = L_\Theta L_f^{j-1} h_i \qquad i=1,\cdots,m,\quad j=1,\cdots,r_i$$

$$\Delta\Omega = [\, L_\Theta \xi_{r+1}, \cdots, L_\Theta \xi_n \,]^T \circ T^{-1}(z,\xi)\,.$$

3. Design of A Robust Fuzzy Sliding Mode Controller and Stability Analysis

In this section, we will synthesize the robust fuzzy sliding mode controller, which combines the sliding mode control with fuzzy control and the simple adaptation laws, to deal with the output tracking problem of MIMO nonlinear systems with higher-order and unmatched uncertainties without the knowledge of the upper bounds on the norm of the uncertainties.

Now, let $y_d = [y_{d1}, \cdots y_{dm}]^T$ be the desired output trajectories, and the following assumption is needed.

Assumption 2: The desired trajectories y_{di}, $i=1,\cdots,m,$ and their first r_i derivatives are uniformly bounded, that is

$$\left\| \left(y_{di}, y_{di}^{(1)}, \cdots, y_{di}^{(r_i)} \right) \right\| \le B_{di} \tag{16}$$

for bounded positive constants B_{di}.

Define the output tracking errors to be

$$e_{ij} = y_i^{(j-1)} - y_{di}^{(j-1)}, \qquad i=1,\cdots,m,\ j=1,\cdots,r_i \tag{17}$$

and $e = [\, e_{11}, \cdots, e_{1r_1}, \cdots, e_{m1}, \cdots, e_{mr_m} \,]^T$. Then the output of the system and its derivatives can be expressed as

$$y_i = h_i = z_{i1}$$

$$y_i^{(1)} = \dot{z}_{i1} = z_{i2} + \Delta \text{¤}_{i1}$$

$$\vdots$$

$$y_i^{(r_i)} = w_i + \Delta\Phi_i \tag{18}$$

where

$$\Delta\Phi_i = \Delta\,\text{¨}_{i1} + \Delta\,\text{¨}_{i2} + \cdots + \Delta\,\text{¨}_{i(r_i-1)} + \Delta\,\text{¨}_{ir_i},$$

and

$\Delta\,\text{¨}_{ij} = \Delta \text{¤}_{ij}^{(r_i-j)}$, for $i=1,\cdots,m$ and $j=1,\cdots,r_i$.

Assumption 3: If the $\Delta \text{¤}_{ij}$ is the function that has continuous derivatives in the domain of definition. These derivatives are bounded by the polynomial which is combined with both $\|e\|^p$ and $\|\text{¦}\|^k$, $p=0,1,\cdots,N$, $k=1,\cdots,M$. That is,

$$\left\|\Delta\,\text{¨}_{ij}\right\| \le \sum_{p=0}^{N} c_{ip}^{j} \|e\|^p + \sum_{k=1}^{M} d_{ik}^{j} \|\text{¦}\|^k,$$

$$i=1,2,\cdots,m, \quad j=1,\cdots,r_i \tag{19}$$

where c_{ip}^{j} and d_{ik}^{i} are unknown positive constants. N and M are positive integers.

Assumption 4: The norm of the uncertainty vector $\Delta\Omega$ satisfies the following condition:

$\| \Delta\Omega \| \le L$, where L is a bounded positive constant.

Then, we can select an error surface vector $S \in R^m$ as below:

$$S = \begin{bmatrix} s_1 \\ \vdots \\ s_m \end{bmatrix} = \begin{bmatrix} e_{1r_1} + c_{11} e_{1(r_1-1)} + \cdots + c_{1(r_1-1)} e_{11} \\ \vdots \\ e_{mr_m} + c_{m1} e_{m(r_m-1)} + \cdots + c_{m(r_m-1)} e_{m1} \end{bmatrix} \tag{20}$$

The sliding surfaces are defined as S=0, where $c_{ij},\ j=1,\cdots,r_i-1,\ i=1,\cdots,m$, are chosen so that the following polynomials:

$$H_i(s_i) = s^{r_i-1} + c_{i1} s^{r_i-2} + \cdots + c_{i(r_i-1)}, \quad i=1,\cdots,m$$

are Hurwitz. Thus, when the output tracking errors reach the sliding surface S=0, i.e.,

$$e_{i1}^{(r_i-1)} + c_{i1} e_{i1}^{(r_i-2)} + \cdots + c_{i(r_i-1)} e_{i1} = 0, \quad i=1,\cdots,m,$$

it implies that the output tracking errors (17) tend to zero as $t \longrightarrow \infty$.

From (8), (17), and (18), it is obvious that the time derivative of S can be obtained as follows:

$$\dot{S} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} + \begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mm} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} - \begin{bmatrix} y_{d1}^{r_1} \\ \vdots \\ y_{dm}^{r_m} \end{bmatrix}$$

$$+ \begin{bmatrix} c_{11} e_{1r_1} + \cdots + c_{1(r_1-1)} e_{12} \\ \vdots \\ c_{m1} e_{mr_m} + \cdots + c_{m(r_m-1)} e_{m2} \end{bmatrix}$$

$$+ \begin{bmatrix} \Delta \text{¤}_{1r_1} + \left(\Delta \text{¤}_{1(r_1-1)}\right)^{(1)} + \cdots + \left(\Delta \text{¤}_{11}\right)^{(r_1-1)} \\ \vdots \\ \Delta \text{¤}_{mr_m} + \left(\Delta \text{¤}_{m(r_m-1)}\right)^{(1)} + \cdots + \left(\Delta \text{¤}_{m1}\right)^{(r_m-1)} \end{bmatrix} \tag{21}$$

In accordance with (19), we will choose the simple adaptive laws to estimate the upper bounds of these higher-order uncertainties. The simple adaptive laws can be written as follows:

$$\dot{\overline{\Psi}}_{1ip}(e) = q_{1ip} |s_i| \, \|e\|^p \quad p=0,1,2,\cdots,N \qquad i=1,\cdots,m$$

$$\dot{\overline{\Psi}}_{2ik}(\text{¦}) = q_{2ik} |s_i| \, \|\text{¦}\|^k \quad k=1,2,\cdots,M \qquad i=1,\cdots,m$$

Now we are in a position to present the robust fuzzy sliding mode controller as follows:

$$u = A^{-1}[\, w - b \,],$$

$$w_i = y_{di}^{(r_i)} - \sum_{j=1}^{r_i-1} c_j e_{r_i-j+1} + k_i u_f^i - p_i s_i, \quad i=1,2,\cdots,m. \tag{22}$$

where p_i is a positive value. k_i is the feedback gain defined by

$$k_i = \sum_{p=0}^{N} \overline{\Psi}_{1ip} \|e\|^p + \sum_{k=1}^{M} \overline{\Psi}_{2ik} \|\text{¦}\|^k$$

$u_f^i,\ i=1,2,\ldots,m$ will be determined by the following IF-THEN rules:

R^j: IF s_i is $\tilde{F}_{s_i}^{j}$, THEN u_f^i is $\tilde{F}_{u_f^i}^{-j}$,

for $j=-2,-1,0,1,2$, $i=1,\cdots,m$

where $\tilde{F}_{s_i}^{j}$ and $\tilde{F}_{u_f^i}^{j}$ are the membership functions of the fuzzy sets, respectively.

Adopting the max-product compositional rule of inference and the method of the singleton fuzzification [6], it is easy to get the membership function of the output fuzzy set. Then using the method of the center of gravity defuzzification, the crisp output u_f^i is calculated by

$$u_f^i = \frac{\int_{-1.5}^{1.5} u_f^i \cdot \text{«}_i(u_f^i)\,\mathrm{d}u_f^i}{\int_{-1.5}^{1.5} \text{«}_i(u_f^i)\,\mathrm{d}u_f^i}, \quad i=1,\cdots,m \tag{23}$$

After some mathematical manipulations, we obtain

$$u_f^i = \begin{cases} 1, & ¢_i \le -1, \\ \dfrac{2¢_i^3 + 9¢_i^2 + 7¢_i + 3}{6¢_i^2 + 9¢_i + 6}, & -1 < ¢_i \le -0.5, \\ \dfrac{2¢_i^3 + 3¢_i^2 - 2¢_i}{6¢_i^2 + 3¢_i + 3}, & -0.5 < ¢_i \le 0, \\ \dfrac{2¢_i^3 - 3¢_i^2 - 2¢_i}{6¢_i^2 - 3¢_i + 3}, & 0 < ¢_i \le 0.5, \\ \dfrac{2¢_i^3 - 9¢_i^2 + 7¢_i - 3}{6¢_i^2 - 9¢_i + 6}, & 0.5 < ¢_i \le 1, \\ -1, & ¢_i > 1, \end{cases} \tag{24}$$

where $i = 1, \ldots, m, \quad ¢_{\mathrm{i}} = s_i / £_i$. From (24), we have

$$u_f^i(t) = -\mathrm{sgn}(s_i(t)), \text{ if } |s_i| \ge £_i$$

Theorem 1: *For the controlled system* (1) *with the control law defined by* (30)*, and under the assumptions* (1)~(4)*, the closed-loop system composed of* (1), (22),(23) *and* (24) *is globally stable, with the tracking errors converging to a neighborhood of zero.*

Proof. Choose the Lyapunov function as follows:

$$V = \frac{1}{2} S^T S + \frac{1}{2} \sum_{p=0}^{N} \left(q_{1p}^{-1} \tilde{\Psi}_{1p}\right)^T \tilde{\Psi}_{1p} + \frac{1}{2} \sum_{k=1}^{M} \left(q_{2k}^{-1} \tilde{\Psi}_{2k}\right)^T \tilde{\Psi}_{2k}$$

where

$$q_{1p}^{-1} = diag\left[\left(q_{11p}\right)^{-1}, \cdots, \left(q_{1mp}\right)^{-1} \right],$$

$$q_{2k}^{-1} = diag\left[\left(q_{21k}\right)^{-1}, \cdots, \left(q_{2mk}\right)^{-1} \right]$$

$$\tilde{\Psi}_{1p} = \left[\tilde{\Psi}_{11p}, \cdots, \tilde{\Psi}_{1mp} \right]^T, \text{and}$$

$$\tilde{\Psi}_{2k} = \left[\tilde{\Psi}_{21k}, \cdots, \tilde{\Psi}_{2mk} \right]^T.$$

Differentiating V with respect to t and Using (21), we obtain

$$\begin{aligned} \dot{V} &= +s_1 k_1 u_f^1 + \cdots + s_m k_m u_f^m + s_1 \Delta\Phi_1 + \cdots + s_m \Delta\Phi_m \\ &+ \sum_{p=0}^{N} \left(q_{1p}^{-1} \tilde{\Psi}_{1p}\right)^T \dot{\tilde{\Psi}}_{1p} + \sum_{k=1}^{M} \left(q_{2k}^{-1} \tilde{\Psi}_{2k}\right)^T \dot{\tilde{\Psi}}_{2k} - p_1 s_1^2 - \cdots - p_m s_m^2 \end{aligned} \tag{25}$$

If $|s_i| > £_i, \quad i = 1, \cdots, m$, (25) can be modified as follow:

$$\begin{aligned} \dot{V} &\le +s_1 k_1 u_f^1 + \cdots + s_m k_m u_f^m + |s_1| \, \|\Delta\Phi_1\| + \cdots + |s_m| \, \|\Delta\Phi_m\| \\ &+ \sum_{p=0}^{N} \left(q_{1p}^{-1} \tilde{\Psi}_{1p}\right)^T \dot{\tilde{\Psi}}_{1p} + \sum_{k=1}^{M} \left(q_{2k}^{-1} \tilde{\Psi}_{2k}\right)^T \dot{\tilde{\Psi}}_{2k} - p_1 s_1^2 - \cdots - p_m s_m^2 \\ &\le -|s_1| k_1 - \cdots |s_m| k_m + |s_1| \|\Delta\Phi_1\| + \cdots + |s_m| \|\Delta\Phi_m\| \\ &+ \sum_{p=0}^{N} \left(q_{1p}^{-1} \tilde{\Psi}_{1p}\right)^T \dot{\tilde{\Psi}}_{1p} + \sum_{k=1}^{M} \left(q_{2k}^{-1} \tilde{\Psi}_{2k}\right)^T \dot{\tilde{\Psi}}_{2k} - p_1 s_1^2 - \cdots - p_m s_m^2 \\ &= \sum_{p=0}^{N} \left[\left(\tilde{\Psi}_{11p} |s_1| \, \|e\|^p - \tilde{\Psi}_{11p} |s_1| \, \|e\|^p \right) + \cdots + \left(\tilde{\Psi}_{1mp} |s_m| \, \|e\|^p - \tilde{\Psi}_{1mp} |s_m| \, \|e\|^p \right) \right] \\ &+ \sum_{k=1}^{M} \left[\left(\tilde{\Psi}_{21k} |s_1| \, \|/\|^k - \tilde{\Psi}_{21k} |s_1| \, \|/\|^k \right) + \cdots + \left(\tilde{\Psi}_{2mk} |s_m| \, \|/\|^k - \tilde{\Psi}_{2mk} |s_m| \, \|/\|^k \right) \right] \\ &- p_1 s_1^2 - \cdots - p_m s_m^2 \\ &= -p_1 s_1^2 - \cdots - p_m s_m^2 \end{aligned} \tag{26}$$

This means that $s_i(t)$ asymptotically converges to the boundary $\left\{ y_i(t) \mid |s_i(t) \le £_i| \right\}$, and the tracking errors are asymptotically bounded in view of (20).

4. An Example and Simulation Results

The Larange-Euler equation of motion for a single-arm manipulator with two rigid links is

$$\begin{aligned} \begin{bmatrix} ²_1 \\ ²_2 \end{bmatrix} &= \begin{bmatrix} \frac{1}{3} m_1 l^2 + \frac{4}{3} m_2 l^2 + m_2 l^2 \cos(¤) & \frac{1}{3} m_2 l^2 + \frac{1}{2} m_2 l^2 \cos(¤) \\ \frac{1}{3} m_2 l^2 + \frac{1}{2} m_2 l^2 + m_2 l^2 \cos(¤) & \frac{1}{3} m_2 l^2 \end{bmatrix} \begin{bmatrix} \ddot{¯} \\ \ddot{¤} \end{bmatrix} \\ &+ \begin{bmatrix} -m_2 l^2 \dot{¤} \sin(¤) & -\frac{1}{2} m_2 l^2 \dot{¤} \sin(¤) \\ \frac{1}{2} m_2 l^2 \dot{¯} \sin(¤) & 0 \end{bmatrix} \begin{bmatrix} \dot{¯} \\ \dot{¤} \end{bmatrix} \\ &+ \begin{bmatrix} \frac{1}{2} m_1 g l \cos(¯) + \frac{1}{2} m_2 g l \cos(¯ + ¤) + m_2 g l \cos(¯) \\ \frac{1}{2} m_2 g l \cos(¯ + ¤) \end{bmatrix} - \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \end{aligned} \tag{27}$$

where the various physical constants are defined as [3] for comparing results:

m_1 : The first link's mass.

m_2 : The second link's mass.

$¯$: The first joint angle, 1 Kg.

$¤$: The second joint angle, 1 Kg.

$²_1$: The first joint torque.

$²_2$: The second joint torque.

d_1, d_1 : The time-varying disturbance, $2 \big/ (1+t)$.

$/$: The link length, 1m.

First, let us put the system's dynamics in a state-space representation, and choose the state vector as $z = [z_{11} \quad z_{12} \quad z_{21} \quad z_{22}]^T = [¯ \quad \dot{¯} \quad ¤ \quad \dot{¤}]^T$, $y_1 = z_{11}$, and $y_2 = z_{21}$.

Then, the state space representation of the system is expressed as

$$\dot{y}_1 = \dot{z}_{11} = z_{12} + \Delta ¤_{11} = z_{11} + «_{11} ¯^2$$

$$\ddot{y}_1 = \dot{z}_{12} = b_1 + \Delta b_1 + (\Delta ¤_{11})' = b_1 + \Delta b_1 + «_{11}(2 \dot{¯})$$

$$\dot{y}_2 = \dot{z}_{21} = z_{22} + \Delta ¤_{21} = z_{22} + «_{21} ¯ ¤$$

$$\ddot{y}_2 = \dot{z}_{22} = b_2 + \Delta b_2 + (\Delta ¤_{21})' = b_2 + \Delta b_2 + «_{21}(¯ \dot{¤} + \dot{¯} ¤)$$

$$e_{ij} = y_i^{(j-1)} - y_{id}^{(j-1)}, \quad i = 1,2, \quad j = 1,2,$$

where

$$y_{1d} = \sin(t) + 0.1 \cdot \sin(3t)$$

$$y_{2d} = 0.1 \cdot \sin(2t) + 0.1 \cdot \sin(4t)$$

From Section 3, we assume that the $b_1, b_2, a_{11}, a_{12}, a_{21}$, and a_{22} are known exactly, and we can obtain the control law (22) where $a_{11} = 8$, $a_{21} = 16$, $p_1 = 8$,

$p_2 = 12$, $q_{10} = 6$, $q_{11} = 6$, $q_{20} = 6$, $q_{21} = 6$, $\pounds_1 = 0.35$, $\pounds_2 = 0.45$, $c_{10} = 2$, $c_{11} = 2$, $c_{20} = 2$, $c_{21} = 2$, $\Psi_{10}(0) = 3$, $\Psi_{11}(0) = 3$, $\Psi_{20}(0) = 3$, $\Psi_{21}(0) = 3$, and u_f^i is defined as (24). We set the uncertain coefficients $\text{«}_{11} = [-0.1 \quad 0.1]$, $\text{«}_{12} = [-0.1 \quad 0.1]$, $\text{«}_{21} = [-0.1 \quad 0.1]$, and $\text{«}_{22} = [-0.1 \quad 0.1]$. The simulation results are given in Figs. 1-2, and the sampling time is chosen as $\Delta t = 0.004$ sec.

5. Conclusion

The output tracking problem of MIMO nonlinear systems with higher-order and unmatched uncertainties has been studied in this paper. Using the Lyapunov method, the proposed robust fuzzy sliding mode control algorithm is proved to be globally stable, and the tracking error converges to a neighborhood of zero. A series of simulation results demonstrate that the robust fuzzy sliding mode controller presented in this paper performs satisfactorily.

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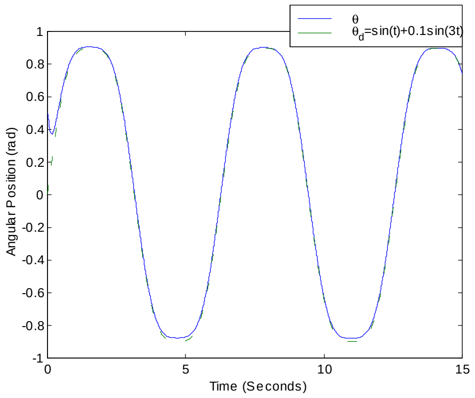

Fig. 1. The tracking trajectory of joint 1

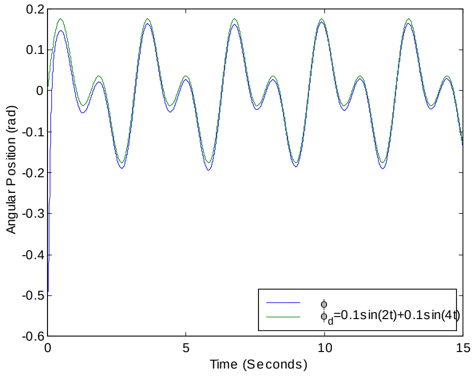

Fig. 2. The tracking trajectory of joint 2