Adaptive Pole-Placement Control of MIMO Stochastic Systems

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Abstract

In this paper, an adaptive pole-placement control algorithm using delayed normalized least mean squares (DNLMS) estimation with inverse logarithm step size is proposed for controlling the multi-input multi-output (MIMO) stochastic systems. The DNLMS estimation is used to identify the plant parameters and then a pole-placement controller is designed and adaptively adjusted using the estimates. Based on the assumptions of a mixing input condition and the satisfaction of a certain law of large numbers, the estimation with inverse logarithm step size has almost sure convergence. Further, by using the perturbation scheme, the control algorithm facilitates the establishment of the adaptive pole-placement control and prevents the closed-loop control system from occurring unstable pole-zero cancellation. An analysis shows that the proposed control algorithm guarantees parameter estimation convergence and system stability in the mean squares sense, with the output of the system approaching zero if there are no uncertainties and disturbances and converging to a neighborhood of zero if they exist. A series of simulations for controlling a mobile robot system are given to illustrate the effectiveness of the proposed scheme. The results show that the proposed control scheme is fairly robust for systems with uncertainties as well as has satisfactory performance characteristics.

1 Introduction

Adaptive pole-placement control algorithm for controlling the multivariable multi-input multi-output (MIMO) stochastic systems has been the subject of investigation for over a decade. The algorithm by adjusting the control parameters adaptively receives attracting considerable attention in various practical applications with better performance than those of constant gain feedback control law, due to its promising potential for the tasks of tackling the presence of unknown parameters or unknown variation in plant parameters and bounded disturbances. To facilitate the control of the system by the algorithm, most of the researches [1] have proceeded along the direction with dead-zone technique. The feature is that when the tracking error is small, it stops to tune the plant parameters and lasts otherwise. Thus, it can prevent the estimates from approaching infinity and guarantee that the tracking error of the system approaches zero when there are no disturbances and uncertainties, and converges to a neighborhood of zero if they exist. Further, Ioannou *et al.* [2] incorporated the σ-modification to the adaptive control law so that the estimates will not drift to infinity in the presence of bounded disturbances and uncertainties. It is seen that the most important issue for adaptive control schemes with self-tuning is to estimate the unknown plant parameters. Among many estimations, least mean squares (LMS) estimation has been used in many fields of applications including communications, system identification, adaptive control, and signal processing because of its simplicity and computation efficiency. In some practical applications, we should use the Delayed LMS (DLMS) estimation, because the signals between states will be delayed. In general, the estimated model is homogeneous if it can represent exactly the controlled system; otherwise, it is nonhomogeneous. In order to obtain good system performance, it is well-known that the step size of the estimation for updating the estimates plays an important role in the convergence of the estimation and stability of the controlled system. The relationship between the step size and convergence speed, and the effect of the constant step size on the stability were studied in [5]. Most researches considered the convergence of the estimation for stochastic systems based on the mean of the weight vector and the excess mean squares error [3]. These estimations are interesting and useful but, from a realization viewpoint, they do not guarantee the convergence of the estimation. In addition, control processes often involve nonnegligible time delays between any particular incident in the behavior of the quantities being controlled, and the result of the operation of the controlling system brought about by this incident. Long, Ling, and Proakis [5, 6] analyzed the convergence properties for the delayed least mean squares (DLMS) estimation based on the independence assumption among the successive inputs. Kabal [4] derived a stability bound to ensure convergence of the mean of the weight vector, where they assumed that the LMS weight vector is statistically independent of the input vector. Long *et al.* [5, 6] considered the case the input vector arises from a tapped delay line implementation to derive a convergence bound for the estimation.

Instead of using the mean of the weight vector and excess mean squares error to prove the convergence of the estimation, some authors proposed the almost sure convergence for the standard LMS estimation with constant step size and with decreasing step size in the nonhomogeneous case [7]. With the assumptions of the mixing input condition and the satisfaction of a certain law of large numbers, Ahn and Voltz [8] demonstrated the DNLMS estimation with a decreasing step size $\mu(k) = a/k$, $a > 0$, had almost sure convergence for the nonhomogeneous case. As for the pole-placement control schemes in the literature, Nassiri-Toussi and Ren [9] proposed the stabilizing adaptive pole-placement control scheme for controlling the MIMO stochastic systems using the weighted extended least squares (WELS) estimation. As for indirect adaptive pole-placement control algorithms, it is seen for certain estimation methods that the indirect adaptive pole-placement control algorithm for deterministic or stochastic system is globally stable in the absence of external persistent excitation if no pole-zero cancellation occurs in the estimated model. In [9], the authors showed that with sufficiently rich external excitations, the parameter estimation with certain perturbation was consistent and the stability of the closed-loop system is hold.

In this paper, an adaptive pole-placement control using delayed normalized least mean squares (DNLMS) estimation with inverse logarithm step size is proposed for controlling the multi-input multi-output (MIMO) stochastic systems. The DNLMS estimation is used to estimate the parameters of the plant model with best curve fitting data on-line such that the control parameters can be adaptively adjusted simultaneously. Based on the assumptions of a mixing input condition and the satisfaction of a certain law of large numbers, the DNLMS estimation with inverse logarithm step size is shown to have almost sure convergence. The convergence property of the estimation is proved for both the nonhomogeneous and homogeneous cases. Moreover, it is shown that the estimation for the homogeneous case gives better performance than that for the nonhomogeneous case, because the estimates will converge to the true value. By using the DNLMS estimation to adaptively tune the parameters of the plant model, an adaptive pole-placement control law is derived for the MIMO stochastic controlled system. If the estimates occur pole-zero cancellations, a certain perturbation technique is used to ensure the coprimeness of the plant model constituted by the estimates. An analysis shows that this algorithm can guarantee parameter estimation convergence and system stability in the mean squares sense. A series of simulations are performed to demonstrate the effectiveness of the proposed control scheme. The results show that the proposed control scheme is fairly robust to uncertainties as well as improved performance characteristics.

2 Pole-placement control algorithm

Let the plant to be controlled be modeled by

$$A(q^{-1})\mathbf{y}_k = q^{-d}B'(q^{-1})\mathbf{u}_k + C(q^{-1})\boldsymbol{\epsilon}_k \tag{1}$$

where $\mathbf{y}_k$, $\boldsymbol{\epsilon}_k \in \Re^p$, $\mathbf{u}_k \in \Re^m$, and q is the forward shift operator. Assume that the output $\mathbf{y}_k$, white noise $\boldsymbol{\epsilon}_k$, and the control input $\mathbf{u}_k$ are processes adapted to the increasing family of σ-fields $\mathcal{F}_k$ subject to the probability space $(\Omega, P, \mathcal{F})$. Let

$$\begin{aligned} A(q^{-1}) &= \mathbf{I}_p + A_1 q^{-1} + \cdots + A_\nu q^{-\nu} \\ B'(q^{-1}) &= B_0 + B_1 q^{-1} + \cdots + B_\nu q^{-\nu} \\ C(q^{-1}) &= \mathbf{I}_p + C_1 q^{-1} + \cdots + C_\nu q^{-\nu} \end{aligned} \tag{2}$$

where $B_0 \neq 0$ and $B(q^{-1}) \triangleq q^{-d}B'(q^{-1})$. Some assumptions on the plant model (1) are made as follows: (A1) The number of outputs, p, is equal to the number of inputs, m. (A2) The noise $\boldsymbol{\epsilon}_k$ is an $\Re^p$-valued $\{\mathcal{F}_k\}$-martingale difference process satisfying

$$\begin{aligned} E\left[\boldsymbol{\epsilon}_k \boldsymbol{\epsilon}_k^T \mid \mathcal{F}_{k-1}\right] &> \mathbf{0} \\ \sup_k \left\{E\left[\|\boldsymbol{\epsilon}_k\|^c \mid \mathcal{F}_{k-1}\right]\right\} &< \infty \end{aligned} \tag{3}$$

for all $k \geq 0$ and some $c > 2$, where $E[\cdot]$ is the expected value of $\cdot$. (A3) $C(q^{-1})$ is taken to have roots on or inside the unit circle. (A4) The input-output transfer function $H(z) = A(z^{-1})^{-1}B(z^{-1})$ is an irreducible MFD, which implies that $A(q^{-1})$ and $B(q^{-1})$ are left coprime and thus the controlled plant is controllable with respect to the control input $\mathbf{u}_k$. Further, it is strictly proper and satisfies

$$\det H(z) \neq 0 \qquad \text{almost all z} \tag{4}$$

In the following, we first assume that $A(q^{-1})$, $B(q^{-1})$, and $C(q^{-1})$ are known. The controller is given by

$$\mathbf{u}_k = -S(q^{-1})R(q^{-1})^{-1}\mathbf{y}_k + G(q^{-1})\mathbf{r}_k \tag{5}$$

or

$$R_L(q^{-1})\mathbf{u}_k = S_L(q^{-1})\mathbf{y}_k + G'(q^{-1})\mathbf{r}_k \tag{6}$$

where $R_L(q^{-1})^{-1}S_L(q^{-1}) = S(q^{-1})R(q^{-1})^{-1}$. The polynomial matrices $R(q^{-1})$, $S(q^{-1})$ and the transfer function matrix $G(q^{-1})$ (or $R_L(q^{-1})$, $S_L(q^{-1})$, and $G'(q^{-1})$) are determined by the pole-placement control design. Here, we let $\mathbf{r}_k$ be the $\Re^m$-valued "deterministic" reference input satisfying the following assumption:

(A5) $\mathbf{r}_k$ is uniformly bounded and is either deterministic ($\mathcal{F}_0$-measurable) or independent of $\boldsymbol{\epsilon}_k$.

From (1) and (6), the closed-loop system can be obtained as:

$$\mathbf{y}_k = B_R(q^{-1})\left(R_L(q^{-1})A_R(q^{-1})+S_L(q^{-1})B_R(q^{-1})\right)^{-1}G'(q^{-1})\mathbf{r}_k + R(q^{-1})\left(A(q^{-1})R(q^{-1})+B(q^{-1})S(q^{-1})\right)^{-1}C(q^{-1})\boldsymbol{\epsilon}_k \quad (7)$$

$$\mathbf{u}_k = A_R(q^{-1})\left(R_L(q^{-1})A_R(q^{-1})+S_L(q^{-1})B_R(q^{-1})\right)^{-1}G'(q^{-1})\mathbf{r}_k - S(q^{-1})\left(A(q^{-1})R(q^{-1})+B(q^{-1})S(q^{-1})\right)^{-1}C(q^{-1})\boldsymbol{\epsilon}_k \quad (8)$$

where $B_R(q^{-1})A_R(q^{-1})^{-1} = A(q^{-1})^{-1}B(q^{-1})$.

It is seen that the pole-placement control objective is to have

$$B_R(q^{-1})\left(R_L(q^{-1})A_R(q^{-1})+S_L(q^{-1})B_R(q^{-1})\right)^{-1}G'(q^{-1}) = B(q^{-1})M(q^{-1})^{-1} \quad (9)$$

where $M(q^{-1})$ is a prespecified stable polynomial matrix such that $M(0) = \mathbf{I}_m$. One can achieve this by letting $B_R(q^{-1}) = B(q^{-1})$, $R_L(q^{-1})A_R(q^{-1}) + S_L(q^{-1})B_R(q^{-1}) = M(q^{-1})$ and $G'(q^{-1}) = \mathbf{I}_m$ and, therefore, computing $(R_L(q^{-1}), S_L(q^{-1}))$ in terms of the right MFD of the system. However, to identify the system, equation (1) corresponding to the left MFD of the plant transfer functions can be transformed readily into a predictor equation and hence can be written as a linear regression model suitable for identification. This suggests that we instead solve the Diophantine equation $A(q^{-1})R(q^{-1}) + B(q^{-1})S(q^{-1}) = P(q^{-1})$, where $P(q^{-1})$ is some stable polynomial matrix such that $P(0) = \mathbf{I}_p$ to find $R(q^{-1})$ and $S(q^{-1})$ in terms of $A(q^{-1})$ and $B(q^{-1})$.

Consider any left MFD $R_L(q^{-1})^{-1}S_L(q^{-1}) = S(q^{-1})R(q^{-1})^{-1}$. Then there exist polynomial matrices, $Q(q^{-1})$ and $Q'(q^{-1})$ both with dimension $(m \times p)$, such that

$$\left(R_L(q^{-1})A_R(q^{-1})+S_L(q^{-1})B_R(q^{-1})\right)Q(q^{-1}) = Q'(q^{-1})P(q^{-1}) \quad (10)$$

Therefore, one cannot have solutions for the pair of Diophantine equations $A(q^{-1})R(q^{-1}) + B(q^{-1})S(q^{-1}) = P(q^{-1})$ and $R_L(q^{-1})A_R(q^{-1}) + S_L(q^{-1})B_R(q^{-1}) = M(q^{-1})$ satisfying $R_L(q^{-1})^{-1}\ S_L(q^{-1}) = S(q^{-1})R(q^{-1})^{-1}$, unless $M(q^{-1})$ and $P(q^{-1})$ meet $M(q^{-1})Q(q^{-1}) = Q'(q^{-1})P(q^{-1})$.

For simplicity, we can take $M(q^{-1}) = \lambda_M(q^{-1})\mathbf{I}_m$ for the pole-placement equation $R_L(q^{-1})A_R(q^{-1}) + S_L(q^{-1})B_R(q^{-1}) = M(q^{-1})$, where $\lambda_M(q^{-1})$ is a polynomial matrix such that every root of $\lambda_M(q^{-1})$ is the same as that of $\det(M(q^{-1}))$. In the following, we then propose the pole-placement control design scheme without adaptability. Let $B_R(q^{-1})A_R(q^{-1})^{-1} = A(q^{-1})^{-1}B(q^{-1})$ be some irreducible right MFD of the input-output transfer function such that $A_R(q^{-1})$ is row-reduced. Then, the pole-placement controller can be written as

$$\mathbf{u}_k = -S(q^{-1})\boldsymbol{\zeta}_k + \lambda_M(q^{-1})\mathbf{z}_k \quad (11)$$

where $R(q^{-1})\boldsymbol{\zeta}_k = \mathbf{y}_k$ and $M(q^{-1})\mathbf{z}_k = \mathbf{r}_k$. The control polynomial matrices $R(q^{-1})$ and $S(q^{-1})$ can be solved by the following pole-placement control scheme:

$$A(q^{-1})R(q^{-1}) + B(q^{-1})S(q^{-1}) = (\lambda_M(q^{-1}) + \Delta)\mathbf{I}_p \quad (12)$$

where $\partial_{r_j}S(q^{-1}) < \partial_{r_j}A_R(q^{-1})$ and $\partial\lambda_M(q^{-1}) \geq \partial A_R(q^{-1}) + \partial B(q^{-1}) - 1$. $\Delta := \Delta(C(q^{-1})) \in \Re$ is chosen such that (i) $\partial\lambda_M(q^{-1}) \geq \partial B(q^{-1}) + \partial A_R(q^{-1}) - 1$, (ii) $\lambda_M(q^{-1})+\Delta$ is a stable polynomial, and (iii) $\det(C(q^{-1}))$ and $\lambda_M(q^{-1})+\Delta$ are coprime. It is seen that the perturbation term Δ in the control scheme (12) is to ensure that the corresponding adaptive control parameters can have unique solution regardless of the external inputs.

Let $\Delta_0, \Delta_1, \cdots, \Delta_{\nu^p} \in \Re$ be ν^p+1 distinct real numbers such that $\lambda_M(q^{-1}) + \Delta_i$ is stable for every i, and define

$$\chi(C(q^{-1}), \Delta) := \left|\det\left(\mathcal{S}\left(\det(C(q^{-1})), \lambda_M(q^{-1}) + \Delta\right)\right)\right| \quad (13)$$

where $\mathcal{S}(\lambda_1, \lambda_2)$ is the Sylvester matrix corresponding to polynomials $\lambda_1(q^{-1})$ and $\lambda_2(q^{-1})$, and $\Delta(\cdot)$ is defined as

$$\Delta(C(q^{-1})) = \begin{cases} 0 \ \text{ if } \chi(C,0) > h_0 (>0) \\ \min\left(\arg\left(\max\limits_{\Delta\in\{\Delta_0,\cdots,\Delta_{\nu^P}\}} \chi(C,\Delta)\right)\right) & \text{otherwise} \end{cases} \quad (14)$$

In what follows, we assume that $A(q^{-1})$, $B(q^{-1})$, and $C(q^{-1})$ are unknown. Hence, the DNLMS estimation with inverse logarithm step size is presented to estimate the plant parameters $\hat{A}(k, q^{-1})$, $\hat{B}(k, q^{-1})$, and $\hat{C}(k, q^{-1})$ for the adaptive pole-placement control design. With the assumptions of the mixing input condition and the satisfaction of a certain law of large numbers, the DNLMS estimation with inverse logarithm step size is shown to have almost sure convergence. The convergence property of the estimation is given both for the homogeneous and nonhomogeneous cases. Consider first the former case, suppose that the controlled plant (1) can be modeled exactly, then we can show that the estimation parameters will converge to the true value matrix. The DNLMS estimation used as an adaptive solution is shown as follows: Give the ARMAX model (1) and express it as the linear regressor form in terms of $\boldsymbol{\phi}_k$:

$$\mathbf{y}_k = \Theta^T\boldsymbol{\phi}_k \quad (15)$$

where

$$\Theta = [-A_1, \cdots, -A_\nu, B_1, \cdots, B_\nu, \mathbf{I}_p, C_1, \cdots, C_\nu]^\top \quad (16)$$

$$\boldsymbol{\phi}_k = [\mathbf{y}_{k-1}, \cdots, \mathbf{y}_{k-\nu}, \mathbf{u}_{k-1}, \cdots, \mathbf{u}_{k-\nu}, \boldsymbol{\epsilon}_k, \boldsymbol{\epsilon}_{k-1}, \cdots, \boldsymbol{\epsilon}_{k-\nu}]^\top \quad (17)$$

where $\boldsymbol{\phi}_k \in \Re^N$ is a regression vector and $\Theta \in \Re^{N\times p}$ the estimation matrix for $N = p(2\nu+1) + m\nu$. The problem is to find Θ such that the delayed normalized least mean squares (DNLMS) error ξ

$$\xi = E\left[\left(\frac{\|\mathbf{y}_k - \Theta^T\boldsymbol{\phi}_k\|}{\|\boldsymbol{\phi}_k\|}\right)^2\right] \quad (18)$$

is minimized. In the absence of knowledge about the statistics of $\mathbf{y}_k$ and $\boldsymbol{\phi}_k$ and since the error signal can not be observed until after some fixed delay for some practical applications, we can solve (18) iteratively by the well-known stochastic gradient algorithm:

$$\Theta(k+1) = \Theta(k) + \mu(k-d)\frac{\boldsymbol{\phi}_{k-d}}{\|\boldsymbol{\phi}_{k-d}\|^2}\left\{\mathbf{y}_{k-d}^T - \boldsymbol{\phi}_{k-d}^T\Theta(k-d)\right\} \quad (19)$$

where $\mu(k)$ is the time-varying step size and is taken as an inverse logarithm. The step size can reflect a trade-off between the amount of weight misadjustment and the speed of adaptation. In this case, it is seen that a small step size gives small misadjustment in the steady state but a slow convergence rate, and vice versa. In order to prove the convergence properties of the DNLMS estimation, we first take the following assumptions of the mixing input condition and the satisfaction of a certain law of large numbers:

(A6) (The Mixing Input Condition): The input vector sequence is mixing in the sense that there exist a finite integer n and an $\alpha > 0$ such that for any constant nonzero N and vector $\mathbf{h}$, the following holds for all k:

$$\frac{1}{n}\sum_{i=0}^{n-1}\left\{\frac{\mathbf{h}^T\boldsymbol{\phi}_{k+i}}{\|\boldsymbol{\phi}_{k+i}\|}\right\}^2 \geq \alpha\|\mathbf{h}\|^2 \quad (20)$$

(A7) (The Law of Large Numbers): The sequences

$$\left\{\frac{\boldsymbol{\phi}_k\boldsymbol{\phi}_k^T}{\|\boldsymbol{\phi}_k\|^2}\right\} \text{ and } \left\{\frac{\boldsymbol{\phi}_k\mathbf{y}_k^T}{\|\boldsymbol{\phi}_k\|^2}\right\}$$

satisfy the law of large numbers in the sense that

$$\lim_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}\frac{\boldsymbol{\phi}_i\boldsymbol{\phi}_i^T}{\|\boldsymbol{\phi}_i\|^2} = E\left[\frac{\boldsymbol{\phi}_i\boldsymbol{\phi}_i^T}{\|\boldsymbol{\phi}_i\|^2}\right] \triangleq \mathbf{R} \quad \text{a.s.} \quad (21)$$

$$\lim_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}\frac{\boldsymbol{\phi}_i\mathbf{y}_i^T}{\|\boldsymbol{\phi}_i\|^2} = E\left[\frac{\boldsymbol{\phi}_i\mathbf{y}_i^T}{\|\boldsymbol{\phi}_i\|^2}\right] \triangleq \mathbf{w} \quad \text{a.s.} \quad (22)$$

respectively, where the symbols a.s. mean almost surely, i.e., save on a set having probability measure zero.

Under (21) and (22), notice that

$$\begin{aligned}\lim_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}\frac{\boldsymbol{\phi}_i\mathbf{y}_i^T - \boldsymbol{\phi}_i\boldsymbol{\phi}_i^T\Theta^*}{\|\boldsymbol{\phi}_i\|^2} &= \mathbf{w} - \mathbf{R}\Theta^* \\ &= \mathbf{0} \text{ a.s.} \quad (23)\end{aligned}$$

Then it is seen that $\Theta^* = \mathbf{R}^{-1}\mathbf{w}$. These assumptions, which do not include the independence assumption, are more realistic; and the almost sure convergence property guarantees convergence of the weight matrix. Let the input signal be mixing, then (21) and (22) are sufficient conditions for almost sure convergence of the DNLMS estimation with

$$\mu(k-d) = \begin{cases} a & \text{if } k-d \leq 1 \\ \frac{a}{\log(k-d)} & \text{otherwise} \end{cases} \quad (24)$$

for any constant $a > 0$. It is seen that the value of a in (24) influences the convergence rate of the weight matrix but it does not influence the convergence property. Nevertheless, the value of a in $\mu(k-d) = a$ if $k-d \leq 1$ influences both the convergence rate and the steady-state error. Note that throughout this paper, we take $a = 1$. Now, let the error estimate matrix $\tilde{\Theta}(k)$ be defined as

$$\tilde{\Theta}(k) = \Theta(k) - \Theta^* \quad (25)$$

Subtracting Θ^* from both sides of (19) and using (25) gives the DNLMS estimation as:

$$\begin{aligned}\tilde{\Theta}(k+1) = \tilde{\Theta}(k) - \mu(k-d)\frac{\boldsymbol{\phi}_{k-d}\boldsymbol{\phi}_{k-d}^T}{\|\boldsymbol{\phi}_{k-d}\|^2}\tilde{\Theta}(k-d) + \\ \mu(k-d)\frac{\boldsymbol{\phi}_{k-d}}{\|\boldsymbol{\phi}_{k-d}\|^2}\left(\mathbf{y}_{k-d}^T - \boldsymbol{\phi}_{k-d}^T\Theta^*\right) \quad (26)\end{aligned}$$

Then, we have $\lim_{k\to\infty}\Theta(k) = \Theta^*$ almost surely both for the homogeneous and nonhomogeneous cases. In particular, $\sup_k\|\Theta(k)\| \leq L < \infty$ almost surely. Hence,

$$\lim_{k\to\infty}\hat{A}(k,q^{-1}) = A(q^{-1}) \quad (27)$$

$$\lim_{k\to\infty}\hat{B}(k,q^{-1}) = B(q^{-1}) \quad (28)$$

$$\lim_{k\to\infty}\hat{C}(k,q^{-1}) = C(q^{-1}) \quad (29)$$

almost surely. In particular, $\|\hat{C}(k,q^{-1})\|_F \leq c < \infty$ almost surely, where c is a constant.

Further, the adaptive pole-placement controller (11) should be rewritten as

$$\hat{\mathbf{u}}_k = -\hat{S}(k,q^{-1})\hat{\boldsymbol{\zeta}}_k + \lambda_M(q^{-1})\mathbf{z}_k \quad (30)$$

where $\hat{R}(k,q^{-1})\hat{\boldsymbol{\zeta}}_k = \hat{\mathbf{y}}_k$, $M\mathbf{z}_k = \mathbf{r}_k$, and $\hat{\mathbf{u}}_k$ and $\hat{\mathbf{y}}_k$ are the estimated model control input and output sequences, respectively. Then the control polynomial matrices $\hat{R}(q^{-1})$ and $\hat{S}(q^{-1})$ can be solved by the following pole-placement control scheme:

$$\hat{A}(k,q^{-1})\hat{R}(k,q^{-1}) + \hat{B}(k,q^{-1})\hat{S}(k,q^{-1}) = (\lambda_M(q^{-1}) + \hat{\Delta}(k))\mathbf{I}_p \quad (31)$$

where $\hat{A}(k,q^{-1})$ and $\hat{B}(k,q^{-1})$ are the estimates of $A(q^{-1})$ and $B(q^{-1})$, respectively, and $\hat{\Delta}(k) \in \{0, \Delta_0, \cdots, \Delta_{\nu^p}\}$ is defined from the definitions of $\Delta(\cdot)$ and $\chi(\cdot,\cdot)$ as

$$\hat{\Delta}(k) = \begin{cases} 0 & \text{if } \chi(\hat{C}(k,q^{-1}),0) > 2h_0 \\ \Delta(\hat{C}(k,q^{-1})) & \text{else if } \chi(\hat{C}(k,q^{-1}),\Delta(\hat{C}(k,q^{-1}))) \geq \\ & (1+\tau_1)\chi(\hat{C}(k,q^{-1}),\hat{\Delta}(k-1)) \ (\tau_1 > 0) \\ \hat{\Delta}(k-1) & \text{otherwise} \end{cases} \quad (32)$$

where the given initial estimate $\hat{C}(0,q^{-1})$ is uniformly stable. Note that $\hat{\mathbf{u}}_k$ will not be well defined if $\hat{A}(k,q^{-1})$ and $\hat{B}(k,q^{-1})$ are not left coprime. Therefore, the DNLMS estimation must guarantee that $\mathcal{S}(\hat{A}(k,q^{-1}),\hat{B}(k,q^{-1}))$, $\forall k$, is nonsingular. If $\hat{A}(k,q^{-1})$ and $\hat{B}(k,q^{-1})$ are not left coprime, we should replace $(\hat{A}(k,q^{-1}),\hat{B}(k,q^{-1}))$ with a certain perturbation of $(\hat{A}(k,q^{-1}),\hat{B}(k,q^{-1}))$ such that

$$\left|\det\left(\mathcal{S}(\hat{A}(k,q^{-1}),\hat{B}(k,q^{-1}))\right)\right| > h_2 \quad (33)$$

for all k and some constant $h_2 > 0$ [9].

In the following theorem, we analyze the stability of the closed-loop system controlled by the proposed adaptive pole-placement control scheme

Theorem 1 : With the Assumptions (A1)–(A4), when the adaptive control algorithm (26) and (30)-(32) is applied to the system (1), it ensures the following statements:

1. The adaptive system is stable in the mean squares sense, i.e.,

$$\sup_n \frac{1}{n}\sum_{k=0}^{n}(\|\hat{\mathbf{y}}_k\|^2 + \|\hat{\mathbf{u}}_k\|^2) < \infty \text{ a.s.} \qquad (34)$$

where $\hat{\mathbf{u}}_k$ and $\hat{\mathbf{y}}_k$ are the actual input and output sequences, respectively.

2. $\lim_{k\to\infty}(\hat{R}(k,q^{-1}), \hat{S}(k,q^{-1})) = (R(q^{-1}), S(q^{-1}))$ almost surely. In particular, $\hat{\mathbf{u}}_k$ and $\hat{\mathbf{y}}_k$ converge in the mean squares sense to

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=0}^{n}(\|\hat{\mathbf{u}}_k - \mathbf{u}_k\|^2 + \|\hat{\mathbf{y}}_k - \mathbf{y}_k\|^2) = 0 \text{ a.s.} \qquad (35)$$

where $\mathbf{u}_k$ and $\mathbf{y}_k$ are the desired input and output sequences, respectively, that would be generated by the ideal control law (11).

3 Example

Consider the nonholonomic mobile robot system shown in Fig. 1. The position of the robot in an inertia Cartesian space $\{O, X, Y\}$ is completely specified by the vector where $\{x_c, y_c\}$ is the coordinate of the center of mass of the vehicle, and is the orientation of the basis $\{C, X_c, Y_c\}$ with respect to the inertia basis. It is assumed that the robot can only move in the direction normal to the axis of the driving wheels, i.e., the mobile base satisfies the conditions of pure rolling and nonslipping:

$$\dot{y}_c\cos(\theta) - \dot{x}_c\sin(\theta) - d\dot{\theta} = 0. \qquad (36)$$

The kinematic equation in terms of its linear velocity and angular velocity can be obtained as

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -d\sin(\theta) \\ \sin(\theta) & d\cos(\theta) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad (37)$$

where $\mathbf{v} = [v \ w]^\top = [v_1 \ v_2]^\top$. It is seen that (37) is called the steering system of the vehicle. The closed-loop kinematic system becomes

$$\begin{aligned} \dot{x} &= (v_{c1} + e_4)\cos(\theta) - d\sin(v_{c2} + e_5) \\ \dot{y} &= (v_{c1} + e_4)\sin(\theta) - d\cos(v_{c2} + e_5) \\ \dot{\theta} &= v_{c2} + e_5 \end{aligned}$$

where $\mathbf{e}_c = [e_4 \ e_5]^\top$ and $\mathbf{v}_c = [v_{c1} \ v_{c2}]^\top$ denote the velocity tracking error and the desired velocity input, respectively.

The DNLMS estimation with inverse logarithm step size $\mu(k) = 1/\log(k)$ is used to estimate the system parameters and the adaptive pole-placement control design is applied to the mobile robot system. We use the same notations as defined in [11]. The purpose of the control scheme is to achieve the trajectory tracking and the velocity tracking. The vehicle parameters are taken as $m = 10kg$, $I = 5kg - m^2$, $R = 0.5m$, $r = 0.05m$, and $v_r = 0.5m/s$. The reference trajectory is a straight line with initial position at $(1, 2)$ and slope 0.4999. The design parameters and the gain matrix are chosen as $K = [2, 4, 0.25]$ and $K_4 = diag\{10, 10\}$, respectively. Here, the error between the actual velocity and the desired velocity is $-0.0571m/s$ after 10 seconds. Fig. 2 shows the response of the trajectory when the step size of the DNLMS estimation is $1/\log(k)$ and the weights of the velocity error and the position error are taken 6 and 4, respectively. After 5 seconds, the actual trajectory is very close to the desired trajectory, too. Fig. 3 shows the response of position errors when the weights of the velocity error and the position error are taken 6 and 4, respectively. Here, Xe and Ye are convergent to $0.0016m$ and $0.0738m$, respectively, after 10 seconds. Fig. 4 shows the response of the actual and desired velocities. Here, the error between the actual velocity and the desired velocity is $-0.0527m/s$ after 10 seconds. If we take the weights of the velocity error and the position error to be 9 and 1, respectively, the velocity error is small but the position errors are large; moreover, the position might be larger and larger. So, if the weight of the position error is too small, the actual path cannot track the desired path well, i.e., either Xe or Ye is large. From the above results, we do have a good control of the mobile robot. Therefore, the adaptive pole-placement control design using DNLMS estimation with step size $\mu(k) = a/\log(k)$ can improve performance characteristics.

4 Conclusion

In this paper, an adaptive pole-placement control algorithm using DNLMS estimation with inverse logarithm step size has been proposed for controlling the MIMO stochastic systems. The DNLMS estimation is used to identify the plant parameters and then a pole-placement controller is designed and adaptively adjusted using the estimates. Based on the assumptions of a mixing input condition and the satisfaction of a certain law of large numbers, the estimation with inverse logarithm step size has almost sure convergence. Further, by using the perturbation scheme, the control algorithm facilitates the establishment of the adap-

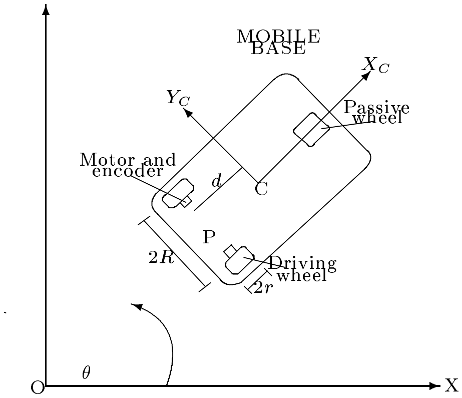

Figure 1: A nonholonomic mobile platform.

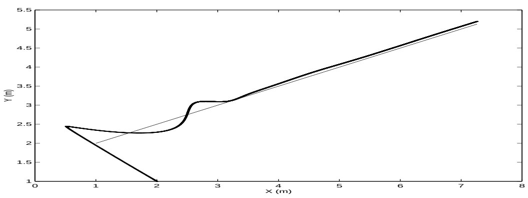

Figure 2: The response of the trajectories: desired (——) and actual (···).

tive pole-placement control and prevents the closed-loop control system from occurring unstable pole-zero cancellation. An analysis shows that the proposed control algorithm guarantees parameter estimation convergence and system stability in the mean squares sense, with the output of the system approaching zero if there are no uncertainties and disturbances and converging to a neighborhood of zero if they exist. A series of simulations for controlling a mobile robot system are given to illustrate the effectiveness of the proposed scheme. From the simulation results, the estimation with step size $\mu(k) = a/\log(k)$ have better performance than that with step size $\mu(k) = a/k$. The results show that the proposed control scheme is fairly robust for systems with uncertainties as well as has satisfactory performance characteristics.

Acknowledge

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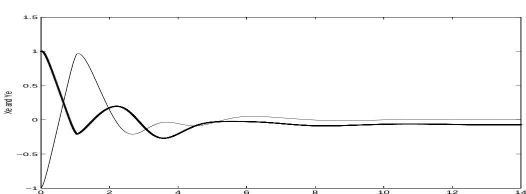

Figure 3: The response of the position errors: Xe (——) and Ye (···).

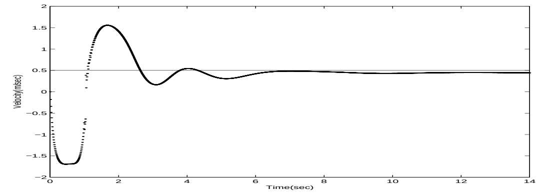

Figure 4: The response of the velocities: desired (——) and actual (···).

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