Estimation based discrete-time sliding control of uncertain nonlinear systems in discrete strict feedback form

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Abstract

In this paper, an on-line discrete-time disturbance estimation method is presented. The proposed method extends the static one-step delay disturbance estimation method by allowing both the estimation accuracy and the error convergence dynamics to be adjusted by the designer. The use of the method in a recursive tracking controller design of an uncertain nonlinear system in discrete strict feedback form and in a system matrix identification problem are examined. A numerical simulation study is also provided.

1 Introduction

In a previous paper [2], we have reported an estimation based continuous-time robust tracking controller design for a class of nonlinear systems with mismatched uncertainties. The equivalent control based disturbance estimators are utilized for on-line disturbance estimation purposes and the estimated values are incorporated into a recursive backstepping controller design in a systematic way.

In this paper, we study an estimation based discrete-time tracking controller design in an attempt to provide a discrete-time counterpart to that of [2]. The literature on the discrete-time on-line disturbance estimation problem is relatively narrow compared to the continuous-time case. Especially, when the disturbance is parameterized by a constant unknown parameter vector, one usually has to impose linear growth conditions on the regression nonlinearities to provide global stability results in case a certainty equivalence term is to be used in the control law for disturbance cancelation [4]. A modified least-squares estimator based on nonlinear data weighting has been proposed in [11] as a remedy to this problem. The incorporation of discrete time parameter or functional estimators into the controller design has also be studied for discrete-time systems in strict feedback form [9], [5]. The strict feedback assumption allows the designer to deal with mismatched uncertainties by a recursive design although it is clearly very restrictive.

The paper is outlined as follows: In Section 2, a discrete-time disturbance estimation method is presented. A recursive discrete-time sliding controller design method which uses the proposed disturbance estimation idea is developed in Section 3. The results of a numerical simulation study are summarized in Section 4. A discrete-time parameter estimator design is given in Section 5. The paper ends in Section 6 with brief concluding remarks.

2 Discrete-Time Disturbance Estimation

2.1 Motivation

Consider a scalar system of the form

$$\dot{x} = u(t) + \delta(t) \tag{1}$$

where $x \in \mathbf{R}$ is the state, $u(t)$ is the control and $\delta(t)$ represents the disturbance for the system. Suppose that the control objective is to stabilize x to zero. If an upper bound on $|\delta(t)|$ is known, the selection of $u = K\ sgn\ x$ with $K > |\delta(t)|$ could easily achieve the control objective. However, if this control is implemented in discrete-time with a sampling time of T, x can only be assured to remain at an $\mathcal{O}(T)$ vicinity of zero. Furthermore, the thickness of the boundary layer in which x is kept is directly related to the gain of the discontinuous control.

As an alternative approach where the sampling issues are taken into considerations during the control design, first discretize Eq. (1) with the assumption that the control is applied through a zero-order-hold so as to obtain

$$x_{k+1} = x_k + Tu_k + T\bar{\delta}_k \tag{2}$$

where $x_k = x(kT)$, $u_k = u(t)$ for $kT \leq t < (k+1)T$,

$$\bar{\delta}_k = \frac{1}{T}\int_{kT}^{(k+1)T} \delta(t)dt \tag{3}$$

and T is the sampling time. Note that, $\bar{\delta}_k$ cannot be computed unless the future values of $\delta(t)$ are known. However, if $\delta(t)$ is smooth $\bar{\delta}_k$ can be predicted by $\bar{\delta}_{k-1}$ which can be computed from

$$\bar{\delta}_{k-1} = \frac{1}{T}[x_k - x_{k-1}] - u_{k-1} \tag{4}$$

with an $\mathcal{O}(T)$ accuracy according to

$$\begin{aligned} \bar{\delta}_k - \bar{\delta}_{k-1} &= \frac{1}{T}\int_{kT}^{(k+1)T} \delta(t)dt - \frac{1}{T}\int_{(k-1)T}^{kT} \delta(t)dt \\ |\bar{\delta}_k - \bar{\delta}_{k-1}| &= 2T\delta_{\max} = \mathcal{O}(T) \end{aligned} \tag{5}$$

where $|d\delta/dt| < \delta_{\max}$.

Incorporating the estimated disturbance to the control law

$$u_k = -\frac{1}{T}x_k - \bar{\delta}_{k-1} \tag{6}$$

one gets

$$x_{k+1} = T[\bar{\delta}_k - \bar{\delta}_{k-1}] = \mathcal{O}(T^2) \tag{7}$$

so that at each sampling instant x is actually forced to an $\mathcal{O}(T^2)$ vicinity of zero. This is an increase in performance compared to the direct discretization of the discontinuous control which would achieve only an $\mathcal{O}(T)$ accuracy. A detailed study of this idea can be found in [7], [8] where it has also been shown that this control leads to an $\mathcal{O}(T^2)$ accuracy in sliding motion for sampled-data systems with control being applied through a zero-order hold.

The main idea of the estimation method presented so far is to calculate the one step previous value of the disturbance using the available quantities and to take it as an estimate for the current disturbance. This results in an $\mathcal{O}(T)$ estimation accuracy provided that the first order derivative of the disturbance is bounded. In Section 2, this idea is extended in two directions: First, if the external disturbance is continuously differentiable to a certain order, using more previous values of the disturbance could actually lead to a better estimation accuracy. Second, the convergence behavior could also be controlled by the designer rather than restricting it to being deadbeat which would then provide additional freedom in shaping the transient behavior of the closed loop system when the estimator is used for controller design purposes.

2.2 Disturbance Estimator Design

Consider a continuous time signal $z(t)$ which is rth-order differentiable; i.e, $|z^{(i)}(t)| \leq Z_i$ for $i = 1, \cdots, r-1$ with some r and constant Z_i's. Let $z_k = z(kT)$ where T denotes the sampling time. Our preliminary objective is to predict z_k using $z_{k-1}, z_{k-2}, \cdots$ without requiring any other information on $z(t)$. Since the only specification known about $z(t)$ is smoothness, an exact predictor cannot be designed, however; it would be desirable to specify an estimation logic which would at least enable the prediction accuracy to be expressed in terms of a free design parameter.

To this end, let's adopt the following notation

$$\begin{aligned} \triangle(q^{-1}) &= 1 - q^{-1} \\ \triangle^2(q^{-1}) &= 1 - 2q^{-1} + q^{-2} \\ &\vdots \\ \triangle^m(q^{-1}) &= \sum_{j=0}^{m}(-1)^j \begin{pmatrix} m \\ j \end{pmatrix} q^{-j} \end{aligned} \tag{8}$$

where q^{-1} is the backward shift operator and

$$\begin{pmatrix} m \\ j \end{pmatrix} = \frac{m!}{(m-j)!j!} \tag{9}$$

By a direct computation, it can be verified that

$$\triangle^i(q^{-1})z_k = \mathcal{O}(T^i) \tag{10}$$

for $i = 1, \cdots, r$. For brevity, only the first two cases are computed below to substantiate the claim

$$|z_k - z_{k-1}| = |\int_{(k-1)T}^{kT} \frac{dz(\tau)}{d\tau}d\tau| \leq Z_1 T \tag{11}$$

$$\begin{aligned} |z_k - 2z_{k-1} + z_{k-2}| &= |\int_{(k-1)T}^{kT}\int_{(k-1)T}^{\tau_1} \frac{d^2 z(\tau_1')}{d\tau_1'^2}d\tau_1' d\tau_1 + \\ &\quad \int_{(k-2)T}^{(k-1)T}\int_{\tau_2}^{(k-1)T} \frac{d^2 z(\tau_2')}{d\tau_2'^2}d\tau_2' d\tau_2| \\ &\leq Z_2 T^2 \end{aligned} \tag{12}$$

As an analogy with the continuous time disturbance estimation technique presented in [3], rewrite the original system of Eq. (2) as follows:

$$\hat{x}_{k+1} = x_k + Tu_k + T\hat{\bar{\delta}}_k \tag{13}$$

where $\hat{\bar{\delta}}$ represents the estimate of $\bar{\delta}$. Subtracting Eq. (13) from Eq. (2), one gets

$$\tilde{x}_{k+1} = T\tilde{\bar{\delta}}_k \tag{14}$$

where $\tilde{\bar{\delta}} = \bar{\delta} - \hat{\bar{\delta}}$ is the estimation error and $\tilde{x} = x - \hat{x}$.

Assume that the external disturbance is rth-order differentiable with an arbitrary r so that

$$\triangle^i(q^{-1})\bar{\delta}_k = \mathcal{O}(T^i) \tag{15}$$

for any $i = 1, \cdots, r$ as shown before. The proposed disturbance estimator filters $\tilde{x}$ according to

$$\hat{\bar{\delta}}_k = \frac{\sum_{j=0}^{m-1} \gamma_j q^{-j}}{\triangle^m(q^{-1})}\tilde{x}_k \tag{16}$$

where $\gamma_0, \cdots, \gamma_{m-1}$'s are free parameters to be selected and m is an integer satisfying $1 \leq m \leq r$. Using Eq. (14) and Eq. (16), the disturbance estimation error can easily be shown to satisfy

$$P(q^{-1})\tilde{\bar{\delta}}_k = R_k \tag{17}$$

where

$$P(q^{-1}) = 1 + \sum_{j=1}^{m}[(-1)^j \begin{pmatrix} m \\ j \end{pmatrix} + T\gamma_{j-1}]q^{-j} \tag{18}$$

$$R_k = \triangle^m(q^{-1})\bar{\delta}_k = \mathcal{O}(T^m) \tag{19}$$

The roots of $P(q^{-1})$ can be placed arbitrarily with proper selections of γ_i's and the order of the disturbance estimator provides additional freedom in adjusting R_k. For a sufficiently smooth $\delta(t)$, an increase in m results in a decrease in the magnitude of R_k since it is $\mathcal{O}(T^m)$ whereas γ_i's can further be used to control the convergence behavior of the estimation error variable. If the control law of Eq. (6) is modified by

$$u_k = -\frac{1}{T}x_k - \hat{\bar{\delta}}_k \tag{20}$$

one gets

$$x_{k+1} = T[\bar{\delta}_k - \hat{\bar{\delta}}_k] = \mathcal{O}(T^{m+1}) \tag{21}$$

so that the magnitude of the boundary layer in which x resides at each sampling instant is $\mathcal{O}(T^{m+1})$. However, it is also worth noting that the intersampling time behavior will still be $\mathcal{O}(T^2)$ if the control is to be implemented through a zero-order-hold. Therefore, it might be needed to complement the new estimator with an higher-order-hold mechanism in the control channel to be able to reduce the deviations of x from zero also between the two sampling instances.

3 Recursive Discrete Time Sliding Control Design

3.1 Problem Statement

Consider the following system in discrete strict feedback form:

$$\begin{aligned} x_{1,k+1} &= x_{1,k} + Tx_{2,k} + T\delta_{1,k} \\ x_{2,k+1} &= x_{2,k} + Tx_{3,k} + T\delta_{2,k} \\ &\vdots \\ x_{n-1,k+1} &= x_{n-1,k} + Tx_{n,k} + T\delta_{n-1,k} \\ x_{n,k+1} &= x_{n,k} + f(x,k) + g(x,k)u + T\delta_{n,k} \end{aligned} \tag{22}$$

where $x_i \in \mathbf{R}$'s are the state variables, $u \in \mathbf{R}$ is the control, $f(x,k)$, $g(x,k)$ are known nonlinear functions, T is the sampling time and δ_i's represent the external disturbances of the system. The control objective is to force the output $y_k = x_{1,k}$ to track a desired bounded reference trajectory $y_{d,k}$ in the presence of unknown δ_i's.

Assumption 1 *Each $\delta_{i,k}$ is in the form of $\delta_i(kT)$ where $\delta_i(t)$ is a bounded external signal which is continuously differentiable to a certain degree $r_i \geq 1$ for $i = 1, \cdots, n$.*

The assumption of being the disturbances of the system totally external smooth signals is obviously very restrictive. However, it should also be noted that this assumption is yet more general than the standard matching condition which does not allow any mismatched disturbance in the system at the first place.

There are a number of different approaches reported in the literatute on the robust control of discrete-time systems in strict feedback form depending on the characterization of the disturbance. For disturbances which can be linearly parameterized by an unknown parameter vector with a known state dependent regression vector, adaptive control theory can be used ([10], [5]). In [9], the disturbances were treated as time-varying parameters independent of the system states. The uncertainty terms were estimated by discrete-time adaptive laws driven by the system states and the estimated disturbances were then used in the controller design.

In this study, we also consider the uncertainties of the system as external time-varying signals as stated in Assumption 1. The disturbance estimation technique of Section 2 is used to estimate δ_i's and the estimated disturbances are incorporated into the control design.

3.2 Estimation of Disturbances

As in Section 2, the disturbance estimators are selected as follows:

$$\hat{\delta}_{i,k} = \frac{\sum_{j=0}^{m_i-1} \gamma_{i,j}q^{-j}}{\triangle^{m_i}(q^{-1})}[x_{i,k} - \hat{x}_{i,k}] \tag{23}$$

for $i = 1, \cdots, n$ where

$$\begin{aligned} \hat{x}_{1,k+1} &= x_{1,k} + Tx_{2,k} + T\hat{\delta}_{1,k} \\ \hat{x}_{2,k+1} &= x_{2,k} + Tx_{3,k} + T\hat{\delta}_{2,k} \\ &\vdots \\ \hat{x}_{n-1,k+1} &= x_{n-1,k} + Tx_{n,k} + T\hat{\delta}_{n-1,k} \\ \hat{x}_{n,k+1} &= x_{n,k} + f(x,k) + g(x,k)u + T\hat{\delta}_{n,k} \end{aligned} \tag{24}$$

and $\gamma_{i,j}$'s, $m_i \leq r_i$'s are free parameters to be selected. The disturbance estimation errors satisfy

$$P_i(q^{-1})\tilde{\delta}_{i,k} = R_{i,k} \tag{25}$$

where

$$P_i(q^{-1}) = 1 + \sum_{j=1}^{m_i}[(-1)^j \begin{pmatrix} m_i \\ j \end{pmatrix} + T\gamma_{i,j-1}]q^{-j} \tag{26}$$

$$R_{i,k} = \triangle^{m_i}(q^{-1})\delta_{i,k} = \mathcal{O}(T^{m_i}) \tag{27}$$

Note that the disturbance estimation error dynamics are independent of the state variables. Furthermore both the estimation accuracy and the convergence behavior can be adjusted as desired via the estimator parameters.

3.3 Controller Development

The control is developed by employing a step-by-step design procedure similar to the continuous time backstepping as in [9]. To this end, define a new set of coordinates

$$\begin{aligned} e_{1,k} &= x_{1,k} - y_{d,k} \\ e_{i,k} &= x_{i,k} - \alpha_{i-1,k} \qquad \text{for } i = 2, \cdots, n \end{aligned} \tag{28}$$

where $\alpha_{i-1,k}$'s are to be determined.

To select $\alpha_{1,k}$, we first rewrite the first part of Eq. (22) in terms of the new coordinate variables:

$$e_{1,k+1} = x_{1,k} + Te_{2,k} + T\alpha_{1,k} + T\delta_{1,k} - y_{d,k+1} \tag{29}$$

The selection of

$$\alpha_{1,k} = \frac{1}{T}\left[(\Omega_1 - 1)x_{1,k} - \Omega_1 y_{d,k} + y_{d,k+1}\right] - \hat{\delta}_{1,k} \tag{30}$$

where Ω_1 is a design constant produces

$$e_{1,k+1} = \Omega_1 e_{1,k} + Te_{2,k} + T\tilde{\delta}_{1,k} \tag{31}$$

and the stability of $e_{1,k}$ in case $e_{2,k}$ and $\tilde{\delta}_{1,k}$ are bounded can be assured by selecting $|\Omega_1| < 1$. At the second step, we first write

$$e_{2,k+1} = x_{2,k} + Te_{3,k} + T\alpha_{2,k} + T\delta_{2,k} - \alpha_{1,k+1} \tag{32}$$

Note that, $\alpha_{1,k+1}$ cannot be computed exactly by the available quantities since it includes $\delta_{1,k}$ dependent terms which emerge from $x_{1,k+1}$ and $\hat{\delta}_{1,k+1}$. To avoid the appearance of messy equations, $\alpha_{1,k+1}$ is expressed in the following compact form

$$\alpha_{1,k+1} = F_1(\bullet) - T\beta_{1,1}\delta_{1,k} \tag{33}$$

where $F_1(\bullet)$ denotes the computable part of $\alpha_{1,k+1}$ and $\beta_{1,1}$ is a known constant which depends on the controller and the estimator parameters. Selecting

$$\alpha_{2,k} = \frac{1}{T}\left[(\Omega_2 - 1)x_{2,k} - \Omega_2\alpha_{1,k} + F_1(\bullet)\right] - \beta_{1,1}\hat{\delta}_{1,k} - \hat{\delta}_{2,k} \tag{34}$$

where Ω_2 is a design constant, one gets

$$e_{2,k+1} = \Omega_2 e_{2,k} + Te_{3,k} + T\beta_{1,1}\tilde{\delta}_{1,k} + T\tilde{\delta}_{2,k} \tag{35}$$

and e_2 can also be guaranteed to be stable for bounded $e_{3,k}$, $\tilde{\delta}_{1,k}$ and $\tilde{\delta}_{2,k}$ with the selection of $|\Omega_2| < 1$.

Continuing with this manner, at the ith step, $\alpha_{i-1,k+1}$ of the previous step is first decomposed into two parts:

$$\alpha_{i-1,k+1} = F_{i-1}(\bullet) - T\sum_{j=1}^{i-1}\beta_{i-1,j}\delta_{j,k} \tag{36}$$

where $F_{i-1}(\bullet)$ represents the computable part of $\alpha_{i-1,k+1}$, $\beta_{i-1,j}$ are known constants and $\alpha_{i,k}$ is selected as follows:

$$\begin{aligned} \alpha_{i,k} = {} & \frac{1}{T}\left[(\Omega_i - 1)x_{i,k} - \Omega_i\alpha_{i-1,k} + F_{i-1}(\bullet)\right] \\ & - \left[\sum_{j=1}^{i-1}\beta_{i-1,j}\hat{\delta}_{j,k}\right] - \hat{\delta}_{i,k} \end{aligned} \tag{37}$$

to obtain

$$e_{i,k+1} = \Omega_i e_{i,k} + Te_{i+1,k} + \left[\sum_{j=1}^{i-1}\beta_{i-1,j}\tilde{\delta}_{j,k}\right] + \tilde{\delta}_{i,k} \tag{38}$$

for $i = 2, \cdots, n-1$.

The result of these coordinate transformations can be summarized as follows:

$$\begin{bmatrix} e_{1,k+1} \\ e_{2,k+1} \\ \vdots \\ e_{n-1,k+1} \end{bmatrix} = \begin{bmatrix} \Omega_1 & T & 0 & \cdots \\ 0 & \Omega_2 & T & \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & & \Omega_{n-1} \end{bmatrix} \begin{bmatrix} e_{1,k} \\ e_{2,k} \\ \vdots \\ e_{n-1,k} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ T \end{bmatrix} e_{n,k} + T\begin{bmatrix} \tilde{\delta}_{1,k} \\ \tilde{\delta}_{2,k} + \beta_{1,1}\tilde{\delta}_{1,k} \\ \vdots \\ \tilde{\delta}_{n-1,k} + \sum_{j=1}^{n-1}\beta_{n-2,j}\tilde{\delta}_{j,k} \end{bmatrix} \tag{39}$$

Note that the system matrix of the e_i variables is in the upper triangular form and its eigenvalues are Ω_i's which are free design parameters. If $e_{n,k}$ can be kept at zero, each $e_i(k)$ converges to boundary layers around zero whose thicknesses are related to the disturbance estimation accuracy with a desired dynamic behavior. Although this process can be continuoued one more step to specify the control law, we stop at the $(n-1)$th step and treat the problem in a discrete-time sliding mode control setting. To this end, $e_{n,k}$ is selected as the natural sliding manifold variable and the control input appears as follows:

$$\begin{aligned} u_k = {} & g^{-1}(x,k)\Bigg[-x_{n,k} - f(x,k) + F_{n-1}(\bullet) - \\ & T\left(\hat{\delta}_{n,k} + \sum_{j=1}^{n-1}\beta_{n-1,j}\hat{\delta}_{j,k}\right) + v_k\Bigg] \end{aligned} \tag{40}$$

$$v_k = \begin{cases} e_{n,k} - Msgne_{n,k} & \text{for} \quad e_{n,k} \notin \mathcal{B} \\ 0 & \text{for} \quad e_{n,k} \in \mathcal{B} \end{cases} \tag{41}$$

where M is a design parameter and $\mathcal{B}$ denotes a boundary layer around $e_{n,k} = 0$. As before, $F_{n-1}(\bullet)$ and $\beta_{n-1,i}$'s result from the parameterization of $\alpha_{n-1,k+1}$ in the following form:

$$\alpha_{n-1,k+1} = F_{n-1}(\bullet) - T\sum_{j=1}^{n-1}\beta_{n-1,j}\delta_{j,k} \tag{42}$$

Considering that the input coefficient matrix will usually be $\mathcal{O}(T)$ in a sampled data framework, M offers flexibility in the trade-off between the transient time duration initial to the quasi-sliding phase and the control magnitude. The control input of Eq. (40)-(41) steers $e_{n,k}$ towards zero by an amount determined by M when $e_{n,k}$ is outside of the boundary layer. When $e_{n,k}$ reaches the boundary layer the discontinuous part of the control is nullified and the control input appears as the ideal *discrete time equivalent control* ([6]) with additional disturbance cancellation terms where $e_{n,k}$ is only affected by $\tilde{\delta}_i$'s whose magnitudes and dynamic behaviors are controllable by the designer via the estimator parameters. Since e_i's have proven to be (ultimately) bounded, the reference trajectory and the disturbances have already been assumed to be bounded at the first place all x_i's are bounded and x_1 is asymptotically forced to a vicinity of the reference trajectory $y_{d,k}$.

4 Numerical Example

This section presents a numerical example on the proposed estimation based recursive tracking controller design. The model used for simulation purposes has the structure of Eq. (22) with $n = 3$, $\delta_{1,k} = \sin(2kT)$, $\delta_{2,k} = \sin(3kT)$, $\delta_{3,k} = \sin(4kT)$ and $T = 0.02$ *sec.* The reference signal to be tracked by x_1 has been selected as $y_{d,k} = 0.5 - \cos(kT)$. Figures 1-4 show the results of two disturbance estimators of different orders with the common parameters: $\Omega_1 = 0.5$, $\Omega_2 = 0.7$, $M = 3$, $\mathcal{B} = \{x : |e_3| < 5\}$. The orders of the disturbance estimators were 2 and 5. As shown in Figure 1-4, x_1 asymptotically tracks the desired reference signal with arbitrarily small tracking errors for both cases. To show the effect of the order of the estimator on the tracking accuracy, the zoomed versions of the tracking errors were also attached. Note that, the tracking accuracy was improved with an increase in the order of the disturbance estimator.

5 Discrete Time Parameter Identification

As another application of the disturbance estimation idea of Section 2, consider

$$x_{k+1} = x_k + TAx_k + \Gamma u_k \tag{43}$$

where $x \in \mathbf{R}^n$ is the state, $u \in \mathbf{R}^m$ is the control, T is the sampling time and A, Γ are constant matrices of appropriate dimensions. The problem considered is to estimate A using the available quantities x and u. As in the disturbance estimation design, we first rewrite Eq. (43) with a new set of variables

$$\hat{x}_{k+1} = x_k + T\hat{A}_k x_k + \Gamma u_k \tag{44}$$

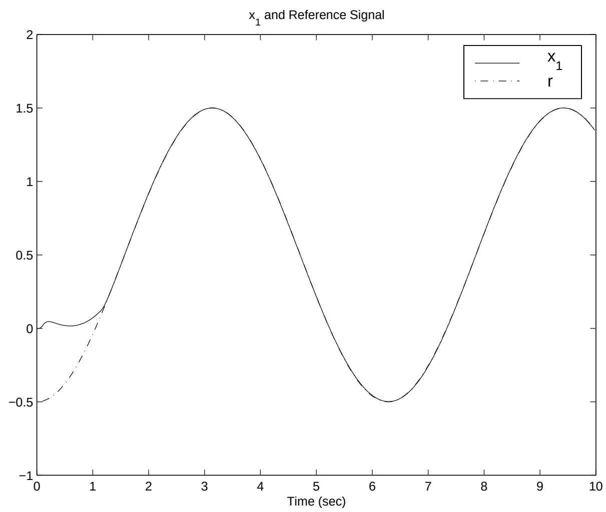

Figure 1: Tracking Plots for $m_1, m_2 = 2$ (Original)

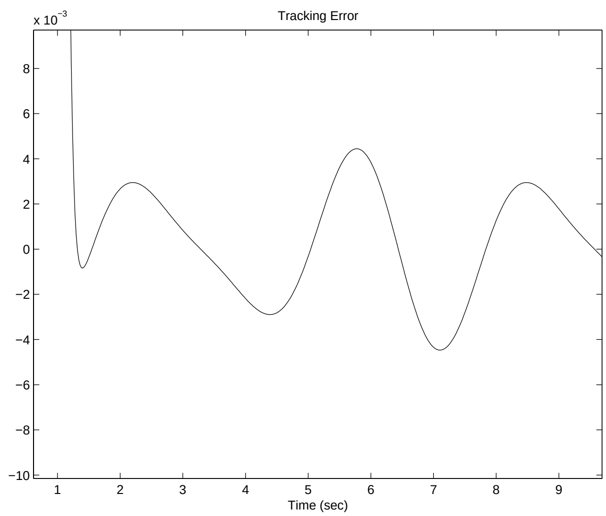

Figure 2: Tracking Plots for $m_1, m_2 = 2$ (Zoomed)

to obtain the unknown parameter estimation error variable $\tilde{A}_k = A - \hat{A}_k$ by another available error variable $\tilde{x}_k = x_k - \hat{x}_k$ according to:

$$\tilde{x}_{k+1} = T\tilde{A}_k x_k \tag{45}$$

The parameter update law is chosen in its standard structure

$$\hat{A}_k = \hat{A}_{k-1} + g\frac{\tilde{x}_k x_{k-1}^T}{x_{k-1}^T x_{k-1}} \tag{46}$$

where g is to be determined. Subtracting A from both sides of Eq. (46) and using Eq. (45), one gets the estimation error dynamics as follows:

$$\tilde{A}_{k+1} = \tilde{A}_k - gT\tilde{A}_k\frac{x_k x_k^T}{x_k^T x_k} \tag{47}$$

Note that $(x_k x_k^T)/(x_k^T x_k) = ee^T$ where $\|e\| = 1$ so that the normalization term of the parameter update law does not cause any singularity problem.

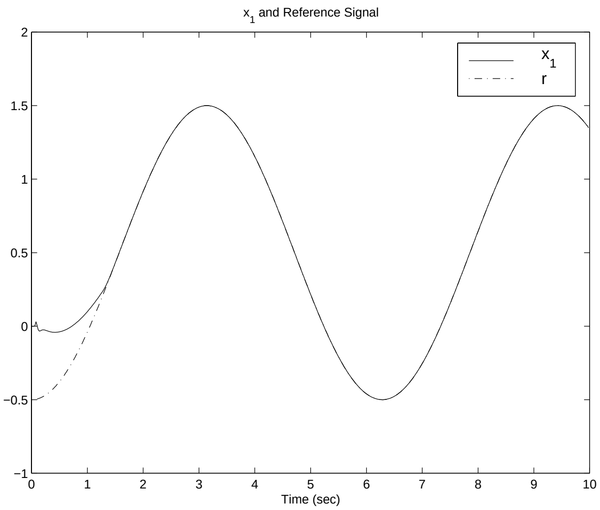

Figure 3: Tracking Plots for $m_1, m_2 = 5$ (Original)

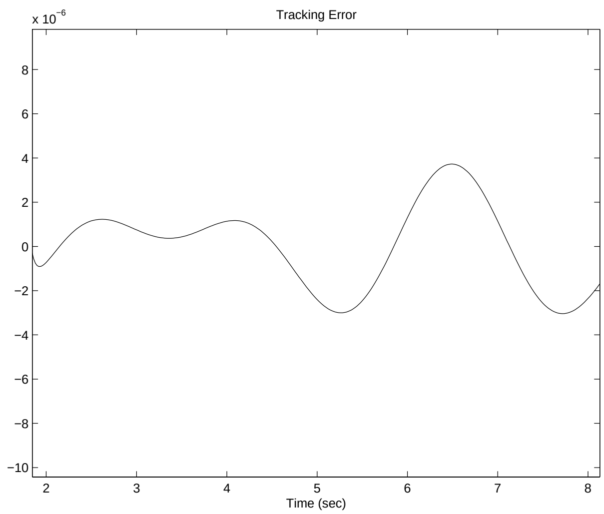

Figure 4: Tracking Plots for $m_1, m_2 = 5$ (Zoomed)

To analyze the stability properties of the estimator, let $V(k) = \mathrm{tr}[\tilde{A}_k \tilde{A}_k^T]$ be the Lyapunov function candidate where tr denotes the standard trace operator and $\Delta V_k = V_{k+1} - V_k$.

By direct substitutions, one gets

$$\Delta V(k) = \mathrm{tr}\left[g^2 T^2 \frac{\tilde{A}_k x_k x_k^T \tilde{A}_k^T}{x_k^T x_k} - 2gT \frac{\tilde{A}_k x_k x_k^T \tilde{A}_k^T}{x_k^T x_k}\right] \quad (48)$$

$$= (g^2 T^2 - 2gT)\|\tilde{A}_k\|^2 \quad (49)$$

Selection of $0 < g < 2/T$ guarantees that V_k is nonincreasing. Since V_k is also nonnegative by definition V_k is bounded ($V_k \in l_\infty$) and so is $\tilde{A}_k$ ($\tilde{A}_k \in l_\infty$). Furthermore, $\tilde{A}_k$ is square summable ($\tilde{A}_k \in l_2$) which directly follows from the summability of ΔV_k. Therefore, $\tilde{A}_k$ converges to zero in time according to Barbalat's lemma.

6 Conclusions

A discrete-time disturbance estimator has been presented. A tracking controller design which utilizes the proposed disturbance estimators in closed loop has been developed for a class of system which can be expressed in discrete strict feedback form. The application of the estimation method to a parameter identification problem has also been reported.

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