Direct Closed-Loop Identification Approach to Unstable Plant

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Abstract

This paper deals with the problem of direct closed-loop identification for unstable plant models disturbed by stochastic noise. The unstable plant is stabilized by a digital feedback controller. Then by introducing the output inter-sampling scheme, the plant model is identified from the inter-sampled input-output data of the plant even though the external reference or the test signal does not hold persistently exciting property. Both time and frequency domain approaches are developed and numerical examples are performed to demonstrate the effectiveness of the proposed approaches.

1 Introduction

It is hardly to perform the identification experiment of unstable plant model in the open loop situation; consequently some closed-loop identification methods have been developed to identify the unstable plant model from closed-loop data. There are several important issues must be considered in unstable plant identification problem: (1) How to design the experiment to guarantee the identifiability; (2) Decorrelation between plant input-output and the disturbance noise; (3) Dealing with the unstable poles in he transfer function. In the indirect closed-loop identification it is considered to reduce the closed-loop identification into a set of stable open loop identification problems [1, 2] using the external test signal with sufficiently persistent excitations. It is claimed that such an external test signal could guarantee the identifiability in the indirect methods. Nevertheless, it increases the variance of control input and output signal, and it always conflicts to the control purpose to keep the plant output round the desired value as close as possible. In many chemical industrial plants, for example, the desired value, i.e. the reference is kept as a constant for a long period. On the other hand, the test signal with high level is often not permitted in some situations. It implies that even though the test signal could be utilized during identification experiment, then the low signal to noise ratio restricts the capacity to improve the identification accuracy in the indirect methods.

In this paper we consider a very restrictive situation in closed-loop identification: Neither the reference signal nor the test signal holds persistently exciting property, i.e. the plant works in normal conditions. Under this situation, the indirect methods are not applicable to the plant model identification; Thus it seems that the direct closed-loop identification approach is the unique potential choice to closed-loop identification. Since the measured external excitations are not available, the identifiability conditions are necessary to ensure that the estimation from the observed data describing the unknown plant well [3, 4]. As it is demonstrated in [4], the identifiability conditions depend on the unknown true plant, the model structure, the identification algorithm and the experiment conditions. In the direct identification methods, the identifiability can be guaranteed by the controller switching technique [5], and the output inter-sampling scheme [6]. However, they are mainly developed for the stable plants. Though the unstable plant case based on the output inter-sampling scheme has also been discussed in [7], it is a special case where the noise is an i.i.d white output noise.

The main purpose of this paper is to extend the output inter-sampling based closed-loop identification approach to the case where the unstable plant is disturbed by colored stochastic noise. Both the time domain and the frequency domain properties of the inter-sampled plant model are studied. Then by using these properties, several identification algorithms are developed. Their effectiveness is demonstrated through some numerical examples.

2 Problem Statement

Assume that the unstable plant is described by a continuous time linear model, and it is stabilized by a digital feedback controller. Further, we assume that the controller is cascaded by a zero-order holder with holding period T so the plant input is a piecewise signal with holding period T. Thus if the plant is also sampled at the interval T, the closed-loop can also be formulated by a discrete-time model corresponding to interval T, as illustrated in Figure 1, where $r(m)$, $u(m)$ and $y(m)$ are the reference, control input, plant output at instant mT respectively, while $e(m)$ is the disturbance noise added to the plant output. In this paper, we will consider the case where the test signal does not hold the persistently exciting property, which is given by 0 without loss of generality. Let the transfer functions of plant and controller be denoted as $G(z^{-1})$ and $C(z^{-1})$ respectively. It is well known that some of the conventional direct methods cannot yield consistent estimates particularly when the controller is of low order when the reference signal $r(m)$ has not enough frequency excitations.

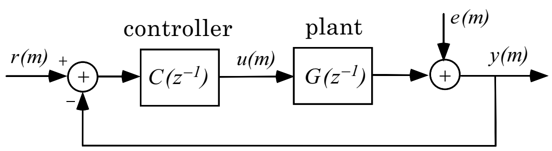

Figure 1: Discrete-time closed-loop model

Provided that the disturbance noise signal $e(\cdot)$ is a stationary stochastic random process with finite 4-th order moment, and it can be modeled as a stable model with interval T or some other shorter intervals following the spectral factorization. Then, the persistent excitation of the plant input can be provided by the output noise even though there are not other external excitations. Following the problem set-up, the discrete-time models of the unknown plant and controller can be given by

$$y(m) = G(z^{-1})u(m) + e(m) \tag{1}$$

$$u(m) = C(z^{-1})(r(m) - y(m)) \tag{2}$$

where

$$G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{b_1 z^{-1} + \cdots + b_{n_G} z^{-n_G}}{1 + a_1 z^{-1} + \cdots + a_{n_G} z^{-n_G}}$$

$$C(z^{-1}) = \frac{S(z^{-1})}{R(z^{-1})} = \frac{s_0 + s_1 z^{-1} + \cdots + s_{n_C} z^{-n_C}}{1 + r_1 z^{-1} + \cdots + r_{n_C} z^{-n_C}}$$

Here n_G and n_C are the orders of the plant and controller respectively, z^{-1} is a backward shift operator with respect to T. We have illustrated the effectiveness of the output inter-sampling to the closed-loop identification problems to enhance the identifiability of the direct methods when the plant is stable. It has been proved that the conventional identifiability conditions are removed in the output inter-sampling based methods even though the controller has low order [6]. In this paper we will extend the identification algorithms into the case where the plant is unstable and is disturbed by colored noise. The output inter-sampling scheme is illustrated in Figure 2, the control interval is given by T and the plant output is sampled at interval $\Delta = T/p$. Here p is an integer denoted as an inter-sampling rate, and the observed plant output is denoted as $y_\Delta(k)$ corrupted by the noise denoted as $e_\Delta(k)$. Let $x(m)$ and $x_\Delta(k)$ are the signal x at instants mT and $k\Delta$ respectively, where x is input, or output, or the disturbance noise signal. Provided that the discrete-time plant model is given by

$$y_\Delta(k) = G_\Delta(q^{-1})u_\Delta(k) + e_\Delta(k) \tag{3}$$

where q^{-1} is the backward shift operator with respect to interval Δ. Supposed that $G_\Delta(q^{-1})$ is characterized by

$$G_\Delta(q^{-1}) = \frac{B_\Delta(q^{-1})}{A_\Delta(q^{-1})} = \frac{b_{\Delta,1}q^{-1} + \cdots + b_{\Delta,n_G}q^{-n_G}}{1 + a_{\Delta,1}q^{-1} + \cdots + a_{\Delta,n_G}q^{-n_G}}$$

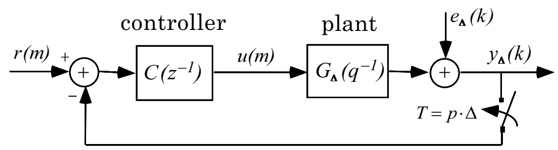

Figure 2: Inter-sampled closed-loop system

3 Preliminaries of Inter-sampling Scheme

The output inter-sampling scheme has some special properties that can be used for the identification problem. In this section we will discuss the properties in both time-domain and frequency domain.

3.1 Inter-sampled Plant Model

Note that in the inter-sampling scheme, $u_\Delta(k) = u(m)$ for $k \in [mp, (m+1)p)$. Then by rewriting the model description of the inter-sampled plant model (3) in state-space realization as

$$\begin{cases} \boldsymbol{x}_\Delta(k+1) = \boldsymbol{A}\boldsymbol{x}_\Delta(k) + \boldsymbol{b}u_\Delta(k) \\ y_\Delta(k) = \boldsymbol{c}^T\boldsymbol{x}_\Delta(k) + e_\Delta(k) \end{cases} \tag{4}$$

then the plant output at instant $mT + j\Delta$ can be given by

$$y_\Delta(mp+j) = \boldsymbol{c}^T(q^p\boldsymbol{I} - A^p)^{-1}\left(\sum_{i=0}^{j-1}\boldsymbol{A}^i\boldsymbol{b}u(m+1) + \sum_{i=j}^{p-1}\boldsymbol{A}^i\boldsymbol{b}u(m)\right) \tag{5}$$

By substituting $z^{-1} = q^{-p}$ into (5), we can write the new input-output relation into the following formulation

$$y_j(m) = G_j(z^{-1})u(m) + e_j(m) \tag{6}$$

where

$$G_j(z^{-1}) = \frac{B_j(z^{-1})}{A(z^{-1})} = \boldsymbol{c}^T \left(\boldsymbol{I} - \boldsymbol{A}^p z^{-1}\right)^{-1} \left(\sum_{i=0}^{j-1} \boldsymbol{A}^i \boldsymbol{b} + \sum_{i=j}^{p-1} \boldsymbol{A}^i \boldsymbol{b} z^{-1}\right) \tag{7}$$

$$y_j(m) = y_\Delta(mp + j), \quad e_j(m) = e_\Delta(mp + j) \tag{8}$$

It is noticed that $G_0(z^{-1})$ is just the transfer function of the plant model of sampling interval T. As shown in Figure 3, only the discrete-time plant output signal at instants mT is added into the feedback controller, which implies that the closed-loop is working in "open loop" at the other instants $mT + j\Delta$ for $j = 1, \cdots, p-1$. If the noise $e_\Delta(k)$ added to the output $y_\Delta(k)$ can be modeled as a stationary stochastic process, it will be claimed that $G_\Delta(q^{-1})$ might be identified from the inter-sampled input-output data of $y_\Delta(k)$ and $u_\Delta(k)$, and the identifiability can be guaranteed since the observation data contain some open loop information.

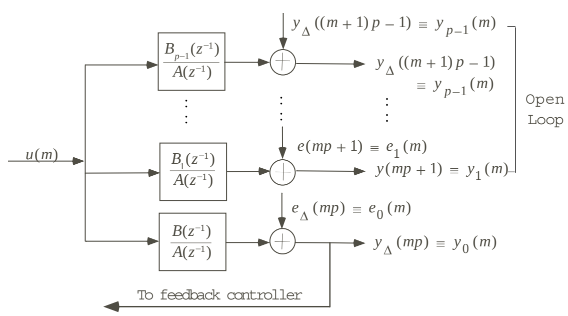

Figure 3: SIMO model structure of inter-sampled plant model

3.2 Frequency Property of Plant Input-Output Signal

Following the spectrum factorization theorem, $e_\Delta(k)$ can be characterized by

$$e_\Delta(k) = H_\Delta(q^{-1})\varepsilon_\Delta(k) = \frac{F_\Delta(q^{-1})}{E_\Delta(q^{-1})}\varepsilon_\Delta(k) \tag{9}$$

where $\varepsilon_\Delta(k)$ is an i.i.d white noise. Note that there exists a polynomial $\bar{E}_\Delta(q^{-1})$ such that the roots of $E(z^{-1}) = E_\Delta(q^{-1})\bar{E}_\Delta(q^{-1})$ are pth power of the roots of $E_\Delta(q^{-1})$, then $e_\Delta(k)$ can be expressed by the following form with respect to the control interval T

$$e_\Delta(k) = \sum\nolimits_{j=0}^{p-1} \left(\frac{F_j(z^{-1})}{E\left(z^{-1}\right)}\varepsilon_\Delta(k-j)\right) \tag{10}$$

where $F_j(p^{-p})$ be a polynomial that contains all the power q^{-lp} of $q^j F_\Delta(q^{-1})\bar{E}_\Delta(q^{-1})$. Further, let the new signal $w(m)$ and $w_\Delta(k)$ be defined by

$$w(m) = \sum\nolimits_{j=0}^{p-1} \left(\frac{F_{p-j}(z^{-1})}{E\left(z^{-1}\right)}\varepsilon_j(m-1)\right)$$

$$w_\Delta(k) = \sum\nolimits_{j=0}^{p-1} \left(\frac{F_{p-j}(q^{-p})}{E\left(q^{-p}\right)}\xi_{\Delta,j}(k)\right)$$

where $F_p(z^{-1}) = zF_0(z^{-1})$, $\varepsilon_j(m) = \varepsilon_\Delta(mp + j)$, $\xi_{\Delta,j}(k)$ is the sequence by holding $\varepsilon_j(m)$ for period T. Without loss of generality, let $r_\Delta(k) = 0$, then $u_\Delta(k)$ can also be written as

$$u_\Delta(k) = \frac{-C(q^{-p})}{1 + C(q^{-p})G(q^{-p})} w_\Delta(k) \tag{11}$$

Consider the statistics property of $\xi_{\Delta,j}(k)$. The correlation function $c_{\xi_{\Delta,j_1}\xi_{\Delta,j_2}}(k, \tau)$ and cross correlation function $c_{\xi_{\Delta,j}\varepsilon_\Delta}(k, \tau)$ can be given by

$$\begin{aligned} c_{\xi_{\Delta,j_1}\xi_{\Delta,j_2}}(k,\tau) &:= E\left\{\xi_{\Delta,j}(k{+}\tau)\xi_{\Delta,j}(k)\right\} \\ c_{\xi_{\Delta,j}\varepsilon_\Delta}(k,\tau) &:= E\{\xi_{\Delta,0}(k+\tau)\varepsilon_\Delta(k)\} \end{aligned} \tag{12}$$

Then $c_{\xi_{\Delta,j_1}\xi_{\Delta,j_2}}(k, \tau)$ and $c_{\xi_{\Delta,j}\varepsilon_\Delta}(k, \tau)$ are almost periodic function in k. Assume that their Fourier series representation with respect to k

$$\mathcal{C}_{(\cdot)_\Delta(\cdot)_\Delta}(\alpha, \tau) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} c_{(\cdot)_\Delta(\cdot)_\Delta}(k,\tau) e^{-i\alpha k} \tag{13}$$

exist, and are nonzero for $\alpha \in \mathcal{A}$, where $\mathcal{A} = \left\{\alpha | \mathcal{C}_{(\cdot)_\Delta(\cdot)_\Delta}(\alpha, \tau) \neq 0, 0 \leq \alpha < 2\pi \ (\text{mod } 2\pi)\right\}$, $(\cdot)_\Delta$ is $\xi_{\Delta,j}$ or ε_Δ. Furthermore, given the cyclic correlation spectra and its Fourier series by

$$\mathcal{S}_{(\cdot)_\Delta(\cdot)_\Delta}(\alpha, \omega) = \sum_{\tau=-\infty}^{\infty} \mathcal{C}_{(\cdot)_\Delta(\cdot)_\Delta}(\alpha, \tau) e^{-i\omega\tau} \tag{14}$$

Let $\mathcal{C}_{\xi_{\Delta,0}\varepsilon_\Delta}(\alpha, 0) := \mathcal{X}$, then from (13) and (14) we have that

$$\begin{aligned} \mathcal{S}_{\xi_{\Delta,j}\varepsilon_\Delta}(\alpha, \omega) &= \sum_{\tau=-\infty}^{\infty} \mathcal{C}_{\xi_{\Delta,j}\varepsilon_\Delta}(\alpha, \tau) e^{-i\omega\tau} \\ &= e^{ij(\alpha-\omega)} \left(1 + e^{-i\omega} + \cdots + e^{-i(p-1)\omega}\right)\mathcal{X} \end{aligned} \tag{15a}$$

$$\begin{aligned} \mathcal{S}_{\xi_{\Delta,j}\xi_{\Delta,j}}(\alpha, \omega) = &\left(1 + e^{-i\omega} + \cdots + e^{-i(p-1)\omega}\right) \\ &\cdot \left(1 + e^{-i(\alpha-\omega)} + \cdots + e^{-i(p-1)(\alpha-\omega)}\right)\mathcal{X} \end{aligned} \tag{15b}$$

On the other hand, following the framework in [8, 9] the cyclic spectra of $w_\Delta(k)$ and the cross spectra of $w_\Delta(k)$,

$e_\Delta(k)$ can be given by

$$\mathcal{S}_{w_\Delta w_\Delta}(\alpha,\omega) = \sum_{j=0}^{p-1} \frac{F_j\left(e^{-ip\omega}\right)F_j\left(e^{-ip(\alpha-\omega)}\right)}{E\left(e^{-ip\omega}\right)E\left(e^{-ip(\alpha-\omega)}\right)}\mathcal{S}_{\xi_{\Delta,j}\xi_{\Delta,j}}(\alpha,\omega) \quad (16a)$$

$$\mathcal{S}_{w_\Delta e_\Delta}(\alpha,\omega) = \sum_{j=0}^{p-1} \frac{F_j\left(e^{-ip\omega}\right)}{E\left(e^{-ip\omega}\right)}H_\Delta\left(e^{-i(\alpha-\omega)}\right)\mathcal{S}_{\xi_{\Delta,j}\varepsilon_\Delta}(\alpha,\omega) \quad (16b)$$

Using $\mathcal{S}_{\xi_{\Delta,j}\varepsilon_\Delta}(\alpha,\omega)$ leads to

$$\mathcal{S}_{w_\Delta e_\Delta}(\alpha,\omega) = H_\Delta(e^{i(\alpha-\omega)})H_\Delta(e^{-i(\alpha-\omega)}) \cdot\left(1+\cdots+e^{-i(p-1)\omega}\right)\mathcal{X} \quad (17)$$

If the disturbance noise is a white i.i.d white noise, i.e. $H_\Delta(q^{-1}) = 1$, we have that

$$\mathcal{S}_{w_\Delta e_\Delta}(\alpha,\omega) = \left(1+\cdots+e^{-i(p-1)\omega}\right)\mathcal{X} \quad (18)$$

Then the cross cyclic spectrum $\mathcal{S}_{u_\Delta e_\Delta}(\alpha,\omega)$ will have the same value for different α.

Further, the cyclic spectra and the cross spectra of plant input output could be given by

$$\mathcal{S}_{u_\Delta u_\Delta}(\alpha,\omega) = \frac{-C(e^{-ip\omega})}{1+C(e^{-ip\omega})G(e^{-ip\omega})} \cdot\frac{-C(e^{-ip(\alpha-\omega)})}{1+C(e^{-ip(\alpha-\omega)})G(e^{-ip(\alpha-\omega)})}\mathcal{S}_{w_\Delta w_\Delta}(\alpha,\omega) \quad (19a)$$

$$\mathcal{S}_{u_\Delta y_\Delta}(\alpha,\omega) = G_\Delta(e^{-i(\alpha-\omega)})\mathcal{S}_{u_\Delta u_\Delta}(\alpha,\omega) + \frac{-C(e^{-ip\omega})}{1+C(e^{-ip\omega})G(e^{-ip\omega})}\mathcal{S}_{w_\Delta e_\Delta}(\alpha,\omega) \quad (19b)$$

The frequency domain approach can be developed by applying the relations in (19).

4 Identification Algorithm

The identification algorithms are given in both time domain and frequency domain.

4.1 PEM Based Algorithm

Traditionally the predictor error model of the plant output is given by

$$\hat{\varepsilon}_\Delta(k) = \frac{1}{\hat{H}_\Delta(q^{-1})}\left(y_\Delta(k) - \hat{G}_\Delta(q^{-1})u_\Delta(k)\right) \quad (20)$$

However, $\hat{\varepsilon}_\Delta(k)$ cannot be calculated stably by using causal filters $\hat{H}_\Delta(q^{-1})$ and $\hat{G}_\Delta(q^{-1})$ since $G_\Delta(q^{-1})$ has unstable poles. If there is external test signal and the signal to noise ratio is high, $\hat{\varepsilon}_\Delta(k)$ could be estimated by stable noncausal filter of $\hat{G}_\Delta(q^{-1})$ [10]. Nevertheless, under the situation where the test signal is not available, the parameteration of PEM algorithm might not converge to the optimal values through noncausal filter. So here we consider the PEM based algorithm using the indirect prediction error method, which is given by [11]. It can be performed by rewriting the plant input-output relation in normal equation form as follows

$$A_\Delta(q^{-1})y_\Delta(k) = B_\Delta(q^{-1})u_\Delta(k) + \bar{H}_\Delta(q^{-1})\varepsilon_\Delta(k) \quad (21)$$

Note that the equation error model $\bar{H}_\Delta(q^{-1})$ contains unstable zeros of $A_\Delta(q^{-1})$, it is a non-minimum phase model. So following [10], the prediction error could be calculated by using a stable filter $\hat{\bar{H}}^*_\Delta(q^{-1})$ whose zeros are equal to the zeros of the estimated $\hat{\bar{H}}_\Delta(q^{-1})$ reflected into the unit circle. Then the optimal predictor error model is given by

$$\hat{\varepsilon}^*(k) = \frac{1}{\hat{\bar{H}}^*_\Delta(q^{-1})}\left(\hat{A}_\Delta(q^{-1})y_\Delta(k) - \hat{B}_\Delta(q^{-1})u_\Delta(k)\right) \quad (22)$$

The criterion of the optimization problem is chosen as follows.

$$V_N(\hat{\boldsymbol{\theta}}^*_\Delta) = \frac{1}{N}\sum_{k=1}^{N}\left(\hat{\varepsilon}^*_\Delta(k,\hat{\boldsymbol{\theta}}^*_\Delta)\right)^2$$

where N is the data number. Then the optimal parameterization of $\hat{A}_\Delta(q^{-1})$, $\hat{B}_\Delta(q^{-1})$ and $\hat{\bar{H}}^*_\Delta(q^{-1})$ can be obtained through minimizing the criterion $V_N(\hat{\boldsymbol{\theta}}^*_\Delta)$ [12]. Furthermore, following the relation given in (7), the parameters of $G(z^{-1})$ are also be calculated.

The identifiability in the proposed method can be interpreted as follows. The prediction error $\hat{\varepsilon}^*_\Delta(k,\hat{\boldsymbol{\theta}}_\Delta)$ is rewritten by

$$\hat{\varepsilon}^*_\Delta(k,\hat{\boldsymbol{\theta}}^*_\Delta) = \frac{1}{\hat{\bar{H}}^*_\Delta(q^{-1})}\left(\tilde{B}_\Delta(q^{-1})u_\Delta(k) + \frac{\tilde{A}_\Delta(q^{-1})B_\Delta(q^{-1})}{A_\Delta(q^{-1})}u_\Delta(k) + \tilde{A}_\Delta(q^{-1})e_\Delta(k) + A_\Delta(q^{-1})e_\Delta(k)\right) \quad (23)$$

where $\tilde{A}_\Delta(q^{-1}) = \hat{A}_\Delta(q^{-1}) - A_\Delta(q^{-1})$, $\tilde{B}_\Delta(q^{-1}) = \hat{B}_\Delta(q^{-1}) - B_\Delta(q^{-1})$ are the error polynomials. Further, following the expression of $u_\Delta(k)$ in (11) yields that

$$\frac{B_\Delta(q^{-1})}{A_\Delta(q^{-1})}u_\Delta(k) + e_\Delta(k) = \frac{-B_\Delta(q^{-1})\bar{A}_\Delta(q^{-1})S(q^{-p})}{A(q^{-p})R(q^{-p}) + B(q^{-p})S(q^{-p})}w_\Delta(k) + e_\Delta(k) \quad (24)$$

where $\bar{A}_\Delta(q^{-1})$ is the polynomial such that $A_\Delta(q^{-1})\bar{A}_\Delta(q^{-1}) = A_\Delta(q^{-p})$. Then for $k = mp + j$, $j = 0, \cdots, p-1$, (24) can be rewritten as

$$\begin{aligned}&\frac{-B_j(q^{-p})S(q^{-p})}{A(q^{-p})R(q^{-p})+B(q^{-p})S(q^{-p})}w_\Delta(k)+e_\Delta(k)\\ &=\frac{B_0(q^{-p})S(q^{-p})e_\Delta(k)-B_j(q^{-p})S(q^{-p})e_\Delta(k-j)}{A(q^{-p})R(q^{-p})+B(q^{-p})S(q^{-p})}\\ &\quad+\frac{A(q^{-p})R(q^{-p})}{A(q^{-p})R(q^{-p})+B(q^{-p})S(q^{-p})}e_\Delta(k) \qquad (25)\end{aligned}$$

On the other hand, $B_j(q^{-p})$ has the property such that

$$q^{-j}B_j(q^{-p}) - B_0(q^{-p}) = A_\Delta(q^{-1})X_j(q^{-1}) \qquad (26)$$

Then (25) has the factor $A_\Delta(q^{-1})$, and it leads to that

$$\begin{aligned}\hat{\varepsilon}^*_\Delta(k,\hat{\boldsymbol{\theta}}^*_\Delta) &= \frac{\bar{H}^*_\Delta q^{-1})}{\hat{\bar{H}}^*_\Delta(q^{-1})}\\ &\cdot\left(\frac{q^{-j}\tilde{B}_\Delta(q^{-1})\bar{A}(q^{-1})S(q^{-p})}{A(q^{-p})R(q^{-p})+B(q^{-p})S(q^{-p})}\right.\\ &\left.+\frac{\tilde{A}_\Delta(q^{-1})X_j(q^{-1})}{A(q^{-p})R(q^{-p})+B(q^{-p})S(q^{-p})}+1\right)\varepsilon^*_\Delta(k) \qquad (27)\end{aligned}$$

where $\varepsilon^*_\Delta(k) = \bar{H}_\Delta(q^{-1})\varepsilon_\Delta(k)/\bar{H}^*_\Delta(q^{-1})$. Note that the coefficients of q^0 in $\tilde{A}_\Delta(q^{-1})$ and $\tilde{B}_\Delta(q^{-1})$ are zero, then following the cyclostationary property of the prediction error $\hat{\varepsilon}^*_\Delta(k,\hat{\boldsymbol{\theta}}^*_\Delta)$, the criterion $V_N(\hat{\boldsymbol{\theta}}^*_\Delta)$ satisfies that

$$\frac{1}{p}\sum_{j=0}^{p-1}E\left(\hat{\varepsilon}^*_\Delta(mp+j,\hat{\boldsymbol{\theta}}^*_\Delta)\right)^2 \geq E\left\{(\varepsilon^*_\Delta(k))^2\right\} \quad (28)$$

(28) has the minimum if and only if $\tilde{A}_\Delta(q^{-1}) = 0$, $\tilde{B}_\Delta(q^{-1}) = 0$, $\hat{\bar{H}}^*_\Delta(q^{-1})/\bar{H}^*_\Delta(q^{-1}) = 1$. It means that the criterion of the PEM algorithm has the minimum at the point where the model parameters are the true ones.

Moreover, similarly as the results given in [6], the Hessian matrix is definitely positive matrix, so the PEM algorithm will converge to the optimal model if the initial values are in some range of the optimal ones.

4.2 Frequency Domain Algorithm

Assume that the controller is linear, then the frequency domain identification of the unstable plant model is performed in the following procedures.

(1) Estimation of $\mathcal{S}_{u_\Delta,u_\Delta}(\alpha,\omega)$ and $\mathcal{S}_{u_\Delta,u_\Delta}(\alpha,\omega)$ for $\alpha \in \mathcal{A} = \{\alpha|\alpha = 2j\pi/p, j = 0, \cdots, p-1\}$.

They are obtained from the collected observation data $y_\Delta(k)$ and $u_\Delta(k)$. The appropriate frequency window function is also chosen to improve the accuracy of spectral estimation.

(2) Estimation of $A(e^{ip\omega})$.

Supposed that the spectral estimates for α_1 and α_2 are obtained. Then following (19) yields that

$$\begin{aligned}&E_\Delta(e^{i(\alpha_1-\omega)})F_\Delta(e^{i(\alpha_2-\omega)})E_\Delta(e^{-i(\alpha_1-\omega)})F_\Delta(e^{-i(\alpha_2-\omega)})\\ &\quad\times\left(\hat{\mathcal{S}}_{u_\Delta,y_\Delta}(\alpha_1,\omega)-G_\Delta(e^{-i(\alpha_1-\omega)})\hat{\mathcal{S}}_{u_\Delta,u_\Delta}(\alpha_1,\omega)\right)\\ &=E_\Delta(e^{i(\alpha_2-\omega)})F_\Delta(e^{i(\alpha_1-\omega)})E_\Delta(e^{-i(\alpha_2-\omega)})F_\Delta(e^{-i(\alpha_1-\omega)})\\ &\quad\times\left(\hat{\mathcal{S}}_{u_\Delta,y_\Delta}(\alpha_2,\omega)-G_\Delta(e^{-i(\alpha_2-\omega)})\hat{\mathcal{S}}_{u_\Delta,u_\Delta}(\alpha_2,\omega)\right)\end{aligned} \qquad (29)$$

Thus when $A(e^{ip\omega})$ has not any reciprocal zeros, $A_\Delta(e^{i(\alpha_1-\omega)})A_\Delta(e^{i(\alpha_2-\omega)})$ can be estimated from (29). Further $\hat{A}_\Delta(e^{ip\omega})$ as well as $\hat{A}_\Delta(e^{i\omega})$ are obtained.

(3) Using the estimate of $A(e^{ip\omega})$ and the knowledge of the controller, the estimate of $(A(e^{ip\omega})R(e^{ip\omega}) + B(e^{ip\omega})S(e^{ip\omega}))E(e^{ip\omega})$, which is denoted as $\hat{\Gamma}(e^{ip\omega})$, can be obtained from $\hat{\mathcal{S}}_{u_\Delta,u_\Delta}(\alpha,\omega)$ in (19a).

(4) Substituting the obtained estimates into (19b) leads to

$$\begin{aligned}\hat{\mathcal{S}}_{u_\Delta y_\Delta}(\alpha,\omega) &= B_\Delta(e^{-i(\alpha-\omega)})\frac{\hat{\mathcal{S}}_{u_\Delta u_\Delta}(\alpha,\omega)}{\hat{A}_\Delta(e^{-i(\alpha-\omega)})}\\ &+\frac{-\hat{A}(e^{-ip\omega})D(e^{-ip\omega})}{\hat{\Gamma}(e^{-ip\omega})}\bar{F}_\Delta(e^{i(\alpha-\omega)})\bar{F}_\Delta(e^{-i(\alpha-\omega)})\\ &\times\sum_{j=0}^{p-1}e^{-ij\omega}\end{aligned} \qquad (30)$$

where

$$\bar{F}_\Delta(e^{i(\alpha-\omega)}) = F_\Delta(e^{i(\alpha-\omega)})\bar{E}_\Delta(e^{i(\alpha-\omega)})$$

Then $B_\Delta(e^{-i(\alpha-\omega)})$ and $\bar{F}_\Delta(e^{i(\alpha-\omega)})$ can be estimated. Moreover, from $\hat{\Gamma}(e^{-ip\omega})$ and $\hat{\bar{F}}_\Delta(e^{i(\alpha-\omega)})$, $H_\Delta(e^{-i\omega})$ is also obtained.

Remarks:

(1) In the frequency domain algorithm, only the spectrum of the disturbance noise is required so it also works for the nonminimum phase model.

(2) The optimization depends on initial values in the PEM algorithm. Sometimes it converges to local minimum when the initial value is different the true ones too much. On the other hand, the spectrum of the plant input-output is finite if the closed-loop is stable even though the plant has unstable poles, and the parameteration of the frequency domain algorithm might moderate the requirement of the initial values.

(3) Compared with frequency estimation, the PEM algorithm can give lower variance whereas the frequency approach is sensitive to the window function.

(4) The main drawback of the proposed methods is that the parameteration depends on the noise model, and the unknown noise model has also to be identified in the same time as the unknown plant model.

5 Numerical Simulations

We consider an unstable plant whose continuous time transfer function is given by

$$G(s) = \frac{0.2165s + 1.9646}{s^2 + 0.6444s - 0.4257} \tag{31}$$

The controller has holding period $T = 1.0$, and is given by

$$u(m) = -0.35y(m) \tag{32}$$

We choose the sampling rate $p = 2$, i.e. $\Delta = T/2 = 0.5$. The true plant model $G(z^{-1})$ and $G_\Delta(q^{-1})$ are given by

$$\begin{aligned} G(z^{-1}) &= \frac{z^{-1} + 0.5z^{-2}}{1 - 1.85z^{-1} + 0.5z^{-2}} \\ G_\Delta(q^{-1}) &= \frac{0.3173q^{-1} + 0.1063q^{-2}}{1 - 1.8164q^{-1} + 0.7246q^{-2}} \end{aligned} \tag{33}$$

Assume the plant is disturbed by noise $e_\Delta(k)$, where

$$\begin{aligned} e_\Delta(k) &= H_\Delta(q^{-1})\varepsilon_\Delta(k) \\ &= (1 - 0.82q^{-1} + 1.5q^{-2})\varepsilon_\Delta(k) \end{aligned} \tag{34}$$

here $\varepsilon_\Delta(k)$ is an i.i.d white noise of $\mathcal{N}(0, 1.0)$. The estimation results of $G_\Delta(q^{-1})$ are summarized in the following table, it shows that both the PEM based algorithm (algorithm I) and frequency algorithm (algorithm II) estimate the plant well.

Table 1: Estimated parameters (10 experiments)

	$a_{\Delta,1}$	$a_{\Delta,2}$	$b_{\Delta,1}$	$b_{\Delta,2}$
True	-1.8164	0.7246	0.3173	0.1063
Alg. I	-1.8100 ±0.0114	0.7166 ± 0.0143	0.3122 ±0.0181	0.1150 ±0.0208
Alg. II	-1.8152 ±0.0356	0.7202 ±0.0444	0.3197 ±0.0523	0.1130 ±0.0648

6 Conclusions

We have demonstrated that by sampling the system output at a higher sampling rate than the input rate, the discrete-time model of the plant can also be given by a single-input multiple-output model structure with common denominator polynomial, and some of the subsystems work in open loop. Moreover, the inter-sampled plant input-output holds cyclostationary property. Making use of these properties, both the time domain and frequency domain approaches have been developed to identify the unstable plant disturbed by colored noise.

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