Robust Output Regulation for Autonomous Vertical Landing[1]

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Abstract

In this paper we consider the design of an autopilot for the autonomous landing of a VTOL air vehicle on a ship whose deck oscillates in the vertical direction due to high sea states. The deck motion is modeled as the superposition of a fixed number of sinusoidal functions of time of unknown amplitude and phase. We design an internal-model based error-feedback dynamic regulator that is robust with respect to uncertainties on the mechanical parameters that characterize the model and secures global convergence.

Keywords: VTOL, Robust Tracking, Output Regulation, Global Stabilization, Saturated Controls, Nonlinear Systems.

1 Introduction

This paper deals with the problem of controlling the motion of a Vertical Take Off and Landing (VTOL) air vehicle in uncertain conditions. The specific problem addressed consists in the design of an autopilot for control of the motion on the vertical/lateral plane, so as to secure smooth landing on the deck of a ship which, due to the high sea states, is subject to large vertical deviations. The problem is cast as a tracking problem but, unlike earlier papers on this subject, in our case the autopilot has no direct information about the required reference trajectory (the position of the deck). On the contrary, it has only access to the tracking errors and their first derivatives. This reflects the real situation faced when the information available for feedback is provided by on-board *passive sensors* (yielding relative distances between the aircraft and the ship) and there is not information on the absolute position of the ship.

To mimic an actual motion of a ship in high seas, the reference trajectory is modeled as a linear combination of a fixed number of sinusoidal functions of time of known frequencies and *unknown* amplitudes and phases (the latter ranging within fixed closed intervals). We address the problem as problem of *robust output regulation* and we solve it by using the methods developed in [1],[5]. In particular we design an error-feedback dynamic regulator which embeds an *internal model* of the reference signal and a stabilizer which, in presence of typical uncertainties which affect the system in question, globally asymptotically stabilize the so-called *zero error manifold*. The last task is achieved by means of a design methodology based on *saturated control laws* which, in view of some aspects related to robustness, can be seen as an extension of well-known powerful results (see [8], [2]) about stabilization of *feedforward systems*.

The paper is organized as follows. In the next section the model of the VTOL aircraft is briefly reviewed and the main problem is precisely stated. Section 3 and section 4 present respectively the design of the internal model and of the stabilizer which characterize the regulator. Section 5 describes simulation results which show the performances of the regulator while section 6 concludes with final remarks.

2 Model and tracking problem

In this section we briefly review the main features of the model describing the simplified planar VTOL dynamics. For a more detailed explanation of the physics behind the system the reader is referred to [3] where an exhaustive list of previous works on the argument is also included.
In fig. 1 the control inputs are the thrust directed out

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the bottom of the aircraft, denoted by T, and the rolling moment produced by a couple of equal forces, denoted by F, acting at the wingtips, whose direction is not perpendicular to the horizontal body axis but tilted by some fixed angle α. If M denotes the mass of the aircraft, J the moment of inertia about the center of mass C, l the distance between the wingtips and g the gravitational acceleration, the motion of the aircraft in the lateral-vertical plane is described by the following 6-*th* dimensional system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\sin(\theta_1)\frac{T}{M} + \cos(\theta_1)\frac{2\sin(\alpha)}{M}F \\ \dot{y}_1 &= y_2 \\ \dot{y}_2 &= \cos(\theta_1)\frac{T}{M} + \sin(\theta_1)\frac{2\sin(\alpha)}{M}F - g \\ \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= \frac{2l}{J}\cos(\alpha)F \end{aligned} \tag{1}$$

where x_1, y_1 and θ_1 denote, respectively, the horizontal and vertical position of the center of mass C and the roll angle of the aircraft with respect to the horizon, while x_2, y_2 and θ_2 their respective velocities. To reflect the

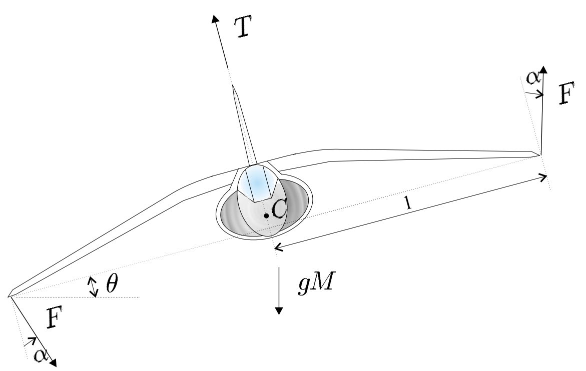

Figure 1: Forces acting on the aircraft.

fact that M, J and α can be affected by uncertainties, we set

$$M = M_0 + M_\Delta \qquad J = J_0 + J_\Delta \qquad \alpha = \alpha_0 + \alpha_\Delta$$

where M_0, J_0, α_0 represent nominal values, while M_Δ, J_Δ, α_Δ are unknown additive quantities which may range within fixed (but not necessarily small) closed intervals. For convenience, we represent these uncertainties as a single point in $\mathbb{R}^3$, as

$$\mu = (M_\Delta, J_\Delta, \alpha_\Delta)\ ,$$

and we denote by $\mathcal{P}$ the compact set within which μ is assumed to range.
The main purpose of the design is to have the aircraft autonomously landing on the deck of a ship *in high seas.* We assume that the position of the landing deck can be thought as a linear combination of a fixed number N of sinusoidal functions of time, whose *amplitudes* and *phases are unknown* constants. Specifically, we model the vertical motion (in the inertial frame) of the landing deck as a function $r(w)$ defined by

$$r(w) = \mathrm{diag}(R_1, \ldots, R_N)w,$$

with $R_i = (1 \quad 0)$, $i = 1, \ldots, N$, where the variable w is generated by an *exosystem*

$$\dot{w} = Sw \qquad w \in \mathbb{R}^{2N} \tag{2}$$

in which

$$S = \mathrm{diag}(S_1, \ldots, S_N), \quad \text{with} \quad S_i = \begin{pmatrix} 0 & \Omega_i \\ -\Omega_i & 0 \end{pmatrix}$$

for $i = 1, \ldots, N$. It is supposed that the initial state of the exosystem $w(0)$ is not known but ranges within fixed compact sets denoted by W.
The goal of the design is to control the aircraft so as to have its output x, θ and y asymptotically tracking the references trajectories

$$\theta_{\mathrm{ref}} = 0 \qquad x_{\mathrm{ref}} = 0 \qquad y_{\mathrm{ref}} = r(w) + H$$

In the latter, H is a *vertical offset*, held constant at some safety value during an initial "learning" phase, in which the feedback law adapts itself to the (unknown) parameters that characterize the ship motion $r(w)$, so as to prevent crashes on the landing deck due to negative vertical errors. Once this phase is over, namely, after the aircraft has learnt how to oscillate vertically at an height H above the landing deck synchronized with the ship, this offset is allowed to gracefully decay to zero so that the aircraft can land smoothly.
We conclude this section describing a few standing hypotheses. Motivated by the abundance of sensors currently available in high performance aircrafts, we assume that the controller has access to the full set of *error variables* and their first derivatives with respect to time, i.e. θ_1, θ_2, x_1, x_2, and e_1, e_2, where $e_1 = y_1 - y_{\mathrm{ref}}$ and $e_2 = \dot{e}_1$. We also assume that the uncertain parameters μ and the initial state of the exosystem $w(0)$ range over known compact sets. So long as the uncertainty α_Δ is concerned, we assume that $|\alpha_\Delta| < \alpha_0$ so that, since α_0 is a small positive angle, $\sin(\alpha) > 0$ and $\cos(\alpha) > 0$ for all α_Δ. Finally, since the thrust input needed to enforce a zero steady state tracking error is given by

$$T = M(g - \ddot{r}(w)), \tag{3}$$

it is easy to realize that a necessary condition to have a positive steady state thrust input (which is an obvious physical requirement) is that

$$\ddot{r}(w) \leq g \qquad \forall\, t < 0. \tag{4}$$

Of course, this restricts the set of admissible initial conditions of the exosystem.

In what follows we cast the problem of nonlinear output regulation, and we solve it using the methods developed in [1] and their robust and "frequency-adaptive" versions recently developed in [5] and [6]. Of course we assume the reader familiar with this theory and we refer to the previous works for the details.

3 Design of the Internal Model

With an eye to the design of the robust stabilizer proposed in [8], we begin by choosing for the controls T and F of (1) the following laws

$$\begin{aligned} T &= \frac{1}{\cos(a\,\mathrm{sat}(\theta_1/a))}[gM_0+u] \\ F &= \frac{J_0}{2\,l\,\cos(\alpha_0)}v \end{aligned} \tag{5}$$

in which $\mathrm{sat}(r)=\mathrm{sgn}(r)\min\{|r|,1\}$ is the standard saturation function, u and v are new inputs (which replace T and F) to be designed, and a is a fixed number satisfying $a<\pi/2$.
Looking at (1)-(5) as a system of the form

$$\begin{aligned} \dot{\mathbf{x}} &= f(\mathbf{x},\mu)+g_u(\mathbf{x},\mu)u+g_v(\mathbf{x},\mu)v \\ \mathbf{y} &= C\mathbf{x}\ , \end{aligned}$$

with $C\mathbf{x}=y_1$, we first solve for $\mathbf{x}=\pi(w,\mu)$, $u=c_u(w,\mu)$ and $v=c_v(w,\mu)$, the *regulator equations*

$$\begin{aligned} \frac{\partial\pi(w,\mu)}{\partial w}S(\varrho)w &= f(\pi(w,\mu),\mu)+g_u(\pi(w,\mu),\mu)c_u(w,\mu) \\ &\quad +g_v(\pi(w,\mu),\mu)c_v(w,\mu) \\ 0 &= C\pi(w,\mu)-r(w)\ . \end{aligned}$$

Simple computations show that

$$c_u(w,\mu)=gM_\Delta+(M_0+M_\Delta)\ddot{r}(w) \qquad c_v(w,\mu)=0\ .$$

Changing the coordinates (y_1,y_2) into

$$(e_1,e_2)=(y_1-r(w)-H,y_2-\dot{r}(w))$$

the so-called *error-system* is obtained which has the form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{[gM_0+u]}{M}\phi_a(\theta_1)+\frac{p_\mu}{M}\cos(\theta_1)v \\ \dot{e}_1 &= e_2 \\ \dot{e}_2 &= \frac{[u-c_u(w,\mu)]}{M}+\frac{[gM_0+u]}{M}\psi_a(\theta_1)+\frac{p_\mu}{M}\sin(\theta_1)v \\ \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= q_\mu v\ . \end{aligned} \tag{6}$$

where the functions $\phi_a(\theta_1)$, $\psi_a(\theta_1)$ are defined as

$$\begin{aligned} \phi_a(\theta_1) &:= \frac{\sin(\theta_1)}{\cos(a\,\mathrm{sat}(\theta_1/a))} \\ \psi_a(\theta_1) &:= \frac{\cos(\theta_1)}{\cos(a\,\mathrm{sat}(\theta_1/a))}-1 \end{aligned}$$

and

$$p_\mu=\frac{J_0\sin(\alpha)}{l\cos(\alpha_0)}, \qquad q_\mu=\frac{J_0\cos(\alpha)}{J\cos(\alpha_0)}\ .$$

We proceed now with the design of the control u, which incorporates an *internal model* of the exogenous inputs. Pick any $2N\times 2N$ Hurwitz matrix F_2, any $2N\times 1$ vector G_2 such that the pair (F_2,G_2) is controllable and a $1\times 2N$ matrix H_2 such that the pair

$$F=\begin{pmatrix} 0 & H_2 \\ -G_2 & F_2 \end{pmatrix}, \qquad G=\begin{pmatrix} 0 \\ G_2 \end{pmatrix} \tag{7}$$

is controllable and F is Hurwitz (the existence of such H_2 derives from standard passivity arguments). Then it is possible to prove ([6]) that there exist a $1\times(1+2N)$ row vector Ψ, of the form

$$\Psi=(\,1 \quad \Psi_2\,)$$

and a map $\bar{\tau}(w,\mu)$ such that the pair $(F+G\Psi,\Psi)$ is observable and the *immersion condition*

$$\frac{\partial\bar{\tau}}{\partial w}Sw=(F+G\Psi)\bar{\tau}(w,\mu), \qquad c_u(w,\mu)=\Psi\bar{\tau}(w,\mu)\ .$$

is satisfied. In view of this we consider, as internal model for our problem, a system of the form

$$\begin{aligned} u &= \Psi\xi+u_{\mathrm{st}} \\ \dot{\xi} &= (F+G\Psi)\,\xi+h(e_1,e_2)\ , \end{aligned} \tag{8}$$

in which $\xi=(\xi_1,\xi_2)\in I\!R\times\mathbb{R}^{2N}$ and $h(e_1,e_2)$ is to be determined. Now consider the change of coordinates which transforms ξ into

$$\chi:=\xi-\bar{\tau}(w,\mu)-GMe_2\ .$$

After some trivial algebra and after choosing

$$h(e_1,e_2)=Gu_{\mathrm{st}}-FGM_0e_2 \tag{9}$$

the subsystem with state variables (χ,e_1,e_2) becomes

$$\begin{aligned} \dot{\chi} &= F\chi+FGM_\Delta e_2-G(\nu+\rho) \\ \dot{e}_1 &= e_2 \\ \dot{e}_2 &= \frac{[\Psi\chi+\Psi GMe_2+u_{\mathrm{st}}]}{M}+\frac{\rho}{M}+\frac{\nu}{M}\ , \end{aligned} \tag{10}$$

in which

$$\nu=p_\mu\sin(\theta_1)v\ , \qquad \rho=[gM_0+\Psi\xi+u_{\mathrm{st}}]\psi_a(\theta_1)\ . \tag{11}$$

In particular note that, by the definition of $\psi_a(\theta_1)$, ρ vanishes whenever $|\theta_1|\le a$.
Choose now the control law

$$u_{\mathrm{st}}=-k_2(e_2+k_1e_1)\ . \tag{12}$$

By means of standard high gain arguments it is possible to show that if k_2 is large enough then system (10) with $\nu=0$ and $\rho=0$, with control law (12) is globally asymptotically stable.

Note also that with this choice of u, the subsystem in question, if we set $\eta^{\mathrm{T}} = (\chi^{\mathrm{T}}, e_1, e_2)$, can be represented in the form

$$\dot{\eta} = A_\eta \eta + B_\eta \nu + P_\eta \rho \qquad (13)$$

with obvious meaning of the matrices A_η, B_η, P_η, in which A_η is a stable matrix.

This subsystem is then coupled with the x, θ subsystem. Simple computations show that, in view of the previous choices and notations, the latter becomes

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & -(g + \ddot{r}(w) + \frac{C_\eta \eta}{M})\phi_a(\theta_1) + \frac{p_\mu}{M}\cos(\theta_1)v \\ \dot{\theta}_1 & = & \theta_2 \\ \dot{\theta}_2 & = & q_\mu v \end{array} \qquad (14)$$

where the coupling with the η-subsystem takes places through the term $C_\eta \eta \phi_a(\theta_1)$ (in which C_η is suitably defined).

The next section will show how to design the control v in order to globally asymptotically stabilize this interconnection.

4 Design of the stabilizer

By the definition of the variable ρ in (11) it is easy to realize that *if* it can be guaranteed that, for some $T_a > 0$,

$$|\theta_1(t)| \leq a, \qquad \text{for all} \qquad t \geq T_a \ , \qquad (15)$$

the overall system can be seen as feedback interconnection of a stable linear system given by

$$\begin{array}{rcl} \dot{\eta} & = & A_\eta \eta + B_\eta p_\mu \sin(\theta_1(t))v \\ \ell & = & \dfrac{\tan(\theta_1(t))}{M} C_\eta \eta + \dfrac{p_\mu \cos(\theta_1(t))}{M} v \end{array} \qquad (16)$$

and system (14) which can be rewritten as

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & -d(t)\varphi(\theta_1)\theta_1 + \ell \\ \dot{\theta}_1 & = & \theta_2 \\ \dot{\theta}_2 & = & q_\mu v \end{array} \qquad (17)$$

having set

$$\varphi(\theta_1) = \frac{\tan(\theta_1)}{\theta_1}, \qquad d(t) = g + \ddot{r}(w). \qquad (18)$$

In view of this it is clear that global asymptotic stability can be achieved if the choice of v renders subsystem (17) *input-to-state stable* with respect to the input ℓ and output v, with a "small" linear gain. We address first the problem of finding a feedback law $v = v(x_1, x_2\theta_1, \theta_2)$ that meets this design goal, namely renders subsystem (17) input-to-state stable, and then we show that this control law makes also (15) fulfilled.

Motivated by powerful well-known results (see [8, 2]) about the stabilization of systems which exhibit a *feedforward structure*, we shall consider for v a feedback law consisting of nested saturation functions. To our purposes, a saturation function is any differentiable function $\sigma : I\!R \to I\!R$ such that $|\sigma'(s)| \leq 2$ for all s, $s\sigma(s) > 0$ for all $s \neq 0$, $\sigma(0) = 0$ and $\sigma(s) = \mathrm{sgn}(s)$ for $|s| \geq 1$. Define the new state variables

$$z_1 := x_1 \qquad z_2 := x_2 + \lambda_1 \sigma(\frac{K_1 z_1}{\lambda_1})$$

$$z_3 := \theta_1 - \lambda_2 \sigma(\frac{K_2 z_2}{\lambda_2}) \qquad z_4 := \theta_2 + \lambda_3 \sigma(\frac{K_3 z_3}{\lambda_3})$$

where λ_i, K_i are positive design parameters, and choose the control law v as

$$v = -\lambda_4 \sigma(\frac{K_4 z_4}{\lambda_4}) \ . \qquad (19)$$

The feedback law thus defined is such that the following result holds.[1]

Proposition 1 *Consider system (17) with control law (19) and assume that*

$$\begin{array}{ccccccc} 0 & < & d_L & \leq & d(t) & \leq & d_U \\ 0 & < & q_L & \leq & q_\mu & & \\ & & 1 & \leq & \varphi(\theta_1) & \leq & \varphi_U \end{array} \qquad (20)$$

for some positive constants d_L, d_U, q_L and φ_U. Then, there exist positive numbers r^* and c^*, positive numbers c_2c_3, c_4 and $\gamma_1, \gamma_3, \gamma_4$ such that, if

$$K_2 = c_2 K_1 \qquad K_3 = c_3 K_1 \qquad K_4 = c_4 K_1 \qquad (21)$$

and

$$\begin{array}{rcl} \lambda_1 & = & \frac{\gamma_1}{K_1}\lambda_2 \\ \lambda_3 & = & \gamma_3 K_1 \lambda_2 \\ \lambda_4 & = & \gamma_4 K_1^2 \lambda_2 \ , \end{array} \qquad (22)$$

system (17) with control law (19) is input-to-state stable, with no restriction on the initial state, restriction $r^* \lambda_2$ on the input ℓ, and

$$\limsup_{t\to\infty} |v(t)| \leq c^* K_1^2 \limsup_{t\to\infty} |\ell(t)| \qquad (23)$$

for all $K_1 > 0$ and $\lambda_2 > 0$.

It is just worth stressing that the previous result leaves the two design parameters λ_2 and K_1 free. These degrees of freedom will be used in order to decrease the gain between the input ℓ and the output v and to make the restriction on ℓ fulfilled from one hand, and to make the crucial condition (15) satisfied on the other.

Returning to the analysis of the stability of the entire

[1] In what follows, we use the notion of input-to-state stability "with restrictions", introduced in [8]. The reader is referred to this paper for a precise definition.

system observe that, if the condition (15) holds, the hypotheses (20) of Proposition 1 hold. Indeed, the standing hypotheses described in Section 2 and in particular the assumption (4) (introduced to avoid negative thrust during the steady state), imply that the function $d(t)$ defined in (18) is bounded (from below and from above) by positive numbers. Likewise, the parameter q_μ is bounded from below and the function $\varphi(\theta_1)$ defined in (18) satisfies $1 \leq \varphi(\theta_1) \leq \tan(a)/a$. We are thus in the condition of using the result of Proposition 1 to impose a "small gain" on the system (17). To this end consider system (16). Using again the hypothesis (15), we deduce that this system is input-to-state stable, with no restriction on the initial state and on the input, and that there exists some number γ^*, independent of μ, such that

$$\limsup_{t\to\infty} |\ell(t)| \leq \gamma^* \limsup_{t\to\infty} |v(t)| \ .$$

Thus to claim the asymptotic stability of the overall system by the small gain theorem we have to impose the fulfillment of the small-gain condition

$$\gamma^* c^* K_1^2 < 1 \tag{24}$$

as well as to make sure that the restriction on the input ℓ to system (17) is respected. Since by construction $\|v(\cdot)\|_\infty \leq \lambda_4$ and $\lambda_4 = \gamma_4 K_1^2 \lambda_2$, this occurs if

$$\gamma^* \gamma_4 K_1^2 < r^* \ . \tag{25}$$

Indeed, both (24) and (25) can be fulfilled by appropriate choice of K_1.
Observe now that the overall system has not finite escape time, and therefore we can claim the global asymptotic stability of the system in question if the critical hypothesis (15) can be fulfilled. As a matter of fact this can be accomplished by a proper choice of the last free design parameter λ_2

Proposition 2 *Consider system (17) with the control law (19). Let K_3, K_4 be fixed positive numbers and suppose*

$$\lambda_3 = \bar{\gamma}_3 \lambda_2 \, , \qquad \lambda_4 = \bar{\gamma}_4 \lambda_2 \tag{26}$$

for some fixed positive numbers $\bar{\gamma}_3, \bar{\gamma}_4$. Then there exists a number λ_2^* such that, for all $\lambda_2 \leq \lambda_2^*$ there exist $T_a > 0$ such that

$$|\theta_1(t)| \leq a \quad \text{for all} \quad t \geq T_a .$$

For reasons of space the proofs of Proposition 1 and 2 cannot be included here (see [7] for full details).

5 Simulation Results

We conclude this paper by presenting simulation results which show the performance of the proposed design in

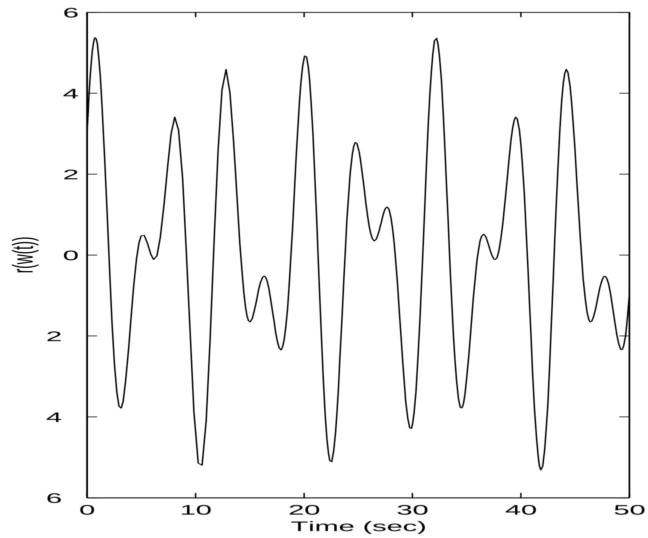

Figure 2: Reference signal (vertical position of the landing deck) in case the initial condition of the exosystem is $w(0) = (2, 2.2, 1, 2.2)$.

case the reference signal $r(w)$ is generated by an exosystem (2) with $N = 2$, namely the superposition of two sinusoidal functions of time whose amplitudes A_1, A_2 and phases φ_1, φ_2 are unknown but ranging within fixed compact sets.
Assuming $\Omega_1 = 1\,rad/sec$ and $\Omega_2 = 1.6\,rad/sec$, we have supposed the reference signals (see fig.2) corresponding to the initial condition for the exosystem: $w(0) = (2, 2.2, 1, 2.2)$. In the design of the controller, the nominal values

$$M_0 = 5 \cdot 10^4 \ Kg, \quad J_0 = 1.25 \cdot 10^4 \ Kg\,m^2 \quad \alpha_0 = 4^o \tag{27}$$

and $l = 5\,m$, $a = \pi/3$, $H = 15\,m$ have been assumed. The internal model has been designed as described in section 3 and the parameters of the high gain control law $u_{\rm st}$ (see (12)) and of the saturated control law (19) have been chosen assuming uncertainties $(M_\Delta, J_\Delta, \alpha_\Delta)$ up to fifty percent of the nominal values (27) and assuming that $\ddot{r}(w(t)) - g \geq 0.5 \ t \geq 0$. The simulation results corresponds to the values of the uncertain parameters $M = 4 \cdot 10^4\,Kg$, $J = 1 \cdot 10^4\,Kg\,m^2$ and $\alpha = 2^o$ and to the aircraft in initial conditions $(x(0), y(0), \theta(0)) = (20, 20, \pi/3)$ and zero velocities.
The two figures fig.3 and fig.4 show the vertical tracking error and the lateral/roll error. To stress the importance of correctly setting the right frequencies in the internal model, we have run the simulation, up to time $t = 50\,sec$, with a wrong internal model (in particular $\Omega_1 = 0.8\,rad/sec$ and $\Omega_2 = 2.8\,rad/sec$). At time $t = 50\,sec$ the right internal model is activated. In both the cases, before $t = 50\,sec$ a large steady-state tracking error is observed, due to the wrong guess of Ω_1, Ω_2. After time $t = 50\,sec$ the tracking error quickly decays to zero, and the aircraft keeps oscillating, at an height of $15\,m$, synchronized with the landing deck. At

time $t = 100\,sec$, when the vertical, lateral and roll errors have become negligible, the vertical offset H, introduced for safety reasons, is let to decay to zero, so that the aircraft lands smoothly on the landing deck.

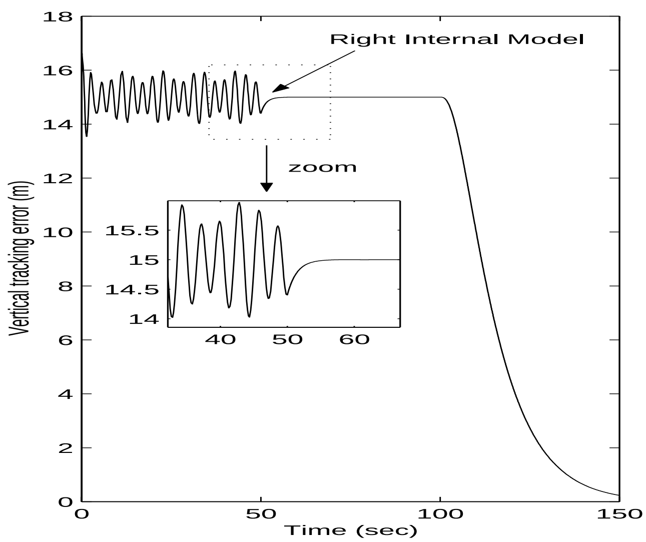

Figure 3: Vertical tracking error. At $t = 50\,sec$ the right internal model is activated and the aircraft oscillates synchronized with the ship. At $t = 100\,sec$ the vertical offset is smoothly steered to zero and the aircraft lands of the platform.

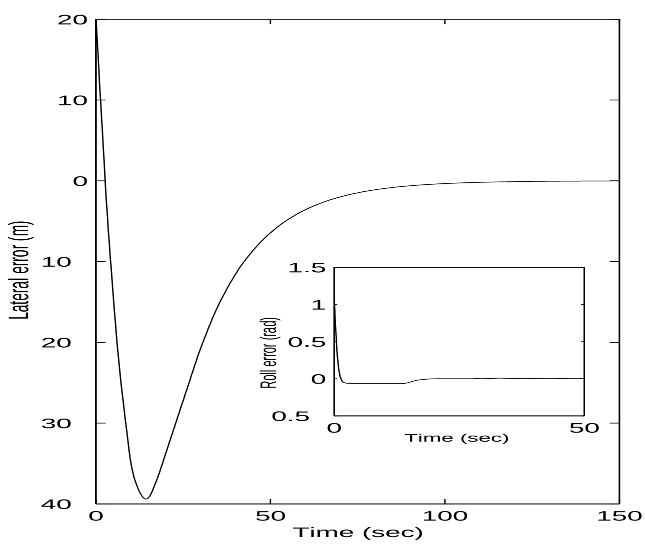

Figure 4: Lateral and roll errors.

6 Conclusions

In this paper we designed an autopilot for VTOL aircraft able to solve the problem of autonomous landing on oscillating platform. The problem has been approached as a robust nonlinear regulator problem and an internal-model based error-feedback dynamic regulator have been shown to solve the problem. The regulator secures global convergence to the zero error manifold and turns out to be robust against typical uncertainties which affect the system.

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