Satisfactory Optimization Control with Fuzzy Constraints and Goals

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Abstract—**This paper presented a satisfactory optimization algorithm with fuzzy goals and fuzzy constraints under the framework of predictive control, it allows for more flexible aggregation of the control objectives than the usual weighting sum of squared errors. Compared to the standard quadratic objective function, the designer has more freedom in specifying the desired process behavior.**

Ⅰ. INTRODUCTION

The main idea of the satisfactory control is to satisfy the multi-requirement of production with limited manipulated degree of freedom, inducing the performance indexes optimization and various soft and hard constraints. The importance for different requirements is defined by decision-maker and guaranteed by the control algorithm, to construct a man-machine cooperative control mode to make user satisfactory. In relevant literature, such as [1,2], the different performance indexes and constraints are definitively described, the weights of all kinds of importance for difficult requirements are only expressed with different coefficients, which undoubtedly make difficult to decide the satisfactory control. In fact, the requirements for the performance indexes and the tolerance for the variable constraints are all fuzzy, only using a simple coefficient to describe them clearly is often impossible. In order to overcome the above disadvantage and improve the exactness of importance for various requirements, we introduce the fuzzy inference into the satisfactory control.

In this paper, we present a satisfactory optimal control with fuzzy constraints and fuzzy goals, make the limited horizon optimal problem in the fuzzy environment become the equivalent definite programming problem.

Ⅱ. DESCRIPTION OF PROBLEM

In the traditional programming, the constraint conditions can not be exceeded and changed, but in the satisfactory optimal control, some of constraints are adjustable, called 'soft constraints'. Thus, every constraint variable can be adjusted within a limit boundary and has a function to reflect the fuzziness of constraint variable boundary defined by decision-maker. We can use the fuzzy variable to describe this case. For fuzzy variable $\tilde{b}$, we define the membership function $\mu(\tilde{b})$, $0 \le \mu \le 1$, which express the degree of membership. $\mu = 1$ indicates the corresponding fuzzy variable belongs to this set, conversely for $\mu = 0$. In fact, we can understand μ as the degree of satisfactory degree. Then the degree of membership is expressed as follows:

$$\mu(b) = \begin{cases} 0, & b < b_{\min} - p_1 \\ 1 - \dfrac{b_{\min} - b}{p_1}, & b_{\min} - p_1 \le b < b_{\min} \\ 1, & b_{\min} \le b \le b_{\max} \\ 1 - \dfrac{b - b_{\max}}{p_2}, & b_{\max} < b \le b_{\max} + p_2 \\ 0, & b_{\max} < b \end{cases} \tag{1}$$

where p_1、p_2 is called fuzzy width or tolerant width, $b_{\min}$、$b_{\max}$ is the expected boundary of fuzzy variable $\tilde{b}$. Obviously, when the fuzzy width is *zero*, it corresponds the 'hard constraint'.

Ⅲ. FUZZY CONSTRAINTS IN SATISFACTORY CONTROL

A. Model-based predictive control

Satisfactory control is actually the predictive control based on the model. It in essence utilizes systems predictive information to optimize the performance index within a finite horizon. In order to overcome the uncertainty, we take the receding horizon strategy in predictive control. The predictive output $\hat{y}(k+i)$, $i = N_1, \cdots, N_2$ is derived from the information at current time t and the future control signal $u(k+i)$, $i = 1, \cdots, N_u$, where $[N_1, N_2]$ is the predictive horizon. The objective to be optimized is:

$$J = \sum_{i=N_1}^{N_2} (\hat{e}(k+i))^2 + \sum_{i=1}^{N_u} \lambda_i (\Delta u(k+i-1))^2 \tag{2}$$

where $\hat{e}(k+i)$ is the predictive error, $\Delta u(k+i-1)$ is the control increment, λ_i is the weight coefficient of the control signal.

The system can be described by the CARIMA model:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + \frac{C(q^{-1})\xi(t)}{\Delta} \tag{3}$$

The predictive equation is:

$$\hat{\boldsymbol{y}} = \boldsymbol{G}\tilde{\boldsymbol{u}} + \boldsymbol{f}$$

where

$$\hat{\boldsymbol{y}} = \left[\hat{y}^T(t+1 \mid t), \ \cdots, \ \hat{y}^T(t+N_2 \mid t)\right]^T,$$

$$\tilde{\boldsymbol{u}} = \left[\Delta u^T(t), \ \cdots, \ \Delta u^T(t+N_u)\right]^T,$$

$$\boldsymbol{f} = \left[f_1^T(t), \ \cdots, \ f_{N_2}^T(t)\right]^T.$$

The control law is:

$$u(t) = u(t-1) + g^T \ (\boldsymbol{w} - \boldsymbol{f}) \qquad (4)$$

where g^T is the former m lines of the matrix $(\mathbf{G}^T\mathbf{G} + \lambda \boldsymbol{I})^{-1}\mathbf{G}^T$, the significance of parameters see also [3].

B. Handling the fuzzy constraints

In this section, we discuss in detail how to deal with the fuzzy boundary optimization. Firstly, consider the control variable u and output variable y in constraint equation. They are all decided by the control increment Δu during the receding horizon optimization in GPC algorithm. The boundary condition can be expressed as:

$$\mathbf{A}\Delta\mathbf{u}\ (t) \le \mathbf{b}(t) \qquad (5)$$

$\mathbf{b}\ (t)$ in equation (5) is transformed from the boundary expression (1), and this transformation is only a series of displacement and inverse, thus under the fuzzy boundary condition, the derived $\tilde{\mathbf{b}}\ (t)$ still has the same form as the nonfuzzy constraints, and the fuzzy width of all fuzzy variables are not changed. It can be expressed as:

$$\mathbf{A}\Delta\mathbf{u}\ (t) \le \tilde{\mathbf{b}}(t) \qquad (6)$$

Ⅳ. OPTIMAL ALGORITHM BASED ON FUZZY PROGRAMMING

From the constraints expression (6) and its optimal performance index (normally in quadratic form), we can get the optimal control acted on the next time. The main difficulty is that the constraints are fuzzy variables. This is a fuzzy programming problem. In fact, the fuzziness of constraint implies an optimal performance index, which is to make the membership degree of fuzzy constraint maximal, that is:

$$\max \quad \tilde{J} = \min_{\Omega} \mu_{\Omega} \qquad (7)$$

where $\tilde{J}$ represents the objective function of fuzzy constraint, Ω represents the fuzzy space of constraint, and μ_Ω is the membership degree of fuzzy variable in Ω, and $\min\limits_{\Omega} \mu_\Omega$ is to minimize the membership degree of all fuzzy variables.

From the performance index (7), we can define the variable μ as:

$$\mu = \min_{\Omega} \mu_{\Omega} \qquad (8)$$

Combined with optimal performance index (12), we have:

$$\begin{aligned} &\max \quad \tilde{J} = \mu \\ &\mu \le \mu_i, \quad \forall \mathbf{b}_i \end{aligned} \qquad (9)$$

from the district $\mathbf{b}_i(t) < \mathbf{A}_i\Delta\mathbf{u}(t) \le \mathbf{b}_i(t) + \mathbf{p}_i$, we can substitute and eliminate μ_i, and then the above inequality can be rewritten as:

$$\mu \le 1 - \frac{\mathbf{A}_i\Delta\mathbf{u}(t) - \mathbf{b}_i(t)}{\mathbf{p}_i}, \quad \forall \mathbf{b}_i \quad \mathbf{A}_i\Delta\mathbf{u}(t) + \mathbf{p}_i\mu \le \mathbf{b}_i(t) + \mathbf{p}_i$$

Defining matrix $\overline{\mathbf{A}}$ 、 $\overline{\mathbf{b}}(t)$ as:

$$\begin{aligned} &\overline{\mathbf{A}} = [\mathbf{A} \quad \mathbf{p}] \\ &\overline{\mathbf{b}}(t) = \mathbf{b}(t) + \mathbf{p} \end{aligned} \qquad (10)$$

and define the optimal variable as:

$$\mathbf{x} = [\Delta u^T(t) \quad \mu]^T \qquad (11)$$

then the fuzzy programming can be transformed as the following standard programming problem:

$$\begin{aligned} &\max \quad \tilde{J} = \mu \\ &\overline{\mathbf{A}}\mathbf{x} \le \overline{\mathbf{b}} \end{aligned} \qquad (12)$$

Algorithm Steps:

(1) Solve the programming problem (12) and get the satisfactory degree μ.

(2) If $\mu = 1$, it means there is a feasible solution without needing to adjust boundary. If $0 < \mu < 1$, it means that the boundary should be adjusted. From the definition of satisfactory degree, we can get the boundary $\mathbf{b}'(t)$ after adjusting: $\mathbf{b}'(t) = \mathbf{b}(t) + \mu\mathbf{p}$. Substituting $\mathbf{b}'(t)$ for $\mathbf{b}(t)$ in (5), and combining (2), we can derive the control action of quadratic programming.

(3) Back to step (1) at the next sample time.

Ⅴ. CONCLUSION

By defining the membership degree of the control objective and system constraint, and using the fuzzy interference, the optimal control problem with constraint, multi-objective multi-degree of freedom can be transferred as a convex optimal problem, so as to utilize the efficient optimal algorithm and guarantee the global optimal solution.

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