Decentralized Stabilization of Fuzzy Large-scale Systems

Feng-Hsiag Hsiao†*, Jiing-Dong Hwang,‡ and Lin-Goei Shiau§

†Department of Electrical Engineering
Chang Gung University, 259
Wen-Hwa 1st Road, Kwei-San
Taoyuan Shian, Taiwan 333, R. O. C.

‡Department of Electronic Engineering
Jin-Wen Institute of Technology, 99
An Chung Road, Hsin Tien
Taipei, Taiwan 231, R.O.C.

§Department of Electrical Engineering
Kun Shan University of Technology
Yung Kang Hsiang, Tainan Shian,
Taiwan 710, R.O.C.

Abstract

A stability criterion in terms of Lyapunov's direct method is derived in this paper to guarantee the asymptotic stability of fuzzy large-scale systems. Based on this criterion and the decentralized control scheme, a set of fuzzy controllers is synthesized via the technique of parallel distributed compensation (PDC) to stabilize a fuzzy large-scale system which consists of a few interconnected subsystems represented by Takagi-Sugeno (T-S) fuzzy models. Finally, a numerical example with simulations is given to illustrate the results.

Key words— Large-scale systems, T-S fuzzy models, parallel distributed compensation.

I. Introduction

A great number of problems today are brought about by the present technology, societal and environmental processes which are highly complex, large in dimension, and stochastic by nature. The field of large-scale systems exists so widely that covers either the fundamental theory of modelling, optimization, and control or certain particular aspects and applications. In addition, large-scale systems analysis, design, and control theory has attained considerable maturity and sophistication and is receiving increasing attention from the theorists and practitioners due to their methodological interests and important real-life applications [1]. In real systems, the large-scale systems include electric power systems, nuclear reactors, aerospace systems, large electric networks, economic systems, process control systems, chemical and petroleum industries, different types of societal systems, and ecological systems. Such systems consist of a number of interdependent subsystems which serve particular functions, share resources, and are governed by a set of interrelated goals and constraints [2]. Recently, many approaches have been used to investigate the stability and stabilization of large-scale systems, as proposed in the literature [3-6].

Fuzzy control has attracted a great deal of attention from both the academic and industrial communities in the past few years, and there have been many successful applications. In spite of the success, there are still many basic issues that remain to be further addressed. Stability analysis and systematic design are certainly among the most important issues for fuzzy control systems. During the last decade, there have been significant research efforts on these issues (see [7-13] and the references therein). However, as far as we know, the stabilization problem of fuzzy large-scale systems remains unresolved.

Hence, a stability criterion in terms of Lyapunov's direct method is derived in this study to guarantee the asymptotic stability of fuzzy large-scale systems. According to this criterion and the decentralized control scheme, a set of fuzzy controllers is synthesized to stabilize a fuzzy large-scale system composed of several interconnected subsystems. Moreover, each subsystem is represented by a Takagi-Sugeno (T-S) type fuzzy

* Author to whom all correspondence should be addressed (fhhsiao@mail.cgu.edu.tw).

model. In this type of fuzzy model, each fuzzy implication is expressed by a linear system model, which allows us to use linear feedback control as in the case of feedback stabilization. The control design is carried out based on the fuzzy model via the concept of PDC scheme. The idea is that a linear feedback control is designed for each local linear model. The resulting overall fuzzy controller, which is nonlinear in general, is a fuzzy blending of each individual linear controller [8, 11].

This paper is organized as follows. First, the T-S fuzzy model is briefly reviewed and the concept of PDC scheme is utilized to design fuzzy controllers. Then, based on Lyapunov's approach, a stability criterion is derived to guarantee the asymptotic stability of fuzzy large-scale systems. Finally, a numerical example with simulations is given to illustrate the results, and the conclusions are drawn.

II. System Description

Consider a fuzzy large-scale system $\boldsymbol{F}$ composed of J interconnected subsystems F_j, $j=1,2,\cdots,J$. The jth isolated subsystem (without interconnection) of $\boldsymbol{F}$ is represented by a T-S fuzzy model which is described by fuzzy IF-THEN rules. The main feature of T-S fuzzy models is to express each rule by a linear state equation, and the ith rule of this fuzzy model is of the following form [8]:

Rule i : IF $x_{1j}(t)$ is M_{i1j} and $\cdots$ and $x_{gj}(t)$ is M_{igj}

$$\text{THEN} \quad \dot{x}_j(t) = A_{ij}x_j(t) + B_{ij}u_j(t) \qquad (2.1)$$

where $x_j^T(t) = [x_{1j}(t), x_{2j}(t), \cdots, x_{gj}(t)]$

$$u_j^T(t) = [u_{1j}(t),\ u_{2j}(t), \cdots, u_{mj}(t)]$$

$i=1,2,\cdots, r_j$ and r_j is the number of IF-THEN rules of the jth subsystem; A_{ij} and B_{ij} are two matrices with appropriate dimensions, $x_j(t)$ is the state vector, $u_j(t)$ is the input vector, M_{ipj} ($p=1,2,\cdots,g$) are the fuzzy sets, and $x_{1j}(t) \sim x_{gj}(t)$ are the premise variables. The final state of this fuzzy dynamic system is inferred as follows:

$$\dot{x}_j(t) = \frac{\sum_{i=1}^{r_j} w_{ij}(t)(A_{ij}x_j(t) + B_{ij}u_j(t))}{\sum_{i=1}^{r_j} w_{ij}(t)}$$

$$= \sum_{i=1}^{r_j} h_{ij}(t)(A_{ij}x_j(t) + B_{ij}u_j(t)) \qquad (2.2)$$

with

$$w_{ij}(t) = \prod_{p=1}^{g} M_{ipj}(x_{pj}(t)),\quad h_{ij}(t) = \frac{w_{ij}(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} \qquad (2.3)$$

in which $M_{ipj}(x_{pj}(t))$ is the grade of membership of $x_{pj}(t)$ in M_{ipj}. In this paper, it is assumed that $w_{ij}(t) \ge 0$, $i = 1, 2, \cdots, r_j$; $j=1,2,\cdots,J$ and $\sum_{i=1}^{r_j} w_{ij}(t) > 0$ for all t. Therefore, $h_{ij}(t) \ge 0$, and $\sum_{i=1}^{r_j} h_{ij}(t) = 1$ for all t.

Based on the above analysis, the jth subsystem F_j with interconnections can be described as follows:

$$F_j : \begin{cases} \dot{x}_j(t) = \sum_{i=1}^{r_j} h_{ij}(t)\left[A_{ij}x_j(t) + B_{ij}u_j(t)\right] + \phi_j(t) & (2.4a) \\ \phi_j(t) = \sum_{\substack{n=1 \\ n \ne j}}^{J} C_{nj}x_n(t), & (2.4b) \end{cases}$$

where C_{nj} is the interconnection matrix between the nth and jth subsystems.

Definition 2.1 [10, 11]: If the pairs (A_{ij}, B_{ij}), $i=1,2,\cdots, r_j$ are controllable, the fuzzy dynamic model (2.2) is called locally controllable.

For the fuzzy controller design, the fuzzy dynamic model (2.2) is assumed to be locally controllable. In the next section, the concept of PDC scheme is utilized to design fuzzy controllers and a stability criterion is proposed to guarantee the asymptotic stability of fuzzy large-scale systems.

III. Parallel Distributed Compensation

On the basis of the decentralized control scheme, a set of fuzzy controllers is synthesized via the technique of parallel distributed compensation (PDC) to stabilize the fuzzy large-scale system $\boldsymbol{F}$. The concept of PDC scheme is that each control rule is distributively designed for the corresponding rule of a T-S fuzzy model. The fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts [9]. Since each rule of the fuzzy model is described by a linear state equation, linear control theory can be used to design the consequent parts of a fuzzy controller. The resulting overall fuzzy controller, nonlinear in general, is achieved by fuzzy blending of each individual linear controller.

Hence, the *j*th fuzzy controller can be described as follows:

Rule *i*: IF $x_{1j}(t)$ is M_{i1j} and $\cdots$ and $x_{gj}(t)$ is M_{igj}

$$\text{THEN} \quad u_j(t) = -K_{ij} x_j(t), \tag{3.1}$$

where i =1, 2,$\cdots$, r_j. The final output of this fuzzy controller is

$$u_j(t) = -\frac{\sum_{i=1}^{r_j} w_{ij}(t) K_{ij} x_j(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} = -\sum_{i=1}^{r_j} h_{ij}(t) K_{ij} x_j(t). \tag{3.2}$$

Substituting Eq. (3.2) into Eq. (2.4), we have the *j*th closed-loop subsystem:

$$\dot{x}_j(t) = \sum_{i=1}^{r_j}\sum_{f=1}^{r_j} h_{ij}(t) h_{fj}(t)[A_{ij} - B_{ij}K_{fj}]x_j(t) + \phi_j(t). \tag{3.3}$$

A stability criterion is given below to guarantee the asymptotic stability of the fuzzy large-scale system ***F***.

Theorem 1: The fuzzy large-scale system ***F*** is asymptotically stable, if there exist positive definite matrices $P_j = P_j^T > 0, \quad j = 1, 2, \cdots, J$ and the feedback gains $K_{ij}'s$ are chosen to satisfy

(I) $\hat{\lambda}_{ij} = \lambda_m(Q_{ij}) - \beta_j > 0$ and $\tilde{\lambda}_{ifj} = \lambda_m(Q_{ifj}) - \beta_j > 0$

for $i = 1, 2, \cdots, r_j$; $i < f \le r_j$; $j = 1, 2, \cdots, J$ (3.4)

or

(II) $$\Lambda_j = \begin{bmatrix} \hat{\lambda}_{1j} & \tilde{\lambda}_{12j} & \cdots & \tilde{\lambda}_{1r_j j} \\ \tilde{\lambda}_{12j} & \hat{\lambda}_{2j} & \cdots & \tilde{\lambda}_{2r_j j} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\lambda}_{1r_j j} & \tilde{\lambda}_{2r_j j} & \cdots & \hat{\lambda}_{r_j j} \end{bmatrix} > 0,$$

for $j = 1, 2, \cdots, J$ (3.5)

where

$$\beta_j = \sum_{\substack{n=1 \\ n \ne j}}^{J} \left(\left\| C_{nj}^T P_j \right\| + \left\| C_{jn}^T P_n \right\| \right),$$

$$Q_{ij} = -[(A_{ij} - B_{ij}K_{ij})^T P_j + P_j(A_{ij} - B_{ij}K_{ij})], \tag{3.6}$$

$$Q_{ifj} = -(G_{ifj}^T P_j + P_j G_{ifj}), \tag{3.7}$$

with

$$G_{ifj} = \frac{(A_{ij} - B_{ij}K_{fj}) + (A_{fj} - B_{fj}K_{ij})}{2},$$

and $\lambda_m(Q_{ij})$ as well as $\lambda_m(Q_{ifj})$ denote the minimum eigenvalues of Q_{ij} and Q_{ifj}, respectively.

Remark 1: In principle, both the conditions Eq. (3.4) and Eq. (3.5) can be used to test the asymptotic stability of the fuzzy large-scale system ***F***. It is therefore reasonable to check the asymptotic stability with either one of the conditions and then, if it fails, to resort the other.

IV. Example

Consider a fuzzy large-scale system composed of three interconnected subsystems which are described as follows:

Subsystem 1:

Rule 1: If $x_{11}(t)$ is M_{111}

Then $\dot{x}_1(t) = A_{11}x_1(t) + B_{11}u_1(t)$

Rule 2: If $x_{11}(t)$ is M_{211}

Then $\dot{x}_1(t) = A_{21}x_1(t) + B_{21}u_1(t)$,

where $x_1^T(t) = [x_{11}(t) \;\; x_{21}(t)]$, $A_{11} = \begin{bmatrix} -29 & 1 \\ 3 & -12 \end{bmatrix}$,

$$A_{21} = \begin{bmatrix} -25 & -4 \\ 5 & -14 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 0.5 \\ -2 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 0.3 \\ 1 \end{bmatrix} \tag{4.1}$$

and membership functions for Rule 1 and Rule 2 are

$$M_{111}(x_{11}(t)) = \frac{1}{1 + \exp[-2x_{11}(t)]},$$

$$M_{211}(x_{11}(t)) = 1 - M_{111}(x_{11}(t)).$$

Subsystem 2:

Rule 1: If $x_{12}(t)$ is M_{112}

Then $\dot{x}_2(t) = A_{12}x_2(t) + B_{12}u_2(t)$

Rule 2: If $x_{12}(t)$ is M_{212}

Then $\dot{x}_2(t) = A_{22}x_2(t) + B_{22}u_2(t)$,

where $x_2^T(t) = [x_{12}(t) \;\; x_{22}(t)]$, $A_{12} = \begin{bmatrix} -30 & 1 \\ -5 & -16 \end{bmatrix}$,

$$A_{22}=\begin{bmatrix}-25 & 1\\ -6 & -13\end{bmatrix},\quad B_{12}=\begin{bmatrix}0.2\\ 2\end{bmatrix},\quad B_{22}=\begin{bmatrix}0.6\\ -3\end{bmatrix} \qquad (4.2)$$

and membership functions for Rule 1 and Rule 2 are

$$M_{112}(x_{12}(t)) = \exp[-x_{12}^2(t)],$$

$$M_{212}(x_{12}(t)) = 1 - M_{112}(x_{12}(t)).$$

Subsystem 3:

Rule 1: If $x_{13}(t)$ is M_{113}

Then $\dot{x}_3(t) = A_{13}x_3(t) + B_{13}u_3(t)$

Rule 2: If $x_{13}(t)$ is M_{213}

Then $\dot{x}_3(t) = A_{23}x_3(t) + B_{23}u_3(t)$,

where $x_3^T(t) = [x_{13}(t)\ \ x_{23}(t)]$, $A_{13}=\begin{bmatrix}-37 & 2\\ 2 & -13\end{bmatrix}$,

$$A_{23}=\begin{bmatrix}-34 & -3\\ 3 & -14\end{bmatrix},\quad B_{13}=\begin{bmatrix}0.8\\ -2\end{bmatrix},\quad B_{23}=\begin{bmatrix}0.9\\ 1\end{bmatrix} \qquad (4.3)$$

and membership functions for Rule 1 and Rule 2 are

$$M_{113}(x_{13}(t)) = \frac{1}{1+\exp[-4x_{13}(t)]},$$

$$M_{213}(x_{13}(t)) = 1 - M_{113}(x_{13}(t)).$$

Moreover, the interconnection matrices among three subsystems are given in the following:

$$C_{21}=\begin{bmatrix}1.5 & -2.1\\ -1 & 3\end{bmatrix},\quad C_{31}=\begin{bmatrix}5 & 4.5\\ 3 & 2.5\end{bmatrix},\quad C_{12}=\begin{bmatrix}2 & -3\\ -1.4 & 1.5\end{bmatrix},$$

$$C_{32}=\begin{bmatrix}1 & -2.4\\ -1.4 & 1.2\end{bmatrix}, C_{13}=\begin{bmatrix}2 & -0.5\\ -0.6 & 0.5\end{bmatrix}, C_{23}=\begin{bmatrix}1 & -1.4\\ 1.2 & -0.3\end{bmatrix}. \qquad (4.4)$$

Therefore, the fuzzy large-scale system can be summarized as

$$F:\begin{cases}\dot{x}_1(t)=\sum_{i=1}^{2}h_{i1}(t)[A_{i1}x_1(t)+B_{i1}u_1(t)]+\phi_1(t) & (4.5a)\\ \dot{x}_2(t)=\sum_{i=1}^{2}h_{i2}(t)[A_{i2}x_2(t)+B_{i2}u_2(t)]+\phi_2(t) & (4.5b)\\ \dot{x}_3(t)=\sum_{i=1}^{2}h_{i3}(t)[A_{i3}x_3(t)+B_{i3}u_3(t)]+\phi_3(t) & (4.5c)\\ \phi_j(t)=\sum_{\substack{n=1\\ n\neq j}}^{3}C_{nj}x_n(t). & (4.5d)\end{cases}$$

Since the pairs (A_{ij}, B_{ij}), $i = 1, 2$; $j = 1, 2, 3$ are all controllable, subsystems $1-3$ are locally controllable. In order to stabilize the fuzzy large-scale system (4.5), three fuzzy controllers which are designed via the concept of PDC scheme are described as follows.

Fuzzy controller of subsystem 1:

Rule 1: If $x_{11}(t)$ is M_{111}

Then $u_1(t) = -K_{11}x_1(t)$, (4.6a)

Rule 2: If $x_{11}(t)$ is M_{211}

Then $u_1(t) = -K_{21}x_1(t)$. (4.6b)

Choosing the closed-loop eigenvalues $(-16, -20)$ for $A_{11} - B_{11}K_{11}$ and the closed-loop eigenvalues $(-22, -17)$ for $A_{21} - B_{21}K_{21}$, we have $K_{11} = [-11.4815\ \ -0.3704]$ and $K_{21} = [-0.5161\ \ 0.1548]$.

Fuzzy controller of subsystem 2:

Rule 1: If $x_{12}(t)$ is M_{112}

Then $u_2(t) = -K_{12}x_2(t)$, (4.7a)

Rule 2: If $x_{12}(t)$ is M_{212}

Then $u_2(t) = -K_{22}x_2(t)$, (4.7b)

Choosing the closed-loop eigenvalues $(-29, -13)$ for $A_{12} - B_{12}K_{12}$ and the closed-loop eigenvalues $(-19, -24)$ for $A_{22} - B_{22}K_{22}$, we have $K_{12} = [-14.2857\ \ -0.5714]$ and $K_{22} = [0.5495\ \ -1.5568]$.

Fuzzy controller of subsystem 3:

Rule 1: If $x_{13}(t)$ is M_{113}

Then $u_3(t) = -K_{13}x_3(t)$, (4.8a)

Rule 2: If $x_{13}(t)$ is M_{213}

Then $u_3(t) = -K_{23}x_3(t)$. (4.8b)

Choosing the closed-loop eigenvalues $(-31, -23)$ for $A_{13} - B_{13}K_{13}$ and the closed-loop eigenvalues $(-20, -16)$ for $A_{23} - B_{23}K_{23}$, we have $K_{13} = [-4.0426\ \ -3.6170]$ and $K_{23} = [-11.7542\ \ -1.4213]$.

In order to satisfy the stability conditions of Theorem 1, the matrices Q_{ij}'s in Eq. (3.6) must be positive definite. Hence, we can obtain the following matrices P_j (j =1, 2, 3) by using LMI optimization algorithms such that Q_{ij}, $i = 1, 2$; $j = 1, 2, 3$ are all positive definite:

$$P_1=\begin{bmatrix}1.5062 & -0.2794\\ -0.2794 & 1.7619\end{bmatrix},\quad P_2=\begin{bmatrix}1.3865 & 0.3153\\ 0.3153 & 1.4738\end{bmatrix},$$

$$P_3 = \begin{bmatrix} 1.3662 & 0.0876 \\ 0.0876 & 1.9350 \end{bmatrix}. \quad (4.9)$$

Substituting Eqs. $(4.1-4.3$, $4.9)$ and the feedback gains $K_{ij}'s$ in Eqs. $(4.6-4.8)$ into Eqs. $(3.6-3.7)$ yields

$$Q_{11} = \begin{bmatrix} 58.9131 & 23.3305 \\ 23.3305 & 45.5584 \end{bmatrix}, \quad Q_{21} = \begin{bmatrix} 77.9267 & -14.5195 \\ -14.5195 & 47.6183 \end{bmatrix},$$

$$Q_{121} = \begin{bmatrix} 80.9133 & -24.5816 \\ -24.5816 & 43.7829 \end{bmatrix}, Q_{12} = \begin{bmatrix} 60.4030 & -23.0437 \\ -23.0437 & 43.0915 \end{bmatrix},$$

$$Q_{22} = \begin{bmatrix} 72.9823 & 17.2892 \\ 17.2892 & 50.8670 \end{bmatrix}, \quad Q_{122} = \begin{bmatrix} 81.8514 & 50.3456 \\ 50.3456 & 39.8423 \end{bmatrix},$$

$$Q_{13} = \begin{bmatrix} 93.3250 & 9.8202 \\ 9.8202 & 77.4496 \end{bmatrix}, Q_{23} = \begin{bmatrix} 61.4088 & -23.0449 \\ -23.0449 & 48.9820 \end{bmatrix},$$

$$Q_{123} = \begin{bmatrix} 80.4472 & 15.3662 \\ 15.3662 & 50.4499 \end{bmatrix}. \quad (4.10)$$

From Eq. (3.5), we have

$$\Lambda_1 = \begin{bmatrix} 2.0797 & 5.6548 \\ 5.6548 & 15.8965 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 7.7627 & -13.0736 \\ -13.0736 & 22.0329 \end{bmatrix},$$

$$\Lambda_3 = \begin{bmatrix} 52.6056 & 23.8213 \\ 23.8213 & 11.1730 \end{bmatrix} \quad (4.11)$$

and the eigenvalues of them are given below:

$$\lambda(\Lambda_1) = 0.0605,\ 17.9157 > 0, \quad (4.12)$$

$$\lambda(\Lambda_2) = 0.0039,\ 29.7917 > 0, \quad (4.13)$$

$$\lambda(\Lambda_3) = 0.3201,\ 63.4586 > 0. \quad (4.14)$$

Although the inequality (3.4) is not satisfied, the matrices Λ_j ($j = 1, 2, 3$) are all positive definite. Therefore, based on condition (II) of Theorem 1, the fuzzy controllers (4.6) –(4.8) can asymptotically stabilize the fuzzy large-scale system (4.5). Simulation results of each subsystem are illustrated in Figs. $4.1-4.3$ with initial conditions, $x_{11}(0) = 1.7$, $x_{21}(0) = -2.2$, $x_{12}(0) = -1.5$, $x_{22}(0) = 2$, $x_{13}(0) = 1.9$ and $x_{23}(0) = -1$.

V. Conclusions

In this paper, a stability criterion is derived from Lyapunov's direct method to ensure the asymptotic stability of fuzzy large-scale systems. In principle, both the conditions of this criterion can be used to test the asymptotic stability of the fuzzy large-scale system. It is therefore reasonable to check the asymptotic stability with either one of the conditions and then, if it fails, to resort the other. On the basis of this stability criterion and the decentralized control scheme, a set of fuzzy controllers is designed via the concept of PDC scheme to stabilize a fuzzy large-scale system. Finally, a numerical example with simulations is provided to demonstrate the results.

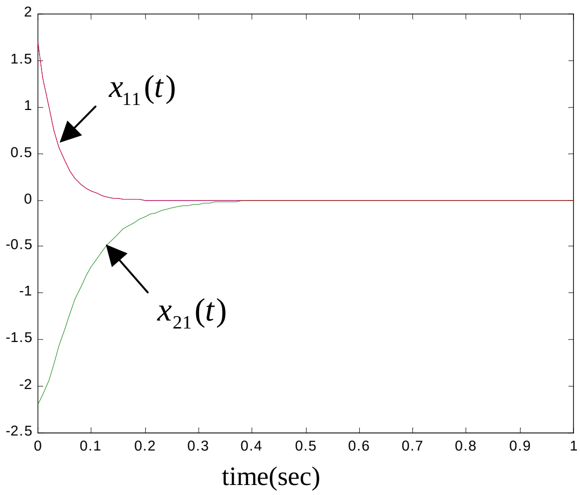

Fig. 4.1 The state response of subsystem 1.

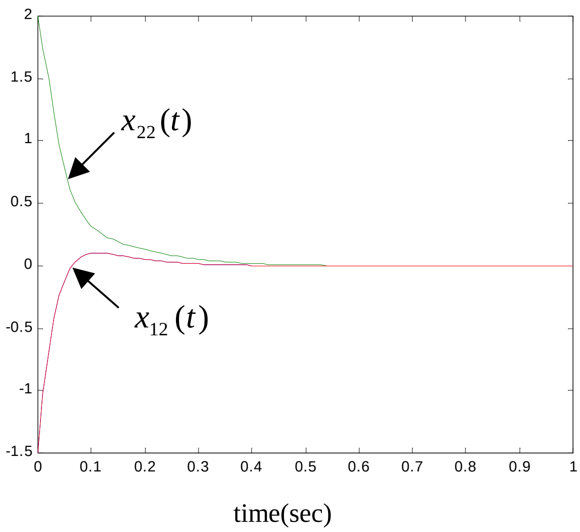

Fig. 4.2. The state response of subsystem 2.

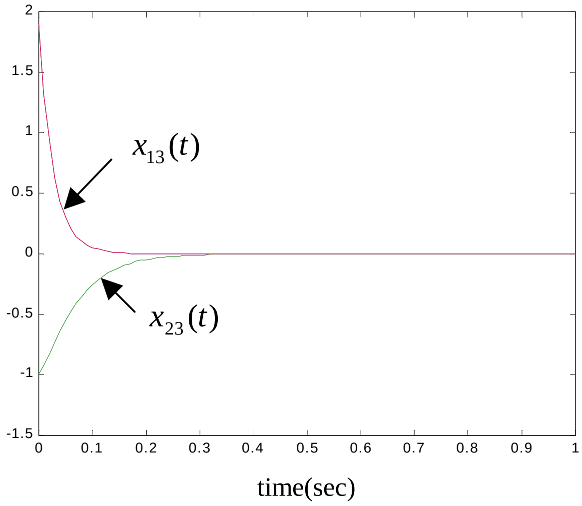

Fig. 4.3. The state response of subsystem 3.

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