Autonomous landing by computer vision: an application of path following in SE(3)

Ruggero Frezza† Claudio Altafini‡

† Dipartimento di Elettronica e Informatica
Università di Padova, Via Gradenigo 6/A 35100 Padova–Italy
frezza@dei.unipd.it
‡ Center for Autonomous Systems and
Division of Optimization and Systems Theory
Royal Institute of Technology
SE-10044 Stockholm, Sweden
altafini@math.kth.se

Abstract

In this paper, we describe a predictive control law for an aircraft autonomous approach to landing based on active vision. The path following problem and the control of the pan-tilt unit that holds the on-board camera are both formulated geometrically in the frameworks of $SE(3)$ and on the sphere S^2.

1 Introduction

In this paper, we propose a predictive control strategy in order to automatically pilot an aircraft during the approach to landing by active vision. The approach to landing consists in following a preassigned landing pattern. Active vision is needed to keep in sight of the camera landmarks which define the pattern. We assume that on board the aircraft there is a camera mounted on a pan-tilt platform.

The control task is to follow the landing pattern while maintaining in sight of a camera the runway or some other specified landmark. The pose of the camera and of the aircraft with respect to the runway is measured applying snakes or active contours [2, 21]. The control of both the aircraft and the pan-tilt unit are formulated in geometric frameworks. The former in $SE(3)$, the special Euclidean group, and the latter on the sphere S^2 in $\mathbb{R}^3$. We expect that this approach will generate "natural" and, in some sense, "optimal" trajectories. Optimality, for example, could stand for mimimum control effort and optimal repartition of the effort among all actuators.

Both the aircraft and the pan-tilt unit can be both considered fully actuated systems if one assumes coordinated flight, which means that the velocity of the aircraft has a non-zero component only along the x body axis. Prediction allows for easy decoupling of the control of the aircraft and of the pan-tilt unit. The approach, mutuated from [10, 8] decouples automatically also lateral and longitudinal control in the sense that the paths followed by the aircraft are independent of its longitudinal velocity.

In [10] the path following control law has been proved stable when applied in $\mathbb{R}^3$. The proof was obtained studying the local dynamics of the desired path with respect to the aircraft. Simulations seem to confirm stability on the manifolds $SE(3)$ and S^2 as well, but we have not proven it yet.

The control strategy mimics what a human pilot would do and we expect it to lead to smooth "comfortable" trajectories. The approach also allows to consider saturations on the inputs, this issue, however, will be discussed in a future paper.

2 Airplane and camera motion models

In this paper we model the aircraft simply as a rigid body [12] and we assume that one,in order to control it, can act directly on the external wrench $[f, \tau]$. The motion of the plane is, therefore, described by a smooth curve on the special Euclidean group $SE(3) = \{(T, R)|T \in \mathbb{R}^3, R \in SO(3)\}$, where $SO(3)$ is the special orhtogonal group of rotation matrices. $SO(3)$ and $SE(3)$ are both matrix Lie groups.

We denote by $g = (T, R)$ a rigid motion which acts on $\mathbb{R}^3$ as follows:

$$\Phi : SE(3) \times \mathbb{R}^3 \longrightarrow \mathbb{R}^3;\ (g, p) \mapsto gp;$$

in coordinates $\quad Rq + T.$

Given an element g_1 of a matrix Lie group G and a tangent vector to G at $g_1 : \dot{g}_1 \in T_{g_1}G$, the push-forward of a left multiplication by g_2, $L_{g_2}g_1$ is given by

$$L_{g_2 *}\dot{g}_1 = g_2\dot{g}_1$$

and similarly for a right multiplication. One can push forward tangent vectors by either left or right multiplications to get left/right invariant representations. Given a tangent vector $\dot{g}$ to G at g, one can obtain a tangent vector at the origin e by left or right multiplication

$$\begin{cases} V^b = g^{-1}\dot{g} \\ V^s = \dot{g}g^{-1} \end{cases}. \tag{1}$$

In the rigid motion case, V^s and V^b are a short way of coding the velocity of the rigid body and are the spatial and body velocity respectively.

The tangent plane to the origin of $SO(3)$ is the set of skew-symmetric 3×3 matrices $so(3)$, the Lie-algebra of $SO(3)$,

$$so(3) = \{\hat{\omega} \in \mathbb{R}^{3\times 3} | \hat{\omega}^T = -\hat{\omega}\} \tag{2}$$

where

$$\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \mapsto \hat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}.$$

Embedding $SE(3)$ in the matrix group $\mathcal{GL}(4)$, one can represent the group action of $SE(3)$ on $\mathbb{R}^3$ as a matrix multiplication where

$$g \doteq \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}, \quad \bar{X} = \begin{bmatrix} X \\ 1 \end{bmatrix}, \quad \bar{X} \mapsto g\bar{X}. \tag{3}$$

are called homogeneous coordinates. The Lie algebra of $SE(3)$ is, then

$$\begin{aligned} se(3) \quad = \quad & \{V \in \mathcal{GL}(4), | V = \begin{bmatrix} \hat{\omega} & v \\ 0_{3\times 1} & 0 \end{bmatrix}, \\ & \hat{\omega} \in so(3), v \in \mathbb{R}^3\}. \end{aligned}$$

The body forces and torques $[f, \tau]$ act on the body velocity as follows

$$\begin{cases} m\dot{v}^b + m\hat{\omega}^b v^b = f \\ \mathcal{I}\dot{\omega} + \hat{\omega}^b \mathcal{I}\omega^b = \tau \end{cases} \tag{4}$$

where m is the mass of the aircraft and $\mathcal{I}$ its matrix of inertia.

The pan-tilt unit that holds the camera has two degrees of freedom consisting of two rotational joints. For the sake of simplicity let us assume that the two rotational axis cross and do so at the center of projection of the camera which is described by a pin-hole model. The camera optical axis moves, therefore, as a ray of S^2, the sphere in $\mathbb{R}^3$. The motion of the camera with respect to the aircraft can, hence, be described by a smooth curve on S^2. We asssume that one can act directly on the velocities of the joints of the pan-tilt unit.

3 Measurements

Measurements are provided by the on-board camera and by resolvers on the joints of the pan-tilt unit. We also assume that the body velocity of the aircraft is known.

One of the control goals is to maintain in sight of the camera specified landmarks that define the landing pattern. Suppose that the control achieves its task so that the perspective projection of the landmark never exits the image plane. We set up a linearly parameterised deformable template [21, 2] approximating with an affine shape model the perspective projection of the contour of the landmark. Then, applying known techniques [2], we recover the pose $g^c(t) \in SE(3)$ of the camera with respect to both the landmark and the landing pattern. Knowing the position and attitude of the camera with respect to the aircraft (given by the joint angles of the pan-tilt unit and the fixed position of its base) one can determine $g(t)$ the position and attitude of the aircraft with respect to the runway (i.e. the pose of the runway in the body frame). At this point one can easily compute at each time t the desired position and attitude of the aircraft on the landing pattern at a given look-ahead distance L.

4 Autonomous approach to landing by active vision

The approach to landing consists in following a preassigned landing pattern. Active vision is needed to keep in sight of the camera a target landmark. The path following problem and the control of the pan-tilt unit are both formulated geometrically. The former in the framework of the special euclidean group $SE(3)$ and the latter of the sphere S^2. An important requirement is that the control law must be a causal output feedback (only measurements up to time t are used to compute the control action at time t) and the state of the system should not be reconstructed.

The control strategy that we shall soon describe is reminiscent of receeding horizon predictive control, but it differs for two important reasons:

a) The control law is predictive since it is designed to follow the desired path at a certain distance in front of the aircraft. The horizon, however, is fixed in terms of space rather than time, decoupling automatically longitudinal from lateral control.

b) The control action is computed by recursive approximations of the desired path with feasible splines in $SE(3)$ and S^2 rather than solving a computationally costly optimal control problem on-line.

The predictive structure allows to decouple easily the control of the aircraft and of the pan-tilt unit.

In [10, 9, 8] the first author proposed a path following control law based on recursive local approximations of the desired path by feasible splines. Where feasible means that the splines are trajectories of the vehicle. The splines depend both on the current state of the aircraft and on the local shape of the contour and are continuously updated based on the current measurements implementing an output feedback control action.

The control strategy consists, schematically, of the following steps:

Measurements: at time t the desired path and its first p space derivatives are measured at look-ahead distance L in body frame coordinates.

Generation of a locally feasible trajectory: which at time t connects the vehicle to the desired path at the look-ahead distance L. We call this trajectory the *connecting contour* γ_c. Trajectories are generated as polynomials or splines in body frame coordinates.

Computation of a control action: that tracks exactly the connecting contour γ_c.

Recursive iteration of the previous steps.

A stability analysis has been carried out in [10] by modeling the dynamics of the desired path with respect to the vehicle body frame.

Here, the task consists in the coordinated control of the aircraft and the pan-tilt unit so that, while the aircraft follows the landing pattern, the runway or some specified landmarks are continuously kept in sight of the camera.

The knowledge of the aircraft longitudinal velocity can be used to set appropriately the look-ahead distance L in order to bound the body forces and torques. The stability analysis, as in $\mathbb{R}^3$ [10] of the entire loop can be carried out in a space framework, i.e. with respect to space variations rather than time derivatives.

We propose the application of a similar strategy on $SE(3)$ and on S^2 to control the aircraft and the pan-tilt unit respectively. In $SE(3)$, for instance, the control strategy consists of the following steps:

Measurements: at time t a deformable template [21] is fit around the image of the runway or of some other landmark.

Generation of a locally feasible trajectory: which at time t connects the aircraft position and pose $g(t) \in SE(3)$ to the desired path at the look-ahead distance L in body frame.

The connecting contour γ_c is generated as a spline in $SE(3)$ [1] based on an extension of the De Casteljau algorithm used on $SO(3)$ and spheres by Crouch et al. [3]. The advantage of such a closed-form formulation is that the calculation of the trajectory is computationally inexpensive.

Computation of the control action: the feasible motion given by the spline γ_c is locally approximated by a constant velocity (a *twist* in $SE(3)$) whose magnitude and orientation can be chosen appropriately in order to avoid saturating effect on the actuators.

Recursive iteration of the previous steps.

The pan-tilt unit is controlled by projecting the trajectory of the aircraft and fixating a given landmark. Prediction allows for decoupling the two control actions.

Obviously, another advantage of this new formulation is that performances can be characterized in a coordinate-free way using the structure of $SE(3)$.

4.1 Generation of the connecting contour in $SE(3)$

Before, we describe the algorithm proposed in [1] we need some further notions on the structure of $SE(3)$.

The isomorphism between $\mathbb{R}^3$ and $so(3)$, together with the exponential map, provides a local coordinatization of $SO(3)$ as follows

$$R \doteq \exp(\hat{\omega}) \in SO(3) \tag{5}$$

$$\exp(\hat{\omega}) = I + \frac{\hat{\omega}}{||\omega||}\sin(||\omega||) + \frac{\hat{\omega}^2}{||\omega||^2}(1-\cos(||\omega||)). \tag{6}$$

This representation is known as the exponential map or the equivalent axis representation.

Similarly the canonical exponential representation of $SE(3)$ is defined as follows

$$\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} = \exp\left(\begin{bmatrix} \hat{\omega} & v \\ 0_{3\times 1} & 0 \end{bmatrix}\right) \tag{7}$$

where

$$\begin{aligned} R &\doteq \exp_{SO(3)}(\hat{\omega}) && (8)\\ T &\doteq A(\hat{\omega})v && (9)\\ A(\hat{\omega}) &\doteq I + \frac{1-\cos(||\omega||)}{||\omega||^2}\hat{\omega} + \\ &\quad + \frac{||\omega|| - \sin(||\omega||)}{||\omega||^3}\hat{\omega}^2. && (10) \end{aligned}$$

Since the matrix $A(\hat{\omega})$ is invertible when $||\omega|| \in (0, \pi)$, one can formally define the following logarithmic maps

$$\begin{aligned} \log_{SO(3)} \ &: \ SO(3) \longrightarrow so(3) \\ &R \mapsto \hat{\omega} = \frac{\theta}{2\sin(\theta)}(R - R^T) \qquad (11) \end{aligned}$$

where θ s.t. $\cos(\theta) = \frac{\text{trace}(R)-1}{2}$, $|\theta| < \pi$;

$$\begin{aligned} \log_{SE(3)} &: SE(3) \longrightarrow se(3) \\ g = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} &\mapsto V = \begin{bmatrix} \hat{\omega} & A^{-1}(\hat{\omega})v \\ 0 & 0 \end{bmatrix} \end{aligned} \quad (12)$$

where

$$A^{-1}(\hat{\omega}) = I - \frac{\hat{\omega}}{2} + \frac{2\sin(||\omega||) - ||\omega||(1+\cos(||\omega||))}{2||\omega||^2 \sin(||\omega||)}\hat{\omega}^2.$$

In order to follow the desired landing pattern, we generate a locally feasible trajectory which, at time t, connects the aircraft position and pose $g(t) \in SE(3)$ to the desired path at the look-ahead distance L. The connecting contour γ_c is generated as a spline in $SE(3)$ based on the extension [1] of the De Casteljau algorithm used also on $SO(3)$ and spheres by Crouch et al. [3].

The connecting contour $\gamma(\cdot) : [0,1] \mapsto SE(3)$ must satisfy the following boundary conditions:

$$\gamma(0) = g_0 = e, \quad \left.\frac{d\gamma}{dt}\right|_{t=0} = \dot{g}_0,$$

$$\gamma(1) = g_f, \quad \left.\frac{d\gamma}{dt}\right|_{t=1} = \dot{g}_f.$$

Transform the boundary values in velocities V_0^1 and V_2^1

$$\dot{g}_0 = 3g_0V_0^1 \qquad \dot{g}_f = 3g_fV_2^1. \quad (13)$$

The velocity V_0^1 is the body velocity of the aircraft at time t, g_f is the position and pose of the landing pattern at a given distance L in front of the aircraft, V_2^1 is the body velocity at which we would like to join it. The look-ahead distance or horizon L is an important control parameter and must be chosen as a function of the longitudinal velocity of the aircraft, of the local shape of the landing path and of input saturations. We will comment on this later.

The algorithms that generates the connecting contour is:

- Compute the "control points"

  $$\begin{aligned} g_1 &= g_0 \exp(V_0^1) \\ g_2 &= g_f \exp(-V_2^1) \end{aligned}$$

  i.e. the points reached by the one parameter arcs at time one.
- Using the logarithmic map, find the velocity V_1^1 such that

  $$g_2 = g_1 \exp(V_1^1).$$

  The velocities V_0^1, V_1^1 and V_2^1 are in $se(3)$ and are constant.
- Construct the velocities $V_0^2(t)$ and $V_1^2(t)$ such that

  $$\begin{aligned} \exp(V_0^2(s)) &= \exp(sV_0^1)\exp((1-s)V_1^1) \quad s \in [0,1] \\ \exp(V_1^2(s)) &= \exp(sV_1^1)\exp((1-s)V_2^1) \end{aligned}$$
- Construct $V_0^3(t)$ such that

  $$\exp(V_0^3(s)) = \exp(sV_0^2(s))\exp((1-s)V_1^2(s)).$$

  The velocities $V_0^2(s)$, $V_1^2(s)$ and $V_0^3(s)$ are the "polynomial" generators.
- The connecting contour is the following "cubic polynomial" in $SE(3)$

  $$\gamma(s) = g_0 \exp(sV_0^1(s))\exp(sV_0^2(s))\exp(sV_0^3). \quad (14)$$

4.2 Control action for the aircraft

The control action at time t should be determined in order to follow the connecting contour (14) at $s = 0$. This is achieved by differentiating twice $\gamma(s)$ at $s = 0$ and computing the body forces and torques $[f, \tau]$ from (4). One can compute the second order covariant derivative of $\gamma(s)$ in $s = 0$ using the result presented in theorem 4 in [3]

$$\frac{D^2}{ds^2}\gamma(s)|_{s=0} = 6\Upsilon_0(V_1^1 - V_0^1)e \quad (15)$$

where Υ_0 is the operator

$$\Upsilon_0 = \left(\int_0^1 \exp(u\,\text{ad}V_0^1)\,du\right)^{-1}$$

The control action is, then, determined using (4), (15) and the derivative of the arc-length s with respect to time which is the longitudinal velocity of the aircraft.

The control law decouples intrinsically longitudinal and lateral controls. The longitudinal control can be set to satisfy other criteria as, for example, staying far enough from the stall velocity of the airplane. Once the forward velocity has been chosen, the lateral control is determined right away. It can be shown that the aircraft trajectories are independent of the longitudinal velocity and therefore the convergence to the desired path only depends on the look-ahead distance L.

In [10] it was shown that, on the plane $\mathbb{R}^2$ and in space $\mathbb{R}^3$, changing L affects the moduli of the eigenvalues of the linearized system which is always stable. Short and long horizons L correspond to fast and slow convergence respectively. It was also shown, surprisingly, that L does not affect the damping factor of the eigenvalues which, instead, depends on the smoothnes of the connecting contour. We still have to prove that something similar happens in $SE(3)$ as well, but the first simulations seem to confirm it.

4.3 Control of the pan-tilt unit

The goal of the control of the pan-tilt unit is to keep always in sight of the camera the runway or some other specified landmark. This can be achieved by fixating a point of the landmark, let's call this point target. The task is then to control the ray of S^2 corresponding to the optical axis of the camera so that it tracks the target as it moves. We proceed as follows: we compute the motion of the target in body frame using the adjoint map on the connecting contour (14) in $SE(3)$

$$\begin{cases} V^b = Ad_{g^{-1}}(V^s) = g^{-1}V^s g \\ \dot{g} = gV^b \end{cases}; \tag{16}$$

we project it onto the sphere S^2 and track it adopting a similar strategy to that has been used to control the aircraft. In aircraft body frame coordinates we can discard the translational part of g: $g = (T, R) \mapsto R$ and, as $SO(3)$ splits into $SO(2)$ and S^2, project $SO(3)$ onto the sphere. Embedding S^2 into $\mathbb{R}^3$: $S^2 = \{p \in \mathbb{R}^3 \,|\, p^T p = 1\}$ we can fix an orthonormal basis $\{e_1, e_2, e_3\}$ and, following e.g. [20], consider the operator

$$\pi : SO(3) \longrightarrow S^2;\; R \mapsto \pi(R) = R\,e_3 = p \tag{17}$$

where e_1, e_2 belong to the tangent plane of the sphere at a given point p and e_3 describes the radial direction. The projected version of (16) looks like

$$\frac{d\pi(R)}{dt} = \dot{R}\,e_3 = R\big((\omega_1 e_1 + \omega_2 e_2) \times e_3\big) = R \begin{bmatrix} \omega_2 \\ -\omega_1 \\ 0 \end{bmatrix}$$

We design a connecting contour by computing a quadratic polynomial in S^2 [3] that connects the position at time t of the optical axis of the camera to the trajectory of the target on the sphere. We impose that at $s = t$ the polynomial is at the current position of the optical axis on the sphere. At time $s = t + \Delta t$, we impose that the polynomial is on the trajectory drawn on S^2 by the projection of the target at the point corresponding at time $t + \Delta t$. In particular, we impose that it *joins* the trajectory and does not crosses it by imposing the first time derivative as well. Since S^2 is a Riemannian manifold one can compute and use directly the geodesics to build the connecting contour with De Casteljau algorithm [3]. We then compute the velocities of the joints of the pan-tilt unit so that it follows the connecting contour. The difference with respect to the control of the aircraft is that this is a *trajectory tracking* rather than a *path following* problem. This means that the connecting contour must be parameterised by time and not by arc-length.

The time horizon Δt is an important control parameter that must be carefully selected considering, in particular, the bandwith of the actuators of the pan-tilt unit and the regularity of the projection of the target trajectory.

References

[1] C. Altafini, "The De Casteljau algorithm on SE(3)", preprint.

[2] A. Blake, M. Isard, "Active Contours", Springer Verlag, 1998.

[3] P. Crouch, F. S. Leite, G. Kun, "Geometric Splines", In *Proc. IFAC 14th Triennal World Congress*, pp. 533–538, Bejing, China, 1999.

[4] E. D. Dickmanns and V. Graefe, "Dynamic monocular machine vision, " *Machine Vision and Application*, vol. 1, pp. 223-240, 1988.

[5] E. D. Dickmanns and V. Graefe, "Applications of dynamic monocular machine vision, " *Machine Vision and Application*, vol. 1, pp. 241-261, 1988.

[6] S. Fürst, S. Werner, D. Dickmanns, E. D. Dickmanns, "Landmark navigation and autonomous landing approach with obstacle detection for aircraft", In *Proc. of AeroSense'97*, Orlando FL, pp. 20–25, April 1997.

[7] R. Frezza and G. Picci, "On line path following by recursive spline updating," In *Proceedings of the 34th IEEE Conderence on Decision and Control,* volume 4, pages 4047–4052, 1995.

[8] R. Frezza, S. Soatto and G. Picci,"A Lagrangian formulation of nonholonomic path following". In "*The Confluence of Vision and Control*", S. Morse et al. (eds), Springer Verlag.

[9] R. Frezza, G. Picci and S. Soatto, "Nonholonomic Model-based Predictive Output Tracking of an Unknown Three-dimensional Trajectory", In *Proceedings of 37th IEEE Conference on Decision and Control,* 1998.

[10] R. Frezza, "Path following for air vehicles in coordinated flight", In *Proc. IEEE-ASME Conf. on Advanced Intelligent Mechatronics AIM'99*, Atlanta, GA, 1999.

[11] J. Hauser, R. Hindman, "Aggressive Flight Maneuvers", in *Proceedings of 36th IEEE Conference on Decision and Control,* pp. 4186–4191, 1997.

[12] T. J. Koo and S. Sastry, "Output Tracking Control Design of a Helicopter Model Based on Approximate Linearization", In *Proceedings of 37th IEEE Conference on Decision and Control,* 1998.

[13] Yi Ma, J. Kosecka and S. Sastry, "Vision guided navigation for nonholonomic mobile robot," In *Proceedings of the 36th IEEE Conderence on Decision and Control,* San Diego, CA, Dec. 1997.

[14] R. Murray, Z. Li and S. Sastry, *A Mathematical Introduction to Robotic Manipulation.* CRC Press Inc., 1994.

[15] P. Rouchon, M. Fliess, J. Lèvine and P. Martin, "Flatness and motion plannning: the car with n trailers". In *Proceedings Int. Conf. ECC'93*, pp. 1518–1522, Groeningen, The Netherlands, 1993.

[16] M. Fliess, J. Lèvine, P. Martin and P. Rouchon, "Design of trajectory stabilizing feedback for driftless flat systems". In *Proceedings Int. Conf. ECC'95*, pp. 1882-1887, Rome, Italy, Sep. 1995.

[17] C. Samson, M. Le Borgne and B. Espiau, *Robot Control the Task Function Approach*. Oxford Engineering Science Series. Clarendon Press 1991.

[18] O. Shakernia, Y. Ma, T. J. Koo, S. Sastry, "Landing an Unmanned Air Vehicle: Vision Based Motion Estimation and Nonlinear Control". *Asian Journal of Control*, 1999.

[19] G. Walsh, D. Tilbury, S. Sastry and J. P. Laumond,"Stabilization of trajectories for systems with nonholonomic constraints". *IEEE Transactions on Automatic Control*, 39(1): pp. 216-222, Jan. 1994.

[20] G. Walsh and S. Sastry,"On reorienting rigid linked bodies using internal motion". *IEEE Transactions on Robotics and Automation*, February 1995.

[21] A. Yuille, P. Hallinan, "Deformable templates", In *Active Vision*,
A. Blake, A. Yuille Eds., pp. 20–38, MIT, 1992.