Universal Fuzzy Controllers for Discrete-Time Systems

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Abstract

In this paper we address the issues of universal fuzzy controllers for discrete time systems. We first present a universal function approximation theorem based on a fuzzy dynamic model. Then we show the results of universal fuzzy controllers for a large class of nonlinear systems.

1. Introduction

Fuzzy logic control (FLC) has recently proved to be a successful control approach for complex nonlinear systems. In many cases it has been suggested as an alternative approach to conventional control techniques.

Recently, there have appeared a number of stability analysis and synthesis results in fuzzy control literature [e.g. 1-3]. The basic idea of these methods is to design a feedback controller for each local model and to construct a global controller from the local controllers in such a way that global stability of the closed loop fuzzy control system is guaranteed.

However, there is still an important question to be answered. That is, whether is there a fuzzy control law which can stabilize a given complex nonlinear system if the system can be stablized? This is the *universal fuzzy controller problem*. This problem is considered in [4] though its proof needs some improvement.

In this paper we will address the problem of universal fuzzy controllers. After presenting dynamic fuzzy models and a universal fuzzy approximator in section 2, we will show the results of the universal fuzzy controller in section 3, which will be followed by some concluding remarks in section 4.

2. Dynamic fuzzy model of a nonlinear system

Consider a general nonlinear discrete-time system described by a state-space model of the form

$$x(t+1) = f(x(t), u(t)) \qquad (2.1)$$

where $x(t) \in \Re^n$ are the state variables, $u(t) \in \Re^p$ are input variables of the system. In this paper we only consider one class of the nonlinear systems, whose function $f(x(t), u(t))$ satisfies the following assumption.

Assumption 2.1: There exists an equilibrium $x_0 = 0 \in \Re^n$ such that $f(0,0) = 0$ and $f \in C^{\rho+1}$ for a given $\rho > 0$, that is, f has the *(ρ+1)th* continuous derivative with respect to x and u on a compact set $X \times U \subset \Re^n \times \Re^p$.

The following dynamic fuzzy model (DFM) will be used to represent the system (2.1).

$$R^l: \quad \text{IF} \quad x_1 \text{ is } F_1^l \text{ AND } .... \; x_n \text{ is } F_n^l$$
$$\text{THEN} \quad x(t+1) = f_l(x(t), u(t)) \qquad (2.2)$$
$$l = 1,2,......,m$$

where R^l denotes the l-th approximation inference rule, $f_l(x(t), u(t))$ is the l-th local model of the nonlinear system (2.1), and $f_l \in C^\rho$, and m is the number of approximation inference rules.

Using a centre-average defuzzifier, product inference and singleton fuzzifier, the dynamic fuzzy model (2.2) can be expressed by the following global model,

$$x(t+1) = \hat{f}(x(t), u(t)) \qquad (2.3)$$

where

$$\hat{f}(x(t), u(t)) := \hat{f}(x(t), u(t)\mu(x)) = \sum_{l=1}^{m} \mu_l(x) f_l(x(t), u(t)) \,,$$

and $\mu_l(x(t))$ is the normalized membership function of the inferred fuzzy set $S_l = \prod_{i=1}^{n} F_i^l$. Each fuzzy set S_l is divided into three regions

$$S_l = S_l^0 \cup \partial S_l \cup S_l^\infty . \qquad (2.4)$$

In this paper we only consider the following class of membership functions. Corresponding to three regions of the fuzzy set S_l, the membership function $\mu_l(t) = \mu_l(x(t))$ satisfies the following conditions,

1) $\sum_{l=1}^{m} \mu_l(t) = 1$. (2.5a)

2) There exists a set of $\bar{x}_l$'s called the centers of S_l's such that

$$\bar{x}_l \in S_l^0, \mu_l(\bar{x}_l) = 1 , \quad l = 1,2,.....,m . \qquad (2.5b)$$

3) For a small enough $\varepsilon_\mu > 0$

$$\mu_l(t) \geq 1 - \varepsilon_\mu, x(t) \in S_l^0,$$
$$\varepsilon_\mu < \mu_l(t) < 1 - \varepsilon_\mu, x(t) \in \partial S_l , \quad l = 1,2,.....,m \qquad (2.5c)$$
$$\mu_l(t) \leq \varepsilon_\mu, x(t) \in S_l^\infty$$

The region S_l^0 is called the dominant region, the region ∂S_l is called the transition region, and the region S_l^∞ is called the inactive region. Such membership functions are called the trapezoid-shaped like membership functions (TSLMF). The typical examples are the trapezoid-shaped function and the triangle-shaped function.

Using the TSLMF we can get the partition of the state space. In this paper we consider the following partition.

Definition 2.1: The state partition is called a well behaviour partition (WBP) if it satisfies

1) Only one of the S_l 's includes the origin, without loss of generality, it is assumed that the origin $x = 0 \in S_1^0$ and $\mu_1(0) = 1$.

2) $X = \bar{S}_1 \cup \bar{S}_2 \cup .... \cup \bar{S}_m$, $\bar{S}_l = S_l^0 \cup \partial\bar{S}_l$,

$\partial\bar{S}_l = \{x \,|\, \mu_l(x) \geq \mu_i, i = 1,2,...,m, i \neq l, x \notin S_l^0\}$

3) $\bar{S}_l, l = 1,2,.....,m$ are closed convex sets.

Let *FM* be the set of all DFM's of the form (2.3). In this paper we consider the following fuzzy control law,

$$u(t) = \hat{g}(x(t), \mu(x)) = \sum_{l=1}^{m} \mu_l(x) g_l(x(t)) \qquad (2.6)$$

where the membership functions in eqn.(2.6) are TSLMF. Let *FC* be the set of all fuzzy controllers of the forms eqn.(2.6).

it has been shown that the fuzzy dynamic models described in eqn.(2.2) or eqn.(2.3) are universal function approximators, that is, given any $f(x,u) \in SS$ there exists an $\hat{f}(x,u,\mu) \in FM$ that will approximate $f(x,u)$ to any degree of accuracy on any compact set.

Theorem 2.1: For any given $f(x,u) \in SS$ on a compact set $X \times U \subset \Re^n \times \Re^p$ and arbitrary $\varepsilon > 0$, there exists an $\hat{f} \in FM$ such that

$$d_\infty(f(x,u) - \hat{f}(x,u)) = \sup_{x \in X, u \in U} (\left\| f(x,u) - \hat{f}(x,u) \right\|) < \varepsilon \,.$$

Corollary 2.1: The approximation DFM $\hat{f}(x,u)$ in *Theorem 2.1* has the following properties, for a given $\varepsilon > 0$

1) there exist a positive constants ε_f such that

$$\left\| f(x,u) - \hat{f}(x,u) \right\| \leq \varepsilon_f \left\| x - \bar{x}_l \right\| \leq \varepsilon x \in \bar{S}_l \,, \qquad (2.7a)$$

$$\left\| f(x,u) - f_l(x,u) \right\| \leq \varepsilon_f \left\| x - \bar{x}_l \right\| \leq \varepsilon x \in \bar{S}_l \,. \qquad (2.7b)$$

3. Universal fuzzy controllers

First, we need the following definitions:

Definition 3.1: The system (2.1) is said to be globally exponentially stabilizable if there exists a feedback control law

$$u(t) = g(x(t)) \qquad (3.1)$$

where $g(\cdot) \in C^{\rho+1}$ such that the closed-loop system

$$x(t+1) = f(x(t), g(x(t))) \hat{=} F_m(x(t)) \qquad (3.2)$$

is globally exponentially stable, that is, there exist positive constants *C>0* and $\lambda < 1$ such that given any initial state *x* the solution of eqn.(3.2) exists for all $t \geq 0$ and satisfies,

$$\| \varphi(t,x) \| \leq C\lambda^t \| x \| \,. \qquad (3.3)$$

We will use *SS* to represent a class of nonlinear systems which satisfy the *Assumption 2.1* and are also exponentially stabilizable.

Definition 3.2: The system (3.2) is said to be semi-globally exponentially stable on a compact set $X \subset \Re^n$ if there exist positive constants *C>0*, $\lambda < 1$, and a region $X_0 \subset X$ such that given any initial state $x \in X_0$ the solution of eqn.(3.2) exists for all $t \geq 0$ and satisfies,

$$\| \varphi(t,x) \| \leq C\lambda^t \| x \|, \; \varphi(t,x) \in X \,. \qquad (3.4)$$

The universal fuzzy controller is then defined as follows.

Definition 3.3: FC is said to be the universal fuzzy controllers, if for any $f \in SS$ there exists a fuzzy control law $\hat{g}(x(t), \mu(t)) \in FC$ such that the closed-loop system

$$x(t+1) = f(x(t), \hat{g}(x(t), \mu(t))) \qquad (3.5)$$

is semi-globally exponentially stable on a compact set $X \times U \subset \Re^n \times \Re^p$.

Theorem 3.1: *FC* is universal fuzzy controllers in the sense of the Definition 3.3.

Proof: Omitted due to space limitation.

4. Conclusions

In this paper we give a universal fuzzy controller theorem for a class of nonlinear systems. The results explain to some extent that why fuzzy control has been successful in indusdrial applications. However, more work is needed to deal with more general nonlinear systems.

References

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