

OPTIMAL FLOW CONTROL FOR MANUFACTURING SYSTEMS MODELLED BY CONTINUOUS PETRI NETS

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Abstract : This work deals with optimal flow control for manufacturing systems that are modelled by continuous Petri nets. The control design consists in defining the time intervals while the machines are working and the time intervals while they are not, in order to minimise the production cost. Results are discussed and compared with the ones obtained with the Wilson model.

Keywords : Continuous Petri nets, manufacturing systems, optimal control, Wilson model, production planning.

1. Introduction

The flow control of manufacturing systems is a complicated problem because such systems exhibit often complex behaviours and non-linear dynamics. Among the existing models [1], Petri nets have resulted in continuous time models, called continuous Petri nets (CtPN) [2], when the considered systems process a great number of parts and when the continuous time description is a good approximation of the system behaviour. Control design for CtPN was recently investigated in order to tune the transition firing speeds such that the marking vector follows a desired trajectory [3, 6]. But economic aspects are not taken into consideration. Others results are related to the minimisation of the production cost [4, 7, 8]. Unfortunately, these results do not concern CtPN.

In comparison with these approaches, our work introduces economic aspects in CtPN models in order to plan the production of manufacturing systems. The proposed control design consists in defining the time intervals while the machines are working and the time intervals while they are not in order to minimise the production cost. Assumptions related to the Wilson model [5] are considered to work out the optimal flow control. Results are discussed and compared with the ones obtained with the discrete Wilson model.

2. Wilson model

Let us consider a planning horizon H subdivided into N time intervals T . At the beginning of each time interval T , a manufacturing order is given. A quantity Q of parts is ordered and manufactured with no delay. Parts are identical. These Q parts are used to satisfy the demand D , assumed to be constant during H . The problem is to determine the number of parts Q manufactured for each order, to minimise the total manufacturing cost C [5]:

$$C = CMA + COR + CST + CBR . \quad (1)$$

The production cost CMA corresponds to the means that are necessary to product a quantity Q of parts. The stocking cost CST corresponds to the means that are necessary to stock a quantity Q of manufactured parts. The ordering cost COR corresponds to the means that are

necessary to order a quantity Q of parts and to start the production. The break-up cost CBR corresponds to the penalty when there is not enough parts in the buffer to satisfy the demand. The following notations are also used. Cc stands as the cost of one order, Cs as the stocking cost for one part during H , Cr as the break-up cost for one part during H , P as the manufacturing cost for one part and $q(t)$ as the number of parts stocked at time t (given in time units (TU)). The production cost is assumed to be constant:

$$CMA = D.P. \quad (2)$$

The ordering cost depends on the number N of times intervals T that divide the planning horizon:

$$COR = Cc.N = \frac{H.Cc}{T} = \frac{D.Cc}{Q} . \quad (3)$$

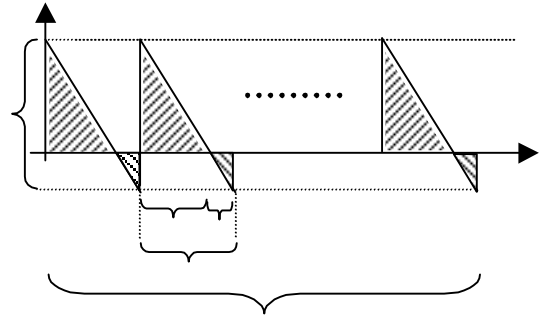


Figure 1 : Wilson model with break-up

Each time interval T is divided into two periods T_1 and T_2 (figure 1). During T_1 the buffer contains enough parts to satisfy the demand : $q(t) > 0$. During T_2 there are not enough parts to satisfy the demand and the buffer content is equal to zero : $q(t) = 0$. The stocking cost and break-up cost are given by equations (4) and (5) :

$$CST = \frac{Cs}{T} \int_0^{T_1} q(t).dt = \frac{Cs.Sm}{2} \left(\frac{T_1}{T_1 + T_2} \right) = \frac{Cs.Sm^2}{2Q} , \quad (4)$$

$$CBR = \frac{Cr}{T} \int_{T_1}^T q(t).dt = \frac{Cr.(Q - Sm).T_2}{2.(T_1 + T_2)} = \frac{Cr.(Q - Sm)^2}{2Q} , \quad (5)$$

where Sm stands for the maximal value of the buffer content. Thus equation (1) results in (6):

$$C = D.P + \frac{D.Cc}{Q} + \frac{Cs.Sm^2}{2Q} + \frac{Cr.(Q - Sm)^2}{2Q} . \quad (6)$$

The optimal values Q^* of Q and Sm^* of Sm are given by:

$$Q^* = \sqrt{\frac{2.D.Cc.\gamma}{Cs}} , Sm^* = \frac{Q^*}{\gamma} , T1^* = \frac{Sm^*}{D} , T2^* = \frac{Q^* - Sm^*}{D} \quad (7)$$

with $\gamma = (Cr + Cs) / Cr$.

3. Flow control for CtPN

CtPN have been deduced from timed Petri nets to represent manufacturing systems that process large numbers of parts. With this model, buffers are represented by places and workstations are represented by transitions. The places marking stands for the buffers content and the transitions firing speed stands for the average production frequencies. Among the existing continuous Petri nets models, CtPN with constant speed and CtPN with variable speed [2], are the most useful approximations of the discrete behaviour of timed Petri nets. Both classes of models have similar behaviours when the marking of the places is not less than 1. In what follows, let us consider a manufacturing workstation modelled with CtPN, (figure 2) such that $m_1 \geq 1$.

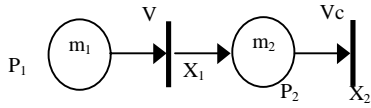


Figure 2 : CtPN model of a workstation

The considered problem consists in defining the periods while the workstation modelled by transition X_1 is working and while it is not in order to satisfy the demand D . Let us assume that : $V.H \geq D$. At the beginning of each time interval T , a manufacturing order is given and a quantity of Q parts is manufactured according the CtPN model. Let us call $Vc=D/H$, the firing speed of X_2 corresponding to the use of the finite product during H .

Each time interval T is divided into 4 periods T_1 , T_2 , T_3 and T_4 , as represented on figure 3. During T_1 , the transition X_1 is fired with the speed V and the content of P_2 increases with speed $V-Vc$. The marking $m_2(t)$ reached the maximal value Sm at the end of T_1 . During T_2 , the transition X_1 is blocked. The place P_2 empties at speed Vc , but P_2 contains enough parts to satisfy the demand. The marking $m_2(t)$ reached the value 0 at the end of T_2 . During T_3 , the transition X_1 is blocked and there is a break-up in the production. During T_4 , parts are manufactured with the speed V in order to satisfy the delayed demand. During T_3 and T_4 , $m_2(t)=0$. The maximal value Rm of the unsatisfied demand is reached at the end of T_3 .

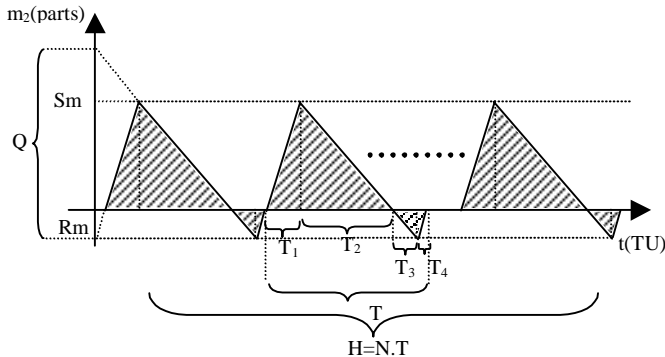


Figure 3 : CtPN production planning with break-up

The manufacturing and ordering costs are defined by (2) and (3). The stocking cost and break-up cost are given by equations (8) and (9) :

$$CST = \frac{Sm.Cs}{2} \cdot \left(\frac{T_1 + T_2}{T} \right) = \frac{Sm^2.Cs}{2Q} \cdot \left(\frac{V}{V - Vc} \right), \quad (8)$$

$$CBR = \frac{Rm.Cr}{2} \cdot \left(\frac{T_3 + T_4}{T} \right) = \frac{Cr.(Q.(V - Vc) - Sm.V)^2}{2.Q.V.(V - Vc)}. \quad (9)$$

Thus, the total cost is given by:

$$C = D.P + \frac{D.Cc.(V - Vc)}{V.(Sm + Rm)} + \frac{Sm^2.Cs}{2.(Sm + Rm)} + \frac{Rm^2.Cr}{2.(Sm + Rm)}.$$

The optimal values Q^* of Q , Sm^* of Sm and Rm^* of Rm are given by (10):

$$Q^* = \alpha \sqrt{\frac{2.D.Cc.\gamma}{Cs}}, Sm^* = \frac{Q^*}{\alpha^2.\gamma}, Rm^* = \frac{Q^*.Cs}{\alpha^2.\gamma.Cr}. \quad (10)$$

with $\alpha = (V / (V - Vc))^{1/2} = (1 - z)^{-1/2}$ and $z = Vc/V \in]0, 1]$. The optimal control design consists to tune the workstation modelled by X_1 at maximal speed V during T_1^* and T_4^* and to stop it during T_2^* and T_3^* :

$$T_1^* = \frac{Sm^*}{V - Vc}, T_2^* = \frac{Sm^*}{Vc}, T_3^* = \frac{Rm^*}{Vc}, T_4^* = \frac{Rm^*}{V - Vc}.$$

4. Conclusions

This work was about the production planning for manufacturing systems modelled by CtPN. The control design was expressed in terms of periods while the workstations are working and of periods while they are not. Let us notice, that the results given by equation (10) are similar to the ones obtained with the discrete Wilson model (7), with the corrective coefficient α such that $Q^*_{(10)} = \alpha.Q^*_{(7)}$ and $Sm^*_{(10)} = Sm^*_{(7)} / \alpha$. When $V \gg Vc$, we have $\alpha \approx 1 + z/2$. The error that corresponds to replace the CtPN solution with the Wilson model solution is approximated by $Vc/2V$.

7. References

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