Robustification of Model Predictive Control

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Abstract

A general procedure leading to an enhancement of robustness of existing Model Predictive Control techniques is proposed. This procedure, which considers additive modeling errors, is illustrated for the case of Cautious Stable Predictive Control. The basic idea is the augmentation of the cost function with an additional term related to a description of the nominal model uncertainty, leading either to a minimization or to a min-max optimization problem, depending on the class of error description being used.

1 Introduction

Model Predictive Control (MPC) has become an area of significant research interest over the last twenty years. This interest has been powered by a stream of successful industrial applications. When focusing on linear (and unconstrained) discrete time transfer function models and quadratic cost functions, some of the best known approaches include the Generalized Predictive Control (GPC) [1], its related algorithms with guaranteed nominal stability as presented e.g. in [2], the *inner loop stabilizing* Stable Predictive Control [3] and the Cautious Stable Predictive Control (CaSC) [4].

There exist several different strategies to robustify the design in the presence of plant modeling errors, depending on how these errors are described.
Structured uncertainties lead in a natural way to a worst-case analysis and therefore to min-max optimization problems assuming either *open-loop* or *closed-loop* control [5].
The robustness issue in the presence of unstructured modeling errors is usually dealt with by enhancing the robustness of existing designs by introducing degrees of freedom based on the Youla parameterization. These parameterizations do not affect the nominal complementary sensitivity and allow the minimization of a robustness cost function derived from the well-known small gain theorem. This two-step procedure leads to a H_∞ problem [3] that can be hard to solve in real time and has led to work on guidelines to choose good and easy to compute suboptimal solutions [6, 7].

In this paper the enhancement of the robustness of the control is achieved by minimizing an augmented MPC cost function. The basic cost function is modified by adding a term which weights the open loop prediction of the output of an additive uncertainty model.
In section §2, the proposed idea is applied to Cautious Stable Predictive Control. To emphasize the main idea, perfect knowledge of the uncertainty is assumed. In section §3 semistructured uncertainties are used and the resulting min-max problem is stated and solved. An example is given in §4, and §5 contains the concluding remarks and further work suggestions.

2 The Basic Idea

Consider a discrete time plant with input $u(t)$, output $y(t)$ and having a nominal model $G_0(z) = \dfrac{z^{-1}b(z^{-1})}{a(z^{-1})}$ with:

$$a(z^{-1}) = 1 + \sum_{i=1}^{n_a} a_i z^{-i}, \quad b(z^{-1}) = \sum_{j=0}^{n_b} b_j z^{-j}. \qquad (1)$$

The plant *true* model is given by $G(z) = G_0(z) + G_\varepsilon(z)$, where $G_\varepsilon(z) \in RH_\infty$ is the additive uncertainty satisfying $G_\varepsilon(1) = 0$.
Further assume that the polynomial $a(z^{-1})$ is factored as $a(z^{-1}) = a^+(z^{-1})a^-(z^{-1})$, where a^- is monic with all its roots (in z) having a modulus less or equal than ρ, where $0 \le \rho \le 1$. The same is done for $b(z^{-1})$. ρ defines a region for *desirable* pole location.
Then, stable predictors for the model (1) are:

$$\hat{u}(t) = \frac{a^+(z^{-1})}{b^-(z^{-1})}c(t), \qquad \hat{y}(t) = \frac{z^{-1}b^+(z^{-1})}{a^-(z^{-1})}c(t), \qquad (2)$$

where $c(t)$ is a finite sequence.

If we define $y_\varepsilon(t) = G_\varepsilon(z)u(t)$, a predictor for $y_\varepsilon(t)$ is given by:

$$\hat{y}_\varepsilon(t) = G_\varepsilon(z)\frac{a^+(z^{-1})}{b^-(z^{-1})}c(t). \qquad (3)$$

This predictor can be truncated, without loosing the essential dynamic features of (3), for a large enough n_h:

$$\hat{y}_\varepsilon(t) = z^{-1}h(z^{-1})c(t),\ h(z^{-1}) = h_0 + h_1 z^{-1} + \cdots + h_{n_h} z^{-n_h},$$

where the coefficients h_i ($i = 0, 1, \ldots, n_h$) are constrained to satisfy $h(1) = 0$.

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With the above setting, it is possible to minimize the cost function given in (4) over future values of $c(t)$ arranged in $\underset{\rightarrow}{\mathbf{c}} = [c(t), c(t+1), \cdots, c(t+n_c-1)]^T$:

$$J(\underset{\rightarrow}{\mathbf{c}}) = \sum_{i=1}^{N_y} (\tilde{r}(t+i) - \tilde{y}(t+i))^2 + \lambda_u \sum_{i=0}^{N_u-1} (\Delta\tilde{u}(t+i))^2$$

$$+ \lambda_\varepsilon \sum_{i=1}^{N_y} (\hat{y}_\varepsilon(t+i))^2, \quad c(t+i) = \frac{r(t+N_y)a^-(1)}{b^+(1)}, \; i \geq n_c, \tag{4}$$

where: $\tilde{r}(t) = a^-(z^{-1})r(t)$, $\tilde{y}(t) = a^-(z^{-1})\hat{y}(t)$, $\Delta\tilde{u}(t) = b^-(z^{-1})\Delta(z^{-1})\hat{u}(t)$, $\Delta(z^{-1}) = 1 - z^{-1}$ and the sequence $r(t+i), i = 1, \ldots, N_y$ is the future reference trajectory.
Given a certain *command horizon* n_c, the control and output horizons are $N_y = n_{b^+} + n_c$ and $N_u = n_{a^+} + n_c + 1$, respectively.[1]
The new tuning parameter λ_ε penalizes the output of the uncertainty model according to its open loop prediction and allows the designer to robustify the design, as illustrated in the example included in section §4. $\lambda_\varepsilon = 0$ corresponds to the CaSC algorithm [4].

3 Semistructured Uncertainties

The assumption that G_ε is perfectly known is impractical, thus in this section G_ε is considered to belong to a family of stable linear models.
Suppose $G_\varepsilon \in \Delta_n(B_\varepsilon, \Theta)$ defined by:

$$\Delta_n(B_\varepsilon, \Theta) = \left\{\theta^T B_\varepsilon \,|\, \theta \in \Theta\right\}, \; \Theta = \left\{\theta \in \mathbb{R}^{n\times 1} \,|\, \theta^T \mathbf{P}\theta \leq 1\right\}$$

$$\mathbf{P} = \mathbf{P}^T > 0, \quad \theta = \begin{bmatrix}\theta_1\\ \theta_2\\ \vdots\\ \theta_n\end{bmatrix}, \quad B_\varepsilon(z) = \begin{bmatrix}B_{\varepsilon 1}(z)\\ B_{\varepsilon 2}(z)\\ \vdots\\ B_{\varepsilon n}(z)\end{bmatrix}. \tag{5}$$

This uncertainty model description contains linear combinations of stable, proper rational transfer functions (basis functions), with coefficients lying in an ellipsoid. Basis functions have been used in system identification and its advantages have been discussed e.g. in [8].
In this case:

$$\hat{y}_\varepsilon(t) = \sum_{j=1}^{n} \theta_j \hat{y}_{\varepsilon j}(t) = \left(\sum_{j=1}^{n} \theta_j B_{\varepsilon j}(z) \frac{a^+(z^{-1})}{b^-(z^{-1})}\right) c(t)$$

$$\approx \left(\sum_{j=1}^{n} \theta_j \tilde{h}_j(z^{-1})\right) c(t-1), \tag{6}$$

where $\tilde{h}_j(z^{-1})$, $\tilde{h}_j(1) = 0$ are the n_h-truncated impulse responses corresponding to $\hat{y}_{\varepsilon j}$.

The minimization of the cost function for the *worst situation* of the open loop prediction of $y_\varepsilon(t)$ is the solution for the min-max problem:

$$\min_{\underset{\rightarrow}{\mathbf{c}}} \max_{\theta\in\Theta} \left\{J_1(\underset{\rightarrow}{\mathbf{c}}) + \lambda_\varepsilon J_2(\underset{\rightarrow}{\mathbf{c}}, \theta)\right\}, \quad \text{with:} \tag{7}$$

$$J_1(\underset{\rightarrow}{\mathbf{c}}) = \sum_{i=1}^{N_y} (\tilde{r}(t+i) - \tilde{y}(t+i))^2 + \lambda_u \sum_{i=0}^{N_u-1} (\Delta\tilde{u}(t+i))^2 \tag{8}$$

$$J_2(\underset{\rightarrow}{\mathbf{c}}, \theta) = \sum_{i=1}^{N_y} (\hat{y}_\varepsilon(t+i))^2 = \sum_{i=1}^{N_y} \left(\sum_{j=1}^{n} \theta_j \hat{y}_{\varepsilon j}(t+i)\right)^2 \tag{9}$$

with $c(t+i)$ constrained as in (4). Note that $J_2(\underset{\rightarrow}{\mathbf{c}}, \theta)$ is a convex functional for any given $\underset{\rightarrow}{\mathbf{c}}$.

4 Example

Let the nominal model be given by $G_0(z) = \frac{2z^{-1}}{1-0.8z^{-1}}$ and suppose that the additive uncertainty is specified according to (5) with $\mathbf{P} = \begin{bmatrix}10/3 & 0\\ 0 & 10/3\end{bmatrix}^2$ and:

$$\begin{aligned} B_{\varepsilon 1}(z) &= \frac{0.99058z^{-1}(1-z^{-1})(1+0.0625z^{-1})}{(1-0.8z^{-1})(1-0.2z^{-1})(1-0.1z^{-1})}, \\ B_{\varepsilon 2}(z) &= \frac{-0.10288z^{-1}(1-10z^{-1})(1-z^{-1})}{(1-0.8z^{-1})(1-0.2z^{-1})(1-0.1z^{-1})}. \end{aligned} \tag{10}$$

The min-max procedure with $\rho = 0, \lambda_\varepsilon = 10, \lambda_u = 1, n_c = 5$ and $n_h = 5$ returns $\theta^\star = [0.223 \;\; 0.201]^T$. The region of uncertainty including the worst cases is shown in figure 1.

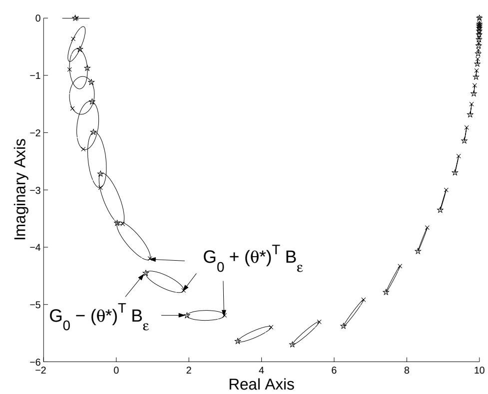

Figure 1: $G_0 + \theta^T B_\varepsilon$, $\theta^T \mathbf{P}\theta = 1$ and $G_0 \pm (\theta^\star)^T B_\varepsilon$

To simplify computations we assume that (2) holds and $\theta^\star$ is constant. In this case, the receding horizon implementation is equivalent to a linear two degree of freedom control loop.

An unstructured uncertainty description can be obtained by fitting a function $G_{\varepsilon\max}(z)$ such that:
$|G_\varepsilon(e^{j\omega})| \leq |G_{\varepsilon\max}(e^{j\omega})|$, $\forall\omega \in [0, \pi]$, $\forall G_\varepsilon(z) \in \Delta_2(\mathbf{B}_\varepsilon, \Theta)$.
A simple trial and error procedure applied on figure 2 allows to fit the rational function $G_{\varepsilon\max}(z) = \frac{0.41(z-1)}{(z-0.8)(z-0.4)(z+0.14)}$.
This is used as an exact description of the uncertainty according to the procedure outlined in section §2 and a controller is computed with the same tuning parameters as in the previous case.

[1] n_{a^+} and n_{b^+} are the degrees of $a^+(z^{-1})$ and $b^+(z^{-1})$, respectively

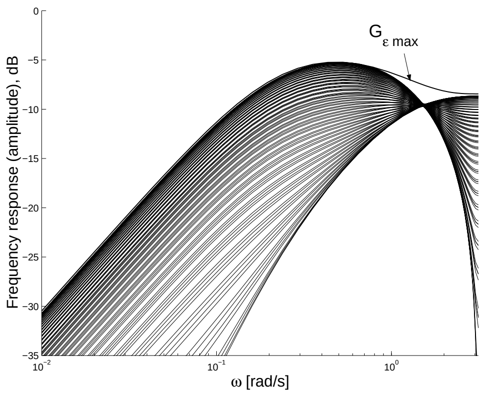

Figure 2: $|\theta^T B_\varepsilon(e^{j\omega})|, \theta^T \mathbf{P}\theta = 1$ and $|G_{\varepsilon\max}(e^{j\omega})|$

Figures 3 and 4 show the performance of two *robust* designs: semistructured uncertainty (solid) and unstructured uncertainty (dashed). Also a standard CaSC controller with $\lambda_u = 5$ (and $\lambda_\varepsilon = 0$) is included (dotted). While Figure 3 shows step response and frequency response of the nominal complementary sensitivity function, T_{yr}, Figure 4 illustrates the *worst case* behavior, corresponding to $G = G_0 - (\theta^\star)^T B_\varepsilon$. In this example we see that, although all designs have similar bandwidths, the *robust* ones behave better. It is also interesting that the unstructured bound yields a good and easy to obtain solution.

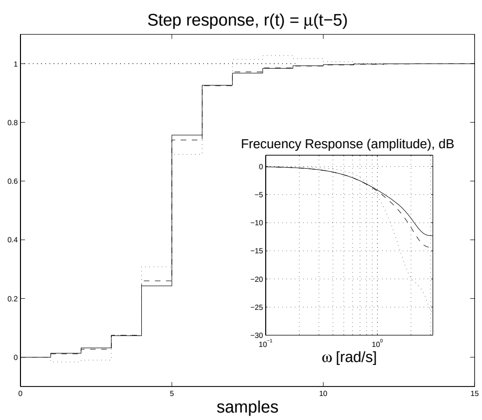

Figure 3: Nominal performance of *robust* and standard designs

5 Conclusions

A new approach to the robust MPC problem has been presented. Unlike many other methods, both the feedback and the reference prefiltering controller are affected by G_ε and the design parameter λ_ε. The proposed method accepts different forms of uncertainty descriptions, allowing the inclusion of different levels of previous knowledge available on the uncertainty (including magnitude and phase information) and can be applied to other MPC strategies as well [9]

Further work should include stability issues, guidelines for selecting λ_ε and connections with other robust MPC algorithms.

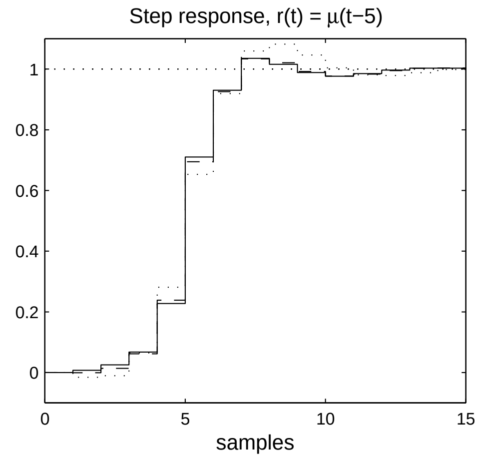

Figure 4: Robustness of the three designs, T_{yr}

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