Asymptotic Stability of Robot Control with Approximate Jacobian Matrix and its Application to Visual Servoing

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Abstract

In order to describe a task for the robot manipulator, a desired path for the end effector is usually specified in task space such as Cartesian space. In the presence of uncertainty in kinematics, it is impossible to derive the desired joint angle from the desired end effector path by solving the inverse kinematics problem. In addition, the Jacobian matrix of the mapping from joint space to task space could not be exactly derived. In this paper, we present feedback control laws for setpoint control of robot with uncertain kinematics and Jacobian matrix from joint space to task space. Sufficient conditions for the bound of the estimated Jacobian matrix and stability conditions for the feedback gains are presented to guarantee the stability of the robot's motion. Simulation results are presented to illustrate the performance of the proposed controllers.

1 Introduction

Many research efforts have been devoted towards the development of control schemes for dynamic control of robotic manipulator. In most of the control methods, the controllers are designed to move the robots along the desired joint angles [1, 2, 3, 4, 5, 6]. In order to move the robot end-effector along a desired path, the exact knowledge of the kinematics is required to solve the inverse kinematics problem to generate the desired paths in joint space. However, when the kinematics of the robot system is uncertain, it is impossible to derive the desired joint angle from the desired end effector path. When the control problem is formulated directly in task space [7, 8, 9], the inverse kinematics problem is replaced by a Jacobian transpose in the control law. However, such schemes still require the exact knowledge of the Jacobian matrix from joint space to task space. In the presence of certain uncertainties, the Jacobian matrix could not be exactly derived. To alleviate the difficulty, a feedback control law with imperfect Jacobian matrix is proposed by Miyazaki and Masutani [11]. The result is valid in a local sense when the initial states belong to a restricted region around the equilibrium state. In the paper [11], a sensor co-ordinate is defined as the task space and the exact knowledge of the Jacobian matrix of the mapping from Cartesian space to sensor space is not required. However, it is assumed that the exact model of manipulator Jacobian matrix of the mapping from joint space to Cartesian space is exactly known. It is also not sure to what extent the uncertainty could be allowed.

Therefore, most research on robot control has assumed that the exact kinematics and Jacobian matrix of the manipulator from joint space to Cartesian space are known. This assumption lead us to several open problems in the development of robot control laws today. In free motion, this implies that the exact lengths of the links, joint offsets and the object or tool which the robot is holding, must be known. When the control problem is extended to the control of multi-fingered robot hands, such assumption also limits its potential applications because the kinematics is usually uncertain in many applications of robot hands. For example, the contact points of the fingers are usually uncertain and changing during manipulation. Similarly, in hybrid position force control, such assumption of exact kinematics also leads us to an open problem on how to control the robot if the kinematics and constraint are uncertain.

To overcome these problems, we proposed approximate Jacobian feedback control laws [12, 13, 14, 15] for setpoint control of robots with uncertainties in the entire kinematics and Jacobian matrix from joint space to task space. It was shown that the end-effector's position converges to the desired position even when the kinematics and Jacobian matrix are uncertain. The proposed controllers [12, 13, 14, 15] require the measurement of a task space by a sensor such as vision systems and the task velocity vector is usually obtained by differentiation of the task vector. In this paper, we propose two new feedback control laws for setpoint control of robots with uncertainties in the kinematics and Jacobian matrix. In these new controllers, instead of using the actual task velocity vector as in [12, 13], we shall estimate the task velocity from the joint velocity and derive condition to guarantee the stability of the robot's motion. The main difficulty of estimating the task velocity from the joint velocity comes from the fact that the Jacobian matrix is uncertain in the presence of kinematic uncertainties. As a result, it is not sure whether the stability of the system could still be guaranteed. The main advantage is that feedback control can be established by measurement of joint velocity using tachometer or by differentiation from the joint angles which is usually less noisy. Sufficient condition for the bound of the estimated Jacobian matrix and stability conditions for the feedback gains shall be presented to gain a further understanding on the stability problem of feedback control with uncertain kinematics. A gravity regressor with uncertain Jacobian matrix is proposed for gravitational force compensation when the gravitational force is uncertain. Applications of the proposed controllers to visual servoing of robots without camera calibrations are discussed and simulation results are presented to illustrate the performances of the proposed controllers.

2 Problem Formulation

The equation of motion for the robotic manipulator with n degrees of freedom is given in joint space as [4]:

$$M(q)\ddot{q} + (B + \frac{1}{2}\dot{M}(q) + S(q,\dot{q}))\dot{q} + g(q) = \tau, \tag{1}$$

where $q \in R^n$ denotes the joint angles of the manipulator, $M(\cdot) \in R^{n\times n}$ is the inertia matrix which is symmetric and positive definite, $B \in R^{n\times n}$ denotes viscous friction matrix, $g(q) \in R^n$ is the gravitational force, $\tau \in R^n$ is the control inputs and $S(q,\dot{q})$ is a skew-symmetric matrix expressed by

$$S(q,\dot{q})\dot{q} = \frac{1}{2}\dot{M}(q)\dot{q} - \frac{1}{2}\{\frac{\partial}{\partial q}\dot{q}^T M(q)\dot{q}\}^T. \tag{2}$$

Let $X \in R^m$ $(m \leq n)$ be a task space vector defined by

$$X = h(q), \tag{3}$$

where $h(\cdot) \in R^n \to R^m$ is generally a nonlinear transformation describing the relation between the joint and task space. Then, the derivative of X is given as

$$\dot{X} = J(q)\dot{q}, \tag{4}$$

where $J(\cdot) = \frac{\partial h(\cdot)}{\partial q} \in R^{m\times n}$ is the Jacobian matrix.

Previously, we have developed the following approximate Jacobian feedback control law [12]:

$$\tau = -\hat{J}^T(q)(K_p s(e) + K_v \dot{X}) + g(q) \tag{5}$$

where $\hat{J}(q) \in R^{m\times n}$ is an approximate Jacobian matrix, $e = X - X_d = (e_1,\cdots,e_m)^T$ is the positional deviation from the desired position $X_d \in R^m$, $s(e) = (s_1(e_1),\cdots,s_m(e_m))^T$, K_p, K_v are positive definite diagonal feedback gains for the position and velocity respectively, $s_i(\cdot)$, $i = 1,\cdots,m$, are saturated functions [4] to be defined. The task space vector X is measured by a sensor such as vision systems, electromagnetic measurement systems and laser tracking systems, and $\dot{X}$ is usually obtained by differentiation of the task vector.

It is important to note that X and $\dot{X}$ could not be calculated from joint space using equations (3) and (4) since the kinematics is uncertain.

In this paper, we propose a new controller based on the estimated task velocity as follows:

$$\tau = -\hat{J}^T(q)K_p s(e) - \hat{J}^T(q)K_v\dot{\hat{X}} + g(q) \tag{6}$$

where

$$\dot{\hat{X}} = \hat{J}(q)\dot{q}.$$

In this case, $\dot{\hat{X}}$ is obtained by differentiation of the joint angles but it introduce a question on whether the stability of the system could still be guaranteed. We assume that $\hat{J}^T(q)$ is chosen so that

$$\|J^T(q) - \hat{J}^T(q)\| \leq p, \tag{7}$$

and p is a positive constant to be defined later.

Our objective is to show the asymptotic stability of the motion under the uncertain Jacobian feedback control law (6) and derive stability conditions to gain a further understanding on the feedback control problem of robot when the kinematics is uncertain.

3 Feedback Control of Robot with Uncertain Jacobian

Let us define a scalar potential function $S_i(\theta)$ and its derivative $s_i(\theta)$ as in [4] with the following properties :

(1) $S_i(\theta) > 0$ for $\theta \neq 0$ and $S_i(0) = 0$.

(2) $S_i(\theta)$ is twice continuously differentiable, and the derivative $s_i(\theta) = \frac{dS_i(\theta)}{d\theta}$ is strictly increasing in θ for $|\theta| < \gamma_i$ with some γ_i and saturated for $|\theta| \geq \gamma_i$, i.e. $s_i(\theta) = \pm s_i$ for $\theta \geq +\gamma_i$ and $\theta \leq -\gamma_i$ respectively where s_i is a positive constant.

(3) There are constants $\bar{c}_i > 0$, $d_i > 0$, $\bar{d}_i(> d_i) > 0$ such that,

$$\bar{d}_i s_i^2(\theta) \geq \theta s_i(\theta) \geq d_i s_i^2(\theta) > 0, \quad S_i(\theta) \geq \bar{c}_i s_i^2(\theta), \tag{8}$$

for $\theta \neq 0$. □

Substituting equation (6) into equation (1), we have

$$M(q)\ddot{q} + (B + \frac{1}{2}\dot{M}(q) + S(q,\dot{q}))\dot{q} + \hat{J}^T(q)(K_p s(e) + K_v\hat{J}(q)\dot{q}) = 0. \tag{9}$$

Let us define a vector y of the form

$$y = \dot{q} + \alpha\hat{J}^+(q)s(e). \tag{10}$$

where $\hat{J}^+(q)$ denotes the pseudo-inverse of $\hat{J}(q)$ such that $\hat{J}(q)\hat{J}^T(q)$ is non-singular and $\hat{J}(q)\hat{J}^+(q) = I$. Then, the inner product of y with equation (9) yields,

$$\frac{d}{dt}V(s(e),\dot{q}) + W(s(e),\dot{q}) = 0, \tag{11}$$

where

$$V(s(e),\dot{q}) = \tfrac{1}{2}\dot{q}^T M(q)\dot{q} + \alpha\dot{q}^T M(q)\hat{J}^+(q)s(e) + \textstyle\sum_{i=1}^m (k_{pi} + \alpha k_{vi})S_i(e_i), \tag{12}$$

$$\begin{aligned} W(s(e),\dot{q}) &= \dot{q}^T(\hat{J}^T(q)K_v\hat{J}(q) + B)\dot{q} + \alpha s(e)^T K_p s(e) \\ &- \dot{q}^T(J^T(q) - \hat{J}^T(q))K_p s(e) - \alpha s(e)^T K_v(J(q) - \hat{J}(q))\dot{q} \\ &+ \alpha\{s(e)^T(\hat{J}^+(q))^T(B - \tfrac{1}{2}\dot{M}(q) + S(q,\dot{q}))\dot{q} \\ &- \dot{s}(e)^T(\hat{J}^+(q))^T M(q)\dot{q} - s(e)^T(\dot{\hat{J}}^+(q))^T M(q)\dot{q},\} \end{aligned} \tag{13}$$

and k_{pi}, k_{vi} denote the i^{th} diagonal elements of K_p and K_v respectively. Since

$$\begin{aligned} &\tfrac{1}{4}\dot{q}^T M(q)\dot{q} + \alpha\dot{q}^T M(q)\hat{J}^+(q)s(e) + \textstyle\sum_{i=1}^m (k_{pi} + \alpha k_{vi})S_i(e_i) \\ &= \tfrac{1}{4}(\dot{q} + 2\alpha\hat{J}^+(q)s(e))^T M(q)(\dot{q} + 2\alpha\hat{J}^+(q)s(e)) - \\ &\alpha^2 s(e)^T(\hat{J}^+(q))^T M(q)\hat{J}^+(q)s(e) + \textstyle\sum_{i=1}^m (k_{pi} + \alpha k_{vi})S_i(e_i) \\ &\geq \textstyle\sum_{i=1}^m \{k_{pi}\bar{c}_i + \alpha(k_{vi}\bar{c}_i - \alpha\lambda_m)\}s_i^2(e_i) \end{aligned}$$

where $\lambda_m \triangleq \lambda_{max}[(\hat{J}^+(q))^T M(q)\hat{J}^+(q)]$. Substituting into equation (12), we have

$$V(s(e),\dot{q}) \geq \frac{1}{4}\dot{q}^T M(q)\dot{q} + \sum_{i=1}^m \{k_{pi}\bar{c}_i + \alpha(k_{vi}\bar{c}_i - \alpha\lambda_m)\}s_i^2(e_i) \geq 0$$

where K_v and α can be chosen so that

$$k_{vi}\bar{c}_i - \alpha\lambda_m > 0. \tag{14}$$

Therefore, the function $V(s(e), \dot{q})$ represents a Lyapunov function candidate for the setpoint control of the robot with uncertain Jacobian matrix. From the last term on the right hand side of equation (13), since $s(e)$ is bounded, there exist constants $c_0 > 0$ and $c_1 > 0$ so that [4]

$$\begin{aligned} &\alpha|s(e)^T(\hat{J}^+(q))^T(B - \tfrac{1}{2}\dot{M}(q) + S(q,\dot{q}))\dot{q} \\ &-\dot{s}(e)^T(\hat{J}^+(q))^T M(q)\dot{q} - s(e)^T(\dot{\hat{J}}^+(q))^T M(q)\dot{q}| \\ &\leq \alpha c_0\|\dot{q}\|^2 + \alpha c_1\|s(e)\|^2. \end{aligned} \tag{15}$$

Substituting this into equation (13) and let $\bar{\Delta} \triangleq J^T(q) - \hat{J}^T(q)$, we have

$$\begin{aligned} W(s(e), \dot{q}) &\geq \dot{q}^T(\hat{J}^T(q)K_v\hat{J}(q) + B - \alpha c_0 I)\dot{q} \\ &+\alpha s(e)^T(K_p - c_1 I)s(e) - \dot{q}^T\bar{\Delta}(K_p + \alpha K_v)s(e) \end{aligned} \tag{16}$$

The existence of a $\bar{\Delta}$ so that $W \geq 0$ can be clearly seen from equation (16). In the following development, we will derive a sufficient condition to guarantee $W \geq 0$. From equation (16), we have

$$\begin{aligned} W(s(e), \dot{q}) \geq &\{\lambda_{min}[\hat{J}^T(q)K_v\hat{J}(q) + B] - \alpha c_0\}\|\dot{q}\|^2 \\ &-p(\lambda_{max}[K_p] + \alpha\lambda_{max}[K_v])\|s(e)\| \cdot \|\dot{q}\| \\ &+\alpha(\lambda_{min}[K_p] - c_1)\|s(e)\|^2, \end{aligned} \tag{17}$$

where b_J denotes the norm bound for $J(q)$.Note that

$$-\|s(e)\| \cdot \|\dot{q}\| \geq -\frac{1}{2}(\|s(e)\|^2 + \|\dot{q}\|^2). \tag{18}$$

Hence

$$\begin{aligned} W(s(e), \dot{q}) \quad \geq \quad &(\lambda_{max}[K_v]l_1 - \alpha c_0)\|\dot{q}\|^2 \\ &+(\lambda_{max}[K_v]l_2 - \alpha c_1)\|s(e)\|^2, \end{aligned} \tag{19}$$

where

$$\begin{aligned} l_1 &= \lambda_1 - \tfrac{p}{2}(a_1 + \alpha), \\ l_2 &= \alpha a_1 \tfrac{\lambda_{min}[K_p]}{\lambda_{max}[K_p]} - \tfrac{p}{2}(a_1 + \alpha), \\ \lambda_1 &= \tfrac{\lambda_{min}[J^T(q)K_v J(q) + B]}{\lambda_{max}[K_v]}, \end{aligned} \tag{20}$$

and $a_1 = \frac{\lambda_{max}[K_p]}{\lambda_{max}[K_v]}$. Hence if

$$\min\{\frac{2\hat{\lambda}_1}{a_1 + \alpha}, \frac{2a_1\alpha\frac{\lambda_{min}[K_p]}{\lambda_{max}[K_p]}}{a_1 + \alpha}\} > p, \tag{21}$$

then $l_1 > 0$ and $l_2 > 0$ and hence K_v can be chosen large enough so that

$$l_1 - \frac{\alpha c_0}{\lambda_{max}[K_v]} > 0, \quad l_2 - \frac{\alpha c_1}{\lambda_{max}[K_v]} > 0 \tag{22}$$

and hence $W \geq 0$. Note that in condition (21), the viscous friction $B\dot{q}$ plays a role in ensuring the robustness of the uncertain Jacobian controller. In practice, such friction is always present in robots or can be added intentionally to the control input τ.

Figure 1 show the graphical illustration of the condition. In order to guarantee the stability of the system with uncertain Jacobian matrix, the allowable bound p of the Jacobian uncertainty $\bar{\Delta} = \hat{J}^T(q) - J^T(q)$ must be less than the minimum of the two curves shown in the figure. From condition (21), if p is small, α can be chosen small and therefore a smaller controller gain is required as seen from equation (22). Note that a larger range of a_1 can be chosen for a smaller p as seen in the figure. Conversely, if p is large, a larger controller gain is required and a narrower range of a_1 is allowed. The condition implies that for a chosen K_v, if the position feedback gain K_p is increase by increasing a_1, then the allowable bound of the Jacobian uncertainty Δ is decreased. This is reasonable because increase K_p amplifies the estimated Jacobian $\hat{J}(q)$ and hence more accuracy on the estimation is required.

As compared with the stability condition in our previous paper [12], we note from condition (21) that a_1 could not be too small with the estimated task space velocity. This could be due to the fact that additional uncertainty is present in the task velocity (estimated by $\hat{J}(q)\dot{q}$) and hence K_v could not be too large.

Note that the intersection of the two curves denotes the maximum bound of the sufficient condition (23). To gain a further insight on the condition, we let $K_v = k_v I$, $K_p = ak_v I$ for simplicity, then condition (21) becomes,

$$\min\{\frac{2\lambda_2}{a + \alpha}, \frac{2a_1\alpha}{a + \alpha}\} > \bar{p} \tag{23}$$

where $\lambda_2 = \lambda_{min}[\hat{J}^T(q)\hat{J}(q) + B/k_v]$ When the two curves intersect, we have $a = \frac{\lambda_1}{\alpha}$ and condition (23) becomes,

$$\bar{p} < \frac{2\alpha\lambda_2}{\alpha^2 + \lambda_2}, \tag{24}$$

The left hand side of equation (24) is maximum when $\alpha = \lambda_2^{\frac{1}{2}}$. Hence, condition (24) becomes,

$$\bar{p} < \lambda_2^{\frac{1}{2}}, \tag{25}$$

which is the maximum allowable bound of the Jacobian uncer

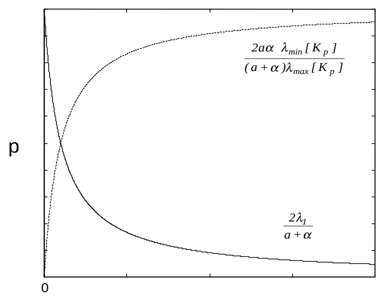

Figure 1: Variation of p with a

We are now in a position to prove the following Theorem:

Theorem 1. *The equilibrium state $(X_d, 0)$ of the closed loop system described by equation (9) is asymptotically stable with uncertain Jacobian matrix $\hat{J}(q)$ if the feedback gains K_p and K_v are chosen to satisfy conditions (14), (21), (22), and $\hat{J}(q)$ is chosen to satisfy condition (7).*

Proof:
Since both $V(s(e), \dot{q})$ and $W(s(e), \dot{q})$ are positive definite, from equation (11), we have

$$\frac{d}{dt}V(s(e), \dot{q}) = -W(s(e), \dot{q}) \leq 0. \quad (26)$$

Hence, V is a Lyapunov function whose time derivative is negative in $(s(e), \dot{q})$. This implies directly the asymptotic stability of the equilibrium state such that, $X_d - X \to 0$, $\dot{q} \to 0$, as $t \to \infty$ for any initial state $X(0)$ and $\dot{q}(0)$.

4 Feedback Control With Uncertain Gravitational Force

Next, we consider the case with uncertain gravitational force compensation and with uncertain Jacobian matrix using the concept of gravity regressor [5, 6, 4]. In our approach, the exact knowledge of the Jacobian matrix is not required in updating the regressor. Note that the gravity term can be completely characterized by a set of parameters $\theta = (\theta_1, \cdots, \theta_p)^T$ [4] as

$$g(q) = Z(q)\theta, \quad (27)$$

where $Z(q) \in R^{n\times p}$ is the gravity regressor. Then, the control input is proposed as

$$\tau = -\hat{J}^T(q)(K_p s(e) + K_v \hat{J}(q)\dot{q}) + Z(q)\hat{\theta}, \quad (28)$$

$$\hat{\theta}(t) = \hat{\theta}(0) - L\int_0^t Z^T(q(\tau))(\dot{q}(\tau) + \alpha\hat{J}^+(q(\tau))s(e(\tau)))d\tau, \quad (29)$$

where $\hat{\theta}(0)$ is the initial estimations at $t = 0$ and $L \in R^{p\times p}$ is a positive definite matrix. Substituting equations (27) and (28) into equation (1), we have

$$M(q)\ddot{q} + (B + \dot{M}(q) + S(q, \dot{q}))\dot{q} + \hat{J}^T(q)(K_p s(e) + K_v\hat{J}(q)\dot{q}) + Z(q)\Delta\theta = 0, \quad (30)$$

where $\Delta\theta = \theta - \hat{\theta}$. The asymptotic stability of the equilibrium state $(X_d, 0)$ with uncertain gravitational force is specified by the following Theorem:

Theorem 2. *The equilibrium state $(X_d, 0)$ of the closed loop system described by equations (30) and (29) is asymptotically stable with uncertain Jacobian matrix $\hat{J}(q)$ and gravitational force if the feedback gains K_p and K_v are chosen to satisfy conditions (14), (21), (22), and $\hat{J}(q)$ is chosen to satisfy condition (7).*

Proof:
Taking inner product of equation (30) with $y = \dot{q} + \alpha\hat{J}^+(q)s(e)$ and using equation (29), we have

$$\frac{d}{dt}\{\tfrac{1}{2}\dot{q}^T M(q)\dot{q} + \alpha\dot{q}^T M(q)\hat{J}^+(q)s(e) + \textstyle\sum_{i=1}^m (k_{pi} + \alpha k_{vi})S_i(e_i) + \tfrac{1}{2}\Delta\theta^T L^{-1}\Delta\theta\} + \dot{q}^T(\hat{J}^T(q)K_v\hat{J}(q) + B)\dot{q}$$
$$+\alpha s(e)^T K_p s(e) - \dot{q}^T(J^T(q) - \hat{J}^T(q))K_p s(e)$$
$$-\alpha s(e)^T(J(q) - \hat{J}(q))K_v\dot{q}$$
$$+\alpha\{s(e)^T(\hat{J}^+(q))^T(B - \tfrac{1}{2}\dot{M}(q) + S(q, \dot{q}))\dot{q}$$
$$-\dot{s}(e)^T(\hat{J}^+(q))^T M(q)\dot{q} - s(e)^T(\dot{\hat{J}}^+(q))^T M(q)\dot{q}, \} = 0,$$

which is equal to

$$\frac{d}{dt}(V(s(e), \dot{q}) + \frac{1}{2}\Delta\theta^T L\Delta\theta) = -W(s(e), \dot{q}). \quad (31)$$

Therefore, by LaSalle's invariance Theorem, we have, $X_d - X \to 0$, $\dot{q} \to 0$ as $t \to \infty$ for any initial state $X(0)$ and $\dot{q}(0)$.

Similar to our previous controllers proposed in [12], the controllers in this paper is dependent on $\hat{J}(q)$ but not $J(q)$. In addition, the analysis is independent of $J^+(q)$ and only dependent on $\hat{J}^+(q)$. This indicates the potential use of designing $\hat{J}(q)$ purposefully so that the control law will not get stall at the singular configurations or regions of the manipulator. That is, instead of using $J(q)$ which is not of full rank at a singular point, it is now possible to design a $\hat{J}(q)$ so that it is of full rank at the singular point. Condition (7) and (21) could also serve as a measure for the passability at singular points [12]). Our approach would widen the feasible workspace of robot since it would be able to pass through the singular points instead of avoiding it. Therefore, the equilibrium state is globally asymptotically stable if the estimate of the Jacobian matrix is carefully chosen to be nonsingular at the singular points.

5 Application to Visual Servoing

If cameras are used to measure the position of the end-effector, the task coordinates is defined as image coordinates. Let r represents the position of the end-effector in Cartesian coordinates and X represents a vector of image feature parameters [16]. The velocity vector $\dot{X}$ is therefore related to $\dot{q}$ as

$$\dot{X} = J_I(r)\dot{r} = J_I(r)J_e(q)\dot{q}, \quad (32)$$

where $J_I(r)$ is the image Jacobian matrix and $J_e(q)$ is the manipulator Jacobian matrix of the mapping from joint space to Cartesian space. Therefore, the controllers described by equation (6) and equation (28) includes image-based position controller [16] with

$$\hat{J}(q) = \hat{J}_I(r)\hat{J}_e(q), \quad (33)$$

where $J(q) = J_I(r)J_e(q)$. It is important to note that in the research of vision-based control [16], no stability result has been obtained for such image-based controller with a general class of uncertainty in the entire Jacobian matrix $J(q)$, taking into consideration the full nonlinear robot dynamics. In [11], a simple rotation transformation matrix $J_I(r) = R(r)$ is considered and $J_e(q)$ is assumed to be exactly known. That is, $\hat{J}(q) = \hat{R}(r)J_e(q)$. Furthermore, it is assumed that the robot is nonredundant and the result is only valid in a local sense. This restricts the potential applications since image-based robots are redundant in general. It is also not sure to what extent the uncertainty of the Jacobian matrix could be allowed.

5.1 Simulation

To illustrate the results, let us consider the 2-link manipulator holding an object as shown in figure 2 [12]. The manipulator Jacobian matrix of the mapping from joint space to Cartesian space $r = [x, y]^T$ for this manipulator is given by :

$$J_e(q) = \begin{bmatrix} -l_1 s_1 - (l_2 + l_o)s_{12} & -(l_2 + l_o)s_{12} \\ l_1 c_1 + (l_2 + l_o)c_{12} & (l_2 + l_o)c_{12} \end{bmatrix} \quad (34)$$

where $s_1 = \sin(q_1)$, $c_1 = \cos(q_1)$, $s_{12} = \sin(q_1 + q_2)$, $c_{12} = \cos(q_1 + q_2)$ and l_1, l_2, l_o denote the lengths of the first, second links and object respectively.

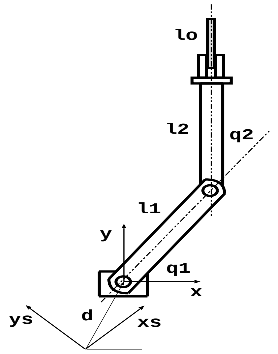

Figure 2: A Two-link Robot Holding an Object

A camera is placing parallel to the plane with certain focal length and some distance away as shown in the figure. We consider the relationship between this vision coordinates $X = [x_s, y_s]^T$ and the Cartesian coordinates given by [12]

$$\begin{bmatrix} x_s \\ y_s \end{bmatrix} = \frac{f_l}{z - f_l} \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

where δ is the rotational angle of the vision coordinates relative to the Cartesian coordinates, $d = (d_x, d_y)^T$ is the offset of the origins of the coordinates, f_l represents the focal length of the camera and z represents the perpendicular distance between the robot and the image form in the camera. The perfect Jacobian of the vision system from the joint to the vision coordinates is as below,

$$J(q) = \frac{f_l}{z - f_l} \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} J_e(q)$$

In the simulation, the exact masses m_1 and m_2 of the links were set to 2 kg, the exact lengths l_1 and l_2 of the links were set to 1 meter, f_l was set as 0.05 m, z was set as 3 m and both d_x and d_y were set to 0 meter. The robot holds an object of $m_o = 0.5$ kg and $l_o = 0.5$ meter.

Effects of Camera Calibration Errors

When the camera model is uncertain due to camera calibration errors, we could only approximate the Jacobian matrix as,

$$\hat{J}(q) = \frac{\hat{f}_l}{\hat{z} - \hat{f}_l} \begin{bmatrix} \cos\hat{\delta} & \sin\hat{\delta} \\ -\sin\hat{\delta} & \cos\hat{\delta} \end{bmatrix} J_e(q)$$

where $\hat{\delta}$ is the estimation of δ, $\hat{f}_l$ and $\hat{z}$ are the estimations of f_l and z respectively.

First, the camera parameters were set as: $\hat{f}_l = f_l = 0.05$ m, $\hat{z} = z = 3$ m and δ was varied. It was observed that the parameter δ has a greater effect on the stability of the robot. The system would become unstable when the error is too large. With $K_v = K_p = 5000I$ $(a = 1)$, Figure 3 shows an unstable response with $\delta = 60^0$.

To stabilize the system, we have to reduce the feedback ratio as stated in the Theorem 1 and illustrated in figure 1. Figures 4 shows a stable response with a is reduced to 0.5 (by increasing K_v) with $K_p = 5000I$, $K_v = 10000I$ and Figure 5 shows a stable response with $K_p = 2500I$, $K_v = 5000I$ (by reducing K_p).

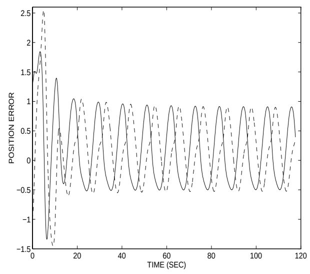

Figure 3: Effects of Large δ

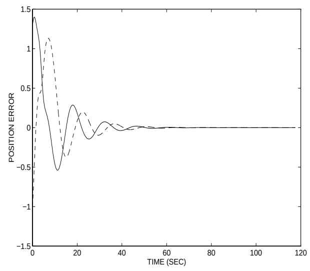

Figure 4: Effects of reducing a by increasing K_v

Effects of Uncertain Kinematics

Next, we consider robots with uncertain kinematics. In addition, the robot is also holding an object of uncertain length or uncertain gripping point. When the kinematics of the robot is uncertain, we could only obtain an approximate manipulator Jacobian matrix as follows:

$$\hat{J}_e(q) = \begin{bmatrix} -\hat{l}_1 s_1 - (\hat{l}_2 + \hat{l}_o) s_{12} & -(\hat{l}_2 + \hat{l}_o) s_{12} \\ \hat{l}_1 c_1 + (\hat{l}_2 + \hat{l}_o) c_{12} & (\hat{l}_2 + \hat{l}_0) c_{12} \end{bmatrix},$$

where $\hat{l}_1$, $\hat{l}_2$ and $\hat{l}_o$ are the estimations of l_1, l_2 and l_o respectively. Simulation result with $\hat{l}_o = 0$ m and $\hat{l}_1 = 0.75$ m, $\hat{l}_2 = 1.25$ m, $K_p = 5000I$, $K_v = 5000I$ is shown in Figure 6.

Overall Effects

When all the uncertainties mentioned above were considered together, the system remains stable so long as the feedback ratio a is tuned properly as in the Theorems. Figure 7 shows a stable response with $\hat{f}_l = 0.1$ m, $\hat{z} = 4$ m, $K_p = 5000I$, $K_v = 30000I$.

Finally, Figure 8 shows the result of using a regressor with $K_p = 5000I$, $K_v = 30000I$, $L = 0.12I$ and $\alpha = 1$. $\hat{\theta}_1(0)$ and $\hat{\theta}_2(0)$ were set to zero.

6 Conclusion

We have proposed feedback control laws for setpoint control of robots with uncertainty in the kinematics from joint space to task space and with uncertain dynamics. We have shown that the task velocity in the controller can be observed by the estimated task velocity $\hat{\dot{X}} = \hat{J}(q)\dot{q}$. The end-effector's position converges to the desired position in the task space even when the kinematics is uncertain. Most important of all, our results can be extended naturally to the control of robot fingers with uncertain Jacobian matrix [14] and hybrid position and force control of robot with uncertain kinematics and constraint [15].

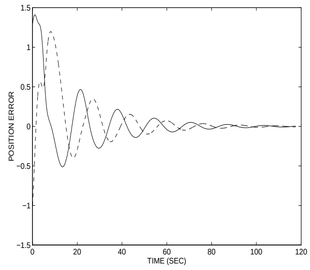

Figure 5: Effects of reducing a by reducing K_p

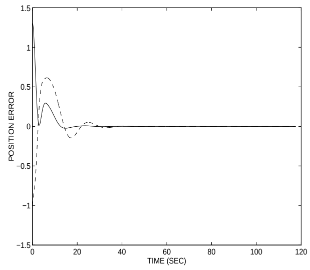

Figure 6: Effect of Uncertain Kinematics

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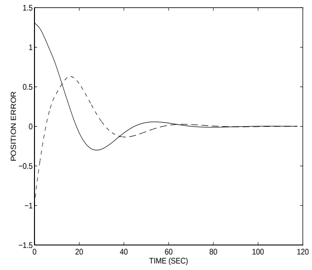

Figure 7: Effects of All uncertainties

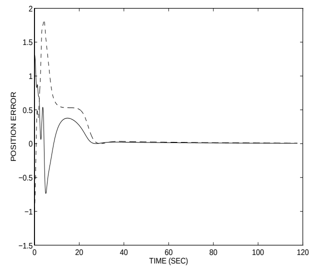

Figure 8: With Gravity Regressor

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