Robust Hurwitz and Schur Stability Test for Interval Matrices*

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Abstract—By relying on a two-dimensional (2-D) face test, we obtain necessary and sufficient condition for the robust Hurwitz and Schur stability of interval matrices. We reveal that it is impossible that there are some isolated unstable points in the parameter space of the matrix family, so the stability of exposed 2-D faces of an interval matrix guarantees stability of the matrix family. Examples are given to demonstrate the applicability of our robust stability test of interval matrices.

Index Terms— Interval matrices, robust stability, test theorem and algorithm

1 Introduction

For an uncertain system, its system matrix can be expressed as an interval matrix or a polytope of matrices, and its robust stability can be determined by testing a family of matrices. However, in contrast to the robust stability analysis of polynomials, there is no possibility to extend Kharitonov's approach to uncertain matrices. Ref. [1, 2] show that the Edge test [3] for a polytope of polynomials can not work for a polytope of matrices and interval matrices. [4, 5] proposed a 2-D face test of matrix family to solve the problem of the stability test for rank-one polytope of matrices. Instead of necessary and sufficient conditions, most presented results about interval matrices are sufficient conditions only [6-8], which merely provide some conservative conclusions for a stable parameter space and the stability of interval matrices. The necessary and sufficient condition of [9] is valid for nonnegative interval matrices, a special class of interval matrix families only.

In this paper, extending the results of [4,5], we give a possible approach to solve the problem of robust stability of interval matrices, whose parameters are stochastically and independently varying in some given range. We show that an interval matrix is a linear affine mapping of its parameters, which means that the exposed faces of parameter space can be mapped into the exposed faces of the interval matrix. For an eigenvalue of an interval matrix, there is a hypersurface of in parameter space, and the hypersurface will intersect at least an exposed face. We further show that the 2-D exposed faces of interval matrices can guarantee stability of entire matrix families. We need not to test higher dimensional faces of interval matrices.

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2 Main results

We first give the definition of interval matrices.

Definition 1: A family of N×N matrices

$$\mathbf{A}(q) = [\underline{\mathbf{A}},\ \overline{\mathbf{A}}], q \in Q = \{q | \underline{q}_{ij} \le q_{ij} \le \overline{q}_{ij}\} \qquad (1)$$

is defined as an interval matrix, where $\underline{\mathbf{A}} = [\underline{q}_{ij}]$ and $\overline{\mathbf{A}} = [\overline{q}_{ij}]$ are fixed matrices, the entries q_{ij} of $\mathbf{A}(q)$ are independent and have uncertainty bounds $\underline{q}_{ij}$ and $\overline{q}_{ij}$, respectively.

Property 1 Interval matrix $\mathbf{A}(q)$ is a linear affine mapping of q.

Proof: Clearly, there exists a linear affine relationship between $\mathbf{A}(q)$ and its parameter set q, since interval matrices satisfy

$$\mathbf{A}(aq^1 + bq^2) = a\mathbf{A}(q^1) + b\mathbf{A}(q^2) \qquad (2)$$

for all $q^1, q^2 \in Q$. Q.E.D.

Since the uncertainty set Q is a hyperrectangle by (1), by Property 1, the $N \times N$ -dimensional interval matrix $\mathbf{A}(q)$ can be viewed as a hyperrectangle in the Euclidean space of maximum dimension N^2(when all the entries q_{ij} of $\mathbf{A}(q)$ are uncertain). The edges and faces of Q are affined into the edges and faces of $\mathbf{A}(Q)$, respectively.

Property 2 The interior of Q can only be affined into the interior of $\mathbf{A}(Q)$.

Proof: Denote *ri C* as relative interior of set *C*, *cl C* as relative closure of set *C*, and *int C* as relative interior of set *C*. Consider $q^0 \in ri\, Q$, and $q^1 \in cl\, Q$ with $q^0 \neq q^1$, for any $x \in (0,1)$, let

$$q(x) = xq^0 + (1-x)q^1.$$

From $q^0 \in ri\, Q$, it follows that there is a ball *B* centered at q^0 with $B \subset Q$. From $q^1 \in cl\, Q$ and the convexity of *Q*, it next follows that

$$B(x) = xB + (1-x)q^1$$

is contained in *Q*. But $B(x)$ is a ball centered at $q(x)$, whence $q(x) \in ri\, Q$. Due to Property 1, we further have

$$\mathbf{A}[q(x)] \in ri\, \mathbf{A}(Q) \quad (3)$$

as desired. Q.E.D.

Property 2 provides us a possibility to determine the stability of interval matrix by testing low dimensional faces of interval matrix.

In this paper, contrary to the definition of exposed edges and 2-D exposed faces of polytopes of polynomials in [3], we define exposed edges and 2-D exposed faces of an interval matrix by corresponding exposed edges and 2-D exposed faces of parameter set *Q*.

The definition of Ref. [3] about exposed edges and 2-D exposed faces of polytopes of polynomials is determined by the boundary of root domain of polytopes of polynomials. However, because the mapping from parameter space to the root domain of complex plane is not one-one, how to identify the exposed edges and 2-D exposed faces of polytopes of polynomials remains an open problem. To avoid the problem, our definitions for exposed edges and 2-D exposed faces of an interval matrix are based on corresponding exposed edges and 2-D exposed faces of parameter set *Q*.

Definition 2: An exposed edge of the interval matrix **A** is defined as

$$\mathbf{A}[Q_1(q_{ij})] = \{q \mid q_{ij} \in [\underline{q}_{ij}, \overline{q}_{ij}]; \; q_{mn} = \underline{q}_{mn} \text{ or } \overline{q}_{mn}, (m,n) \neq (i,j)\} \quad (4)$$

i.e. let one entry q_{ij} of the interval matrix be free, the other entries are fixed to their upper bounds or lower bounds, then the matrix with one free parameter is an exposed edge of the given interval matrix.

We have to notice that the stability of all exposed edges of an interval matrix does not guarantee stability of the entire interval matrix [1], which is different from interval polynomials.

Similar to Definition 2, the exposed faces of an interval matrix in the parameter space can be determined by the following definition.

Definition 3: An exposed 2-D face of the interval matrix $\mathbf{A}(q)$ is defined as

$$\begin{aligned} Q_2(q_{ij}, q_{kl}) = \{q \mid\; & q_{ij} \in [\underline{q}_{ij}, \overline{q}_{ij}]; \\ & q_{kl} \in [\underline{q}_{kl}, \overline{q}_{kl}],\ for\ kl \neq ij; \\ & q_{mn} = \underline{q}_{mn} \text{ or } \overline{q}_{mn}, \\ & for\ (m,n) \neq (i,j) \text{ and } (m,n) \neq (k,l)\} \end{aligned} \quad (5)$$

i.e. let two entries of the interval matrix be free, the other entries are fixed to their upper-bounds or lower bounds, then the matrix with two free parameters is an exposed face of the given interval matrix.

The number of exposed faces of an interval matrix depends on the number of uncertain parameters in the matrix, here we have considered the possible case of some parameters being fixed. Suppose the number of uncertain parameters in an interval matrix to be $2 \le K \le N \times N$, then the number of exposed faces of the interval matrix is

$$N_F = 2^{K-2} \frac{K(K-1)}{2} \quad (6)$$

The characteristic polynomial associated with interval matrix $\mathbf{A}(q)$ in Eq. (1) is

$$P(\lambda, q) = \det(\lambda I - \mathbf{A}(q)) \quad (7)$$

In the complex plane **C**, we need define the root domain of the characteristic polynomial (7) as follows.

Definition 4: Consider any $\mathbf{A}(q), q \in Q$, then $\mathbf{D}(\mathbf{A}(q)) \subset \mathbf{C}$ is the eigenvalue domain if

$$\mathbf{D}(\mathbf{A}(q)) = \{\lambda : \mathrm{P}(\lambda, q) = 0,\ q \in Q\} \quad (8)$$

From the definition, we can further define $\partial\mathbf{D}$ as the boundary of $\mathbf{D}(\mathbf{A}(q))$.

The question we need to answer is whether $\partial\mathbf{D}$ is contained in the eigenvalue domain of 2-D exposed faces of $\mathbf{A}(Q)$, i.e.

$$\partial\mathbf{D}(\mathbf{A}(Q)) \subset \bigcup_{\mathrm{i}} \mathbf{D}[\mathbf{A}(Q_2(\mathrm{i}))], q \in Q \quad (9)$$

Definition 5: The interval matrix defined by (1) is Hurwitz stable, if all the roots of $P(\lambda, q)$ in (8) for all $q \in Q$ are in the open left-half plane.

Definition 6: The interval matrix defined by (1) is Schur stable, if all the roots of $P(\lambda, q)$ in (8) for all $q \in Q$ are inside the unit disk.

Interval matrices have other important properties, from which we can obtain some useful results.

Property 4: $D(\mathbf{A}(Q))$ is compact.

Proof: The characteristic polynomial of an interval matrix has the following relation with its roots

$$P(\lambda, q) = \det(\mathbf{A}(q) - \lambda I) \qquad (10)$$

Since Q is compact (closed and bounded), and $\mathbf{A}(q)$ is affine mapping of q by Property 1, $D(\mathbf{A}(Q))$ is also compact.

Q.E.D.

Lemma 1: If $\lambda_0 \in D(\mathbf{A}(Q))$, there is at least one exposed face $\mathbf{A}(Q_2(i))$ of $\mathbf{A}(Q)$ such that

$$\lambda_0 \in D[\mathbf{A}(Q_2(i))], i \in \{1, \ldots, N_F\} \qquad (11)$$

where $\mathbf{A}(Q_2(i))$ are precisely the exposed faces of $\mathbf{A}(q)$, and N_F is the number of the exposed faces.

Proof: Assuming that $\mathbf{A}(Q)$ has K $(K \geq 3)$ uncertain parameters, since $\lambda_0 \in D(\mathbf{A}(Q))$, from Eq.(7), we get a set of parameters,

$$L(q) = \{q \mid \det[\lambda_0 I - \mathbf{A}(q)] = 0\} \qquad (12)$$

in the parameter space Q.

If λ_0 is real, solve Eq.(12), $L(q)$ we get is a (K-1)-dimensional hypersurface in Q; if λ_0 is complex, then $L(q)$ is a (K-2)-dimensional hypersurface in Q.

Since $\dim(Q) = K \geq 3$, $\dim(L(q) \cap aff(Q)) \geq 2$, where $aff(Q)$ is the affine hull of Q. Because q_{ij} are independent, if λ_0 is real, $L(q)$ is a (K-1)-dimensional hypersurface in Q, and if λ_0 is complex, $L(q)$ is a (K-2)-dimensional hypersurface, which imply that the set $L(q) \cap aff(Q)$ must pierce the relative boundary of Q. Since the relative boundary of Q is the union of some (K-1)-dimensional hyperrectangles

$$\partial Q = \bigcup_i Q_{K-1}(i) \qquad (13)$$

in parameter space, at least one of the relative boundary of $Q_{K-1}(\cdot)$ satisfies $\lambda_0 \in D(\mathbf{A}(Q_{K-1}))$ due to Property 2. Repeat the preceding argument, if $\dim(Q_m) \geq 3$ m=K-2, K-3, ...,3, the set $L(q) \cap aff(Q_m)$ must pierce the relative boundary of $Q_m(\cdot)$, Since $\mathbf{A}(q)$ is affine mapping of q, $\mathbf{A}[L(q) \cap aff(Q_m)]$ must pierce the relative boundary of $\mathbf{A}[Q_m(\cdot)]$. This procedure will ultimately produce a two-dimensional surface $Q_2(\cdot)$ for which $\lambda_0 \in D(\mathbf{A}(Q_2))$, $Q_2(\cdot)$ belongs to the relative boundary of $Q_3(\cdot)$. However, $\mathbf{A}(Q_2(\cdot))$ is an exposed face of $\mathbf{A}(Q)$.

Q.E.D.

Remarks:

In the above proof, we can prove that finally $L(q) \cap aff(Q_3)$ must pierce the 2-D face $Q_2(\cdot)$, since $L(q)$ is a (K-1)-dimensional hypersurface in Q and $L(q)$ is half open due to the entries q_{ij} of $\mathbf{A}(q)$ are independent. If the entries q_{ij} are dependent, we can not ensure that $L(q) \cap aff(Q_3)$ must pierce at least one 2-D face $Q_2(\cdot)$. Here it is critical that the entries q_{ij} of $\mathbf{A}(q)$ are independent.

Since $L(q) \cap aff(Q_3)$ is a curve and it is half-open, it may not intersect any vertex or edge of $Q_2(\cdot)$, but it had to pierce some 2-D face $Q_2(\cdot)$. Therefore, generally, the stability of vertices and edges of an interval matrix does not guarantee the stability of entire interval matrix.

From Lemma 1, we can further obtain the following lemma.

Lemma 2:

$$\partial D(\mathbf{A}(Q)) \subset \bigcup_{i=1}^{N_F} D(\mathbf{A}(Q_2(i)) \qquad (14)$$

where $Q_2(i)$ are precisely the exposed faces of $\mathbf{A}(q)$, and N_F is the number of the exposed faces.

Based on the above results, we can propose and prove the following theorem.

Theorem 1: An interval matrix is Hurwitz stable if and only if all its exposed faces are Hurwitz stable.

Proof: The necessity is obvious, since all of its exposed faces of an interval matrix are sub-sets of the interval matrix.

Sufficiency: It is impossible that in $\mathbf{A}(Q)$ there exists isolated unstable point

$$\mathbf{A}(q_0) \in \mathbf{A}(\overline{Q}), \overline{Q} = Q - \partial Q \qquad (15)$$

where ∂Q is the relative boundary of Q. Assume that λ_0 is an unstable eigenvalue of $\mathbf{A}(q_0)$, and it is real, from Lemma 1, there exists a (K-1)-dimensional hypersurface $L(q)$ in Q, such that $L(q) \cap aff(Q_3)$ must pierce at least one 2-D face $Q_2(\cdot)$, which means that corresponding to λ_0, there exists a subset of matrices, $\mathbf{A}[L(q)]$, which is a hypersurface from the interior point $\mathbf{A}(q_0)$ to an exposed face $\mathbf{A}(Q_2(\cdot))$ of $\mathbf{A}(Q)$, instead of the isolated unstable point $\mathbf{A}(q_0)$ only.

Thus, the isolated unstable point $\mathbf{A}(q_0)$ can only occur on one of exposed 2-D faces of $\mathbf{A}(q)$, but it is also impossible since it will contradict the condition of the theorem: the exposed faces of $\mathbf{A}(q)$ matrix are Hurwitz stable.

Now, we need exclude the possibility that there exists a continuous unstable curve $D(\mathbf{A}(q_c)), q_c \in \overline{Q}$. We only need show that if there exists such unstable curve, it will intersect $D(\mathbf{A}(Q_2(j))$, where

$$D(\mathbf{A}(Q_2(j)) \subset \bigcup_{i=1}^{N_F} D(\mathbf{A}(Q_2(i)),\ j \in \{1,\ldots,N_F\}$$

…(16)

Since the unstable curve $D(\mathbf{A}(q_c)) \subset D, q_c \in \overline{Q}$, by Lemma 2, the eigenvalue domain *D* will have a part in the right-half plane. Since *D* is compact due to Property 3, to keep the property, some part of the boundary $\partial\mathbf{D}$ will intersect the right-half plane, due to Lemma 2, which implies that there exists at least one unstable exposed face of $\mathbf{A}(q)$, i.e. $D[\mathbf{A}(Q_2(j))]$ in Eq.(14) intersects the right-half plane. However, it contradicts the theorem's assumption that all its exposed faces are Hurwitz stable . Q.E.D.

On the same line of Theorem 1, we can obtain and prove a theorem for the Schur case.

Theorem 2: An interval matrix is Schur stable if and only if all of its exposed faces are Schur stable.

The proof is trivial, since it can be obtained by slight modification for the proof of Theorem 1, where the problem becomes whether eigenvalue domain *D* has a part outside unit disk.

It is worth to point that, the conditions of Theorem 1 and Theorem 2 for interval matrices can not be extended to the uncertain matrices with more general multilinear or dependent parameters. For latter, the affine property (Property 1) of interval matrices can not hold always.

3 Application

To verify our results in last section to be correct, according to Theorem 1, we only need to confirm following condition

$$\max_{q\in Q_2(\cdot)}\{\mathrm{Re}\lambda[\mathbf{A}(Q_2(\cdot))]\} = \max_{q\in Q}\{\mathrm{Re}\lambda[\mathbf{A}(Q(q))] \qquad (17)$$

This condition means that the eigenvalues of interval matrix $\mathbf{A}(q)$ with maximum of real parts can be found on some of exposed 2-D faces of $\mathbf{A}(q)$.

Example 1: Test whether the following interval matrix, which is taken from [1], satisfies condition (16):

$$\mathbf{A}(q_1,q_2,q_3,q_4) = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & q_3 \\ 0 & -0.7115 & q_4 \end{bmatrix} \qquad (18)$$

where

$$q_1 \in [-2.478,-1.4471], q_2 \in [-0.0518,-0.0194],$$
$$q_3 \in [2.0,3.437], q_4 \in [-0.0026,-0.0012].$$

The parameter space of interval matrix (18) is a 4-dimensional one. In the 4-D parameter space, by Eq.(6), we know that interval matrix (18) has 24 exposed faces. From definition 3, we can obtain all the exposed faces. Testing the exposed faces and $\mathbf{A}(q_1,q_2,q_3,q_4)$ in Eq.(18) by a numerical algorithm we developed, we find

$$\max_{q\in Q_2(\cdot)}\{\mathrm{Re}\lambda[\mathbf{A}(Q_2(\cdot))]\} = \max_{q\in Q}\{\mathrm{Re}\lambda[\mathbf{A}(Q(q))]$$
$$= -0.0103$$

satisfies condition (17), and all the faces are Hurwitz stable. Due to Theorem 1, the interval matrix (18) is Hurwitz stable. The result is in agreement to that of [1].

The results of Example 1 confirm that Theorem 1 is correct, we need not check every point of interior of parameter space for an interval matrix, it is impossible and not necessary; since the stability of exposed faces of an interval matrix can guarantee the stability of its interior points.

We are also interested in answering whether our results can solve some open problems in [1]. Therefore we use our theorems in last section to test the stability of some interval matrices, the stability of which can not be determined by edge test of interval matrices.

Example 2: Test the Hurwitz stability of the following interval matrix with 4 uncertain parameters

$$\mathbf{A}(q) = \begin{bmatrix} -1 & 0 & [-1,1] \\ 0 & -1 & [-1,1] \\ [-1,1] & [-1,1] & 0.1 \end{bmatrix} \qquad (19)$$

Calculating the number of exposed faces by Eq.(6), we know that interval matrix (19) has 24 exposed faces. We get one of exposed faces by virtue of Definition 3,

$$\mathbf{A}[Q_2(q_{13},q_{23})]=\begin{bmatrix}-1 & 0 & q_{13}\\ 0 & -1 & q_{23}\\ 1 & 1 & 0.1\end{bmatrix} \quad (20)$$

for $q_{13}\in[-1,1], q_{23}\in[-1,1]$.

Since the exposed face (20) has some eigenvalues on the right-half plane,

$$\max_q\{\mathrm{Re}\lambda[\mathbf{A}(Q_2(q))]\}=1.0674,$$

interval matrix (19) is not Hurwitz stable due to Theorem 1.

Example 3: Test the Schur stability of the following interval matrix, which is derived from Example 2 of [10]:

$$\mathbf{A}(q)=\begin{bmatrix}[-0.3,-1.4] & [0.2109,0.4109] & [0.1229,0.329]\\ [-0.15,0.05] & [-0.22,-0.18] & [-0.5,1.1]\\ [-0.45,-0.25] & [0.2,0.4] & [-0.18,0.2]\end{bmatrix}$$
...(21)

The parameter space Q of interval matrix (21) is a 9-dimensional one and of 4608 exposed faces. Applying Theorem 2 to test all the exposed faces, we find that the faces are Schur stable, so the interval matrix in (21) is Schur stable. This result is in agreement to that of [10].

4 Further Discussions

Our further question to be discussed is whether the 2-D face stability test for interval matrices can be used to the stability test for the polytopes of matrices? In [4,5], we have given a positive answer, but we had to notice that there exist some differences between the two kinds of matrix families. The continuous time dynamic systems which involve the polytopes of matrices are as following

$$\frac{d\mathbf{x}}{dt}=\left(\mathbf{A}_0+\sum_{k=1}^{K}q_k\mathbf{A}_k\right)\mathbf{x}, \mathbf{x}(0)=\mathbf{x}_0 \quad (22)$$

and discrete time dynamic systems system as follows

$$\mathbf{x}(n+1)=\left(\mathbf{A}_0+\sum_{k=1}^{K}q_k\mathbf{A}_k\right)\mathbf{x}(n), \mathbf{x}(0)=\mathbf{x}_0 \quad (23)$$

where the matrices $\mathbf{A}_k\in R^{N\times N}$ are known , and uncertain parameters

$$q_K=[\underline{q}_k,\overline{q}_K], k=1,...,K \quad (24)$$

We study the problem of robust stability of the systems (22) and (23) with uncertain parameters (24). This problem in fact is to study robust Hurwitz stability for system (22), or Schur stability for system (23), of the perturbed matrix given by following definition.

Definition 7: The perturbed matrix in Eq.(22) or Eq.(23) is define

$$\mathbf{A}(q_1,...,q_K)=\mathbf{A}_0+\sum_{k=1}^{K}q_k\mathbf{A}_k, q_K=[\underline{q}_k,\overline{q}_K]$$
…(25)

where the matrices $\mathbf{A}_k\in R^{N\times N}$ are known, the uncertain parameters $q_k, k=1,...,K$ are independent due to (24). We denote by Q^K the set of $q=[q_1,...,q_k]$, the elements of Q^K called vectors.

We had to notice that different from interval matrix, the uncertain matrix family (25) belongs to a kind of polytopes of matrices, for which we have following two stability test theorems[4,5].

Theorem 3: The perturbed matrix in Eq.(25) is Hurwitz stable if and only if all its 2-D faces are Hurwitz stable.

Theorem 4: The perturbed matrix in Eq.(25) is Schur stable if and only if all its 2-D faces are Schur stable.

The proofs of Theorem 3 and 4 can refer [4, 5, and 11].

Compared with Theorem 1and Theorem 2, we can find that here, we have not mention the exposed 2-D faces, since every 2-D face of a polytope of matrices is its exposed 2-D face. Therefore, we need test all 2-D faces of a polytope of matrices, while it is not necessary for an interval matrix.

Now, we give an algorithm to construct the 2-D faces. The 2-D faces $\mathbf{A}(Q_3(\cdot))$ in Theorem 3 and 4 can be expressed as

$$\mathbf{A}(Q_3)=\mathbf{A}_iq_i+\mathbf{A}_jq_j+\mathbf{A}_kq_k, \mathbf{A}_i, \mathbf{A}_j, \mathbf{A}_k\in\{\mathbf{A}_m\}, i\neq j\neq k \quad (26)$$

where

$$q_i\geq 0, q_j\geq 0, q_k\geq 0, q_i+q_j+q_k=1 \quad (27)$$

$\mathbf{A}(Q_3(\cdot))$ under the constraint (27) in fact is a matrix triangle in $\mathbf{R}^{N\times N}$, the matrix triangle is the smallest 2-D face. We can express $\mathbf{A}(Q_3(\cdot))$ in a simpler form

$$\mathbf{A}(Q_3) = y[x\mathbf{A}_j + (1-x)\mathbf{A}_k] + (1-y)\mathbf{A}_i \qquad (28)$$

for all x,y∈[0,1], $i,j,k \in \{1, ..., M\}$, where $\mathbf{A}_i, \mathbf{A}_j, \mathbf{A}_k \in \{\mathbf{A}_m\}, i \neq j \neq k$.

Example 4**:** Consider the following polytope of matrices, which is taken from Ref. [2],

$$\mathbf{A}[q_1,...,q_4)] = \begin{bmatrix} 0 & 1 & -q_1 & -q_2 \\ -1 & 0 & -q_3 & -q_4 \\ q_1 & q_3 & -1 & 0 \\ q_2 & q_4 & 0 & -1 \end{bmatrix} \qquad (29)$$

where $q_k = [-1,1], k = 1,...,4$.

To test the stability of the polytope of matrices given by Eq.(29), we need find all its vertix matrices, then according to Eq.(27) or Eq.(28), construct 2-D face with different combination of three vertex matrices, then test the stability of all 2-D faces. We find that some of 2-D faces are unstable, for example, the following 2-D face

$$\mathbf{A}(Q_3) = y[x\mathbf{A}(1,1,1,1) + (1-x)\mathbf{A}(-1,-1,-1,-1,)] + (1-y)\mathbf{A}(1,1,-1,-1)$$

for all x,y∈[0,1], with eigenvalue on imaginary axis, so the polytope of matrices (29) is not Hurwitz stable due to Theorem 3, the result is in agreement to that of [2].

We can find that though the parameters of the polytope of matrices in Example 4 are dependent, the 2-D face test can detect the unstable point in parameter space. Example 4 also shows that the difference between interval matrices and polytopes of matrices, for latter, we need test their all 2-D faces, the definition of exposed 2-D faces of interval matrices is not suitable to the case of polytopes of matrices.

We had to point that to guarantee the test results to be reliable, the numerical realization of 2-D test algorithm should adopt the grids to be tiny enough, since the 2-D face test itself is not a finite test. However, since we need not test high dimensional faces, which are no longer with two parameters only, the 2-D face test can greatly reduce computation amount in the stability test procedure of interval matrices and polytopes of matrices.

5 Conclusions

In this paper, we presented necessary and sufficient conditions for the robust Hurwitz and Schur stability of interval matrices. Different from uncertain polynomials, we show that the robust Hurwitz and Schur stability of interval matrices can be ensured by the stability of their 2-D exposed faces.

We give an algorithm to obtain the exposed faces of an interval matrix. Examples verify our results to be valid. Different from the stability test of interval matrices, for a polytope of matrices, we need test its all 2-D faces to determine its stability.

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