Design of a STT Missile Autopilot Using Functional Inversion and LMI Approach[1]

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Abstract

In this paper, we present a novel systematic approach for the autopilot design of STT missiles. First, the nonlinear model of a STT missile is partially linearized via functional inversion techniques and then, the additional set-point tracking controller can be designed by the well-known LMI approach. The stabilization conditions are given in terms of LMI's.

1. Introduction

In this paper, we attempt to design a high performance autopilot for short-range surface-to-air STT (Skid-to-Turn) missiles. First, the pitch and yaw dynamics are still coupled due to the effect of bank angle. Second, the dynamics of STT missiles are highly nonlinear. Moreover, the aerodynamics cannot be described in closed form but is usually available only in look-up table form. Furthermore, it has been known that the input-output dynamic characteristics from the control fin deflection to the missile acceleration of tail-controlled missiles are of nonminimum phase. There have been a lot of approaches to design an autopilot such as the conventional linear perturbation technique[3], gain-scheduling approach[1, 2] and the well-known input/output (I/O) feedback linearization technique[4]-[7]. Unfortunately, however, it cannot be applied to the autopilot design for STT missiles. It is mainly because its direct application to acceleration control of missiles cannot guarantee internal stability since it can leave the hidden or zero dynamics unstable. In this paper, the nonlinear STT missile model is partially linearized using funtional inversion method and then the stabiliziong controller is designed using LMI techniques [8]. LMI(Linear matrix inequality) technique [8] can effectively handle the remaining nonlinearities and provide the systematic procedure to design a controller for STT missiles.

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2. Partial linearization of a STT missile model

The 3-dimensional motion of a STT missile then can be described by the following nonlinear ordinary differential equations [9, 10]. The meanings of the symbols used throughout the paper are the same as given at *Nomenclature* section in [9].

-Yaw Dynamics

$$\begin{aligned} \dot{V} &= -Ur + \frac{QS}{m}C_y(\tan^{-1}(\tfrac{V}{U}), \delta_r, \tfrac{V_M}{V_s}, \tan^{-1}(\tfrac{V}{W})) \\ \dot{r} &= \frac{QSD}{I_M}C_n(\tan^{-1}(\tfrac{V}{U}), \delta_r, \tfrac{V_M}{V_s}, \tan^{-1}(\tfrac{V}{W})) \\ A_y &= \frac{QS}{m}C_y(\tan^{-1}(\tfrac{V}{U}), \delta_r, \tfrac{V_M}{V_s}, \tan^{-1}(\tfrac{V}{W})) \ , \end{aligned} \tag{1}$$

-Pitch Dynamics

$$\begin{aligned} \dot{W} &= Uq + \frac{QS}{m}C_z(\tan^{-1}(\tfrac{W}{U}), \delta_q, \tfrac{V_M}{V_s}, \tan^{-1}(\tfrac{V}{W})) \\ \dot{q} &= \frac{QSD}{I_M}C_m(\tan^{-1}(\tfrac{W}{U}), \delta_q, \tfrac{V_M}{V_s}, \tan^{-1}(\tfrac{V}{W})) \\ A_z &= \frac{QS}{m}C_z(\tan^{-1}(\tfrac{W}{U}), \delta_q, \tfrac{V_M}{V_s}, \tan^{-1}(\tfrac{V}{W})) \ . \end{aligned} \tag{2}$$

Here, the functions C_y,C_z,C_m,C_n are the aerodynamic coefficients and are obtained in look-up table form through wind-tunnel experiments. The first step in our autopilot design for STT missiles is to transform the missile dynamics in (1) and (2) into a "partial-linearized" system by using the partial-linearizing controller proposed in [9]. As shown in [9], the functions C_y and C_z are invertible with respect to δ_r and δ_q, respectively. Thus, there exist the inverse mappings K_y, K_z of C_y and C_z with respect to δ_r and δ_q and the following partial linearized model can be obtained[9].

$$\begin{aligned} \dot{x}_1 &= A_1x_1 + B_1\hat{v}_y + G_1u_a \\ \dot{x}_2 &= A_2x_2 + B_2\hat{v}_z + G_2u_b \\ y_1 &= \hat{A}_y \ = \ C_1x_1 + \hat{v}_y \\ y_2 &= \hat{A}_z \ = \ C_2x_2 + \hat{v}_z \end{aligned} \tag{3}$$

where $\hat{x}(t) \triangleq \frac{x(\frac{t}{V_M})}{V_M}$ for $x = V, W, q, r$, $\hat{x}(t) \triangleq \frac{x(\frac{t}{V_M})}{V_M^2}$ for $x = v_y, v_z, A_y, A_z$, and

$$\begin{aligned} x_1 &= [\hat{V} \ \hat{r}]^T, \quad x_2 \ = \ [\hat{W} \ \hat{q}]^T, y_1 = \hat{A}_y, y_2 = \hat{A}_z \\ u_a &= H_a(\hat{V}, \hat{W}), \ u_b \ = \ H_b(\hat{V}, \hat{W}) \\ A_1 &= \begin{bmatrix} 0 & 0 \\ 0 & -hv \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 \\ 0 & hv \end{bmatrix}, h_v = \frac{(l_f - l_g)m}{I_M} \\ B_1 &= \begin{bmatrix} 1 \\ -hv \end{bmatrix}, G_1 \ = \ \begin{bmatrix} 0 \\ -\frac{\rho}{2} \end{bmatrix}, C_1 \ = \ [0 \ \ 1] \end{aligned}$$

$$B_2 = \begin{bmatrix} 1 \\ hv \end{bmatrix}, G_2 = \begin{bmatrix} 0 \\ \frac{\rho}{2} \end{bmatrix}, \; C_2 = [0 \quad -1]. \qquad (4)$$

Note that the nonlinearities in the system in (3) are only due to the term related to the function H and the variables of the system (3) are time-scaled by $\frac{t}{V_M}$.

3. Design of a controller using LMI techniques

In this section, we design a set-point tracking controller for the linearized STT missile model, (3), using LMI techniques. For given acceleration commands $\hat{A}_y^C$ and $\hat{A}_z^C$, we consider the following form of dynamic output feedback controllers.

$$\text{Yaw Controller} \; : \; \dot{\hat{x}}_{c_1} = A_{c_1}\hat{x}_{c_1} + B_{c_1}\{\hat{A}_y - \hat{A}_y^C\}, \quad \hat{v}_y = C_{c_1}\hat{x}_{c_1} + D_{c_1}\{\hat{A}_y - \hat{A}_y^C\} \qquad (5)$$

$$\text{Pitch Controller} \; : \; \dot{\hat{x}}_{c_2} = A_{c_2}\hat{x}_{c_2} + B_{c_2}\{\hat{A}_z - \hat{A}_z^C\}, \quad \hat{v}_z = C_{c_2}\hat{x}_{c_2} + D_{c_2}\{\hat{A}_z - \hat{A}_z^C\} \qquad (6)$$

where $x_{c_i} \in R^k, \; i = 1, 2$. Note that the controller is fully decoupled. As shown in [9], we assume that the nonlinearities H_a and H_b which satisfy some sector condition [9]. That is,there exist nonnegative constants $K_{min}, \; K_{max}$ such that the following inequalities hold.

$$\{\begin{bmatrix} \tilde{H}_a \\ \tilde{H}_b \end{bmatrix} - K_{min}\begin{bmatrix} \tilde{V} \\ \tilde{W} \end{bmatrix}\}^T\{\begin{bmatrix} \tilde{H}_a \\ \tilde{H}_b \end{bmatrix} - K_{max}\begin{bmatrix} \tilde{V} \\ \tilde{W} \end{bmatrix}\} \leq 0 \qquad (7)$$

where the definitions of $\tilde{H}_a$ and $\tilde{H}_b$ are given in [9]. Now, we provide a sufficient condition of the existence of stabilizing controllers for the system (3).

Theorem 1 : *If there exist positive definite matrices X_i and Y_i satisfying the follwing LMI's for given K_{min} and K_{max} and each $i = 1, 2$, there exists a stabilizing controller for the system (3).*

$$\begin{bmatrix} (B_i^T)^+ & 0 \\ 0 & I_2 \end{bmatrix}^T \begin{bmatrix} X_iA_i^T + A_iX_i + K_{min}\{G_iD_iX_i + X_iD_i^TG_i^T\} & G_i + \frac{1}{2}(K_{max} - K_{min})X_iD_i^T \\ \{G_i + \frac{1}{2}(K_{max} - K_{min})X_iD_i^T\}^T & -1 \end{bmatrix} \begin{bmatrix} (B_i^T)^+ & 0 \\ 0 & I_2 \end{bmatrix} < 0$$
$$\begin{bmatrix} C_i^+ & 0 \\ 0 & I_2 \end{bmatrix}^T \begin{bmatrix} A_i^TY_i + Y_iA_i + K_{min}\{Y_iG_iD_i + D_i^TG_i^TY_i\} & Y_iG_i + \frac{1}{2}(K_{max} - K_{min})D_i^T \\ \{Y_iG_i + \frac{1}{2}(K_{max} - K_{min})D_i^T\}^T & -1 \end{bmatrix} \begin{bmatrix} C_i^+ & 0 \\ 0 & I_2 \end{bmatrix} < 0 \qquad (8)$$
$$\begin{bmatrix} X_i & I_2 \\ I_2 & Y_i \end{bmatrix} \geq 0$$

The sufficient condition given by (8) can be checked by the well-known convex optimization techniques. The stabilizing controllers are obtained using the method suggested in [8].

4. Conclusion

In this paper, we present a novel autopilot design methodology, where functional inversion techniques and LMI approach are introduced. When the nonlinearities in the partially linearized model satisfy some Lipschitz-like conditions, the existence of stabilizing controllers can easily be checked using LMI approach and the existing design methods can be applied to the design of such controllers. While the existing control method can not easily estimate the stability margins for the nonlinear systems, the proposed method based on LMI techniques can easily measure the stability margins of the control systems through simple search algorithms.

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