Edge Test for Domain Stability of Polytopes of Two-dimensional (2-D) Polynomials*

Yang Xiao
Institute of Information Science, Northern Jiaotong University
Beijing 100044, P.R. China, E-mail: yxiao@center.njtu.edu.cn

Abstract

The necessary and sufficient conditions of domain stability of polytopes of 2-D polynomials have been established. The conditions are based on edge theorem and convex directions. The number of edges to be tested can be reduced by a testing set constructed by us. An example has been given to demonstrate the applicability of our new approach.

Indexing terms—Polytopes of two-dimensional polynomials, robust domain stability

1. Introduction

There exists a class of 2-D systems, such as linear partial differential equation systems and linear partial difference equation systems, with uncertain parameters or subject to random disturbances. Different from 1-D case, the state variables of the systems are two-dimensional, and the characteristic polynomials of the systems are of two complex variables. The asymptotic stability problem of uncertain 2-D systems can be concluded as that of robust stability of the characteristic polynomials with uncertain parameters. Since the root domain of 2-D polynomials is in two-dimensional complex number space C^2, classical stability analysis is not suitable to that of 2-D polynomials [1].

Based on extreme point test of interval 1-D polynomials, [2-4] get some results of about robust Hurwitz stability of diamond family of bivariate polynomials and interval bivariate polynomials. Applying the perturbation radius, [5] establishes sufficient conditions of domain stability for polytopes of 2-D polynomials. [7,8] show that the idea of edge test of 1-D polynomials [6] can be extended to 2-D, and get necessary and sufficient conditions of Hurwitz and Schur stability test of polytopes of bivariate polynomials, respectively. In this paper, based on the results of [7-10], we simplify the edge test in 2-D case by using a testing set, which adopts the phase conditions of introduced the test of convex direction. The test set greatly reduces the number of edges to be tested.

2. Auxiliary Results

Similar to 1-D case, we can define several types of polytopes of bivariate polynomials.

Definition 1: The polytopes of bivariate polynomials are defined as

$$B(s_1,s_2,q)=\sum_{n=0}^{N}\sum_{m=0}^{M} b_{mn}(q)\, s_1^{M-m} s_2^{N-n},(s_1,s_2)\in C^2 \quad (1)$$

where

$$b_{mn}(q)=\sum_{k=1}^{K}(a_k q_k + c_k),\, q_k \in [\underline{q}_k,\overline{q}_k] \quad (2)$$

a_k and c_k are real fixed parameters, and q_k are uncertain parameters.

From Eq.(2), we can see that $b_{mn}(q)$ depends (affines) linearly on **q**,

$$\begin{aligned}\mathbf{q} &= [q_1,q_2,\ldots,q_L] \\ &\in Q=\{q{:}q_i^- \le q_i \le q_i^+, i=1,2,\ldots,K\}\end{aligned} \quad (3)$$

and the bounding set Q is obtained by assuming an upper bound and a lower bound for each component q_i of **q**.

Definition 1 is a basic one for polytopes of bivariate polynomials, from the definition, we can easily derive some the definitions of other kinds of polytopes of bivariate polynomials.

Definition 2: If the parameter space Q in Definition 1 is a unit simplex

$$Q=\{\mathbf{q}\in R^K{:}q_i \ge 0,\, for\ i=1,\ldots,K\ and\ \sum_{i=1}^{K} q_i = 1\} \quad \ldots(4)$$

and $B_i(s_1,s_2)$, $i=1,\ldots,K$ are fixed polynomials, we can define a kind of polytopes of bivariate polynomials as

$$B(s_1,s_2,\mathbf{q})=\sum_{i=1}^{K} q_i B_i(s_1,s_2) \quad (5)$$

where

$$B_i(s_1,s_2)=\sum_{n=0}^{N}\sum_{m=0}^{M} b_{mn}^{i} s_1^{M-m} s_2^{N-n} \quad (6)$$

* This work is supported by NSFC of China.

We also can see that $B(s_1,s_2,\mathbf{q})$ in Definition 2 is a convex hull, and we write

$$B(s_1,s_2,\mathbf{q})=Conv[B_i(s_1,s_2)] \qquad (7)$$

which consists of all convex combinations of the $B_i(s_1,s_2)$.

Definition 3: If the parameter space *Q* in Definition 2 is

$$Q=Conv\{q_i\} \qquad (8)$$

where q_i denotes the extreme point of *Q*, *i=1,2,...,K*, and $B_i(s_1,s_2)$, *i=1,...,K* are fixed polynomials, we can define a kind of polytopes of bivariate polynomials as

$$B(s_1,s_2,\mathbf{q})=B_0(s_1,s_2)+\sum_{i=1}^{K} q_i B_i(s_1,s_2),\mathbf{q}\in Q \qquad (9)$$

where *Q* is defined by Eq.(8), and the fixed polynomial $B_i(s_1,s_2)$ has the same form as Eq.(6).

If $B_i(s_1,s_2)$ and $B_j(s_1,s_2)$ are vertex polynomials of a polytope of bivariate polynomials, we can further define the edge of the polytope.

Definition 4: An edge of a polytopes of bivariate polynomials is defined as

$$E_{ij}(s_1,s_2,x)=xB_i(s_1,s_2)+(1-x)B_j(s_1,s_2) \qquad (10)$$

for all $x\in[0,1]$, $i,j\in\{1, ..., M\}$, $i\neq j$, M is the number of vertex polynomials of the polytope, $B_i(s_1,s_2)$ and $B_j(s_1,s_2)$ are vertex polynomials of the polytope of bivariate polynomial.

The domain stability problem of a family of bivariate polynomials concerns the root domain of the family.

Definition 5: A polytope of bivariate polynomials is Hurwitz stable, if

$$B(s_1,s_2,\mathbf{q})\neq 0,\ for\ all\ \mathrm{Re}\, s_1\geq 0, \mathrm{Re}\, s_2\geq 0 \qquad (10)$$

Definition 6: A polytope of bivariate polynomials is Schur stable, if

$$B(s_1,s_2,\mathbf{q})\neq 0,\ for\ all\ |s_1|\geq 1, |s_2|\geq 1 \qquad (11)$$

Different from 1-D case, the root domain of polytope of bivariate polynomials is in C^2. We call C^2, the set of ordered 2-tuples of complex numbers the 2-dimensional complex number space.

Ref. [7,8] have found a simple approach to test the Hurwitz stability and Schur stability of polytopes of bivariate polynomials, the approach can be expressed by following two theorems.

Theorem 1[7]: The polytope of bivariate polynomials given by Definition 1-3 is Hurwitz stable if and only if
(a) the exposed edges of $B(s_1,1,\mathbf{q})$ are Hurwitz stable; and
(b) for all $\omega_1\in R$, the exposed edges of $B(j\omega_1,s_2,\mathbf{q})$ are Hurwitz stable.

The proof is given in [7].

Theorem 2[8]: The polytope of bivariate polynomials given by Definition 1-3 is Schur stable if and only if

(a) the exposed edges of $B(s_1,1,\mathbf{q})$ are Schur stable; and
(b) for all $\omega_1\in[0,2\pi]$, the exposed edges of $B(e^{j\omega_1},s_2,\mathbf{q})$ are Schur stable.
The proof is given in [8].

Condition (a) of Theorem 1 and 2 can be checked by presented 1-D edge test [6]. Condition (b) of Theorem 1 and 2 can be tested by the approach of [7,8]. Different from 1-D case, the test of is Condition (b) is two parameters' one. As pointed by [7,8], because the mapping from parameter space to C^2 is not one-one, it is difficult to identify the exposed edges of $B(j\omega_1,s_2,\mathbf{q})$ and $B(e^{j\omega_1},s_2,\mathbf{q})$, [7,8] had to test all the edges of a polytope of bivariate polynomials. We will discuss how to reduce the number of the edges in next section.

3. Main Results

The concept of convex direction of a polytope of 1-D polynomials has been proposed in [9]. The 1-D result of [9] can be extended to polytope of bivariate polynomials [10].

Lemma 1: For all $\omega_1\in R$, a 2-D polynomial $B(j\omega_1,s_2)$ is a convex direction in Hurwitz case iff

$$\frac{\partial \arg B(j\omega_1,j\omega_2)}{\partial \omega_2}\geq 0,\ for\ all\ \omega_2\in R \qquad (12)$$

Since for any given $\omega_1\in R$, the 2-D polynomial $B(j\omega_1,s_2)$ can be regarded as a 1-D polynomial with complex coefficients, the proof can refer to [10].

Lemma 2: For all $\omega_1\in[0,2\pi]$, a 2-D polynomial $B(e^{j\omega_1},s_2)$ is a convex direction in Schur case iff

$$\frac{\partial \arg B(e^{j\omega_1},e^{j\omega_2})}{\partial \omega_2} \geq \frac{N}{2}, \text{ for all } \omega_2 \in [0,2\pi] \quad (13)$$

where N is the highest degree of variable s_2 in $B(e^{j\omega_1},s_2)$. Since for any given $\omega_1 \in [0,2\pi]$, the 2-D polynomial $B(e^{j\omega_1},s_2)$ can be regarded as a 1-D polynomial with complex coefficients, the proof can refer to [10].

Lemma 3: If for given $\omega_1 \in R$,

$$x_0 B_i(j\omega_1,j\omega_{20}) + (1-x_0)B_j(j\omega_1,j\omega_{20}) = 0 \quad (14)$$

then

$$\left.\frac{\partial \arg[B_i(j\omega_1,j\omega_2) - B_j(j\omega_1,j\omega_2)]}{\partial \omega_2}\right|_{\omega_2=\omega_{20}}$$

$$= x_0 \left.\frac{\partial \arg[B_i(j\omega_1,j\omega_2)]}{\partial \omega_2}\right|_{\omega_2=\omega_{20}} + (1-x_0)\left.\frac{\partial \arg[B_j(j\omega_1,j\omega_2]}{\partial \omega_2}\right|_{\omega_2=\omega_{20}} \quad (15)$$

Since for given $\omega_1 \in R$, the 2-D polynomial $B(j\omega_1,s_2)$ can be regarded as a 1-D polynomial with complex coefficients, the proof can refer to [9].

Lemma 4: If for given $\omega_1 \in [0,2\pi]$,

$$x_0 B_i(e^{j\omega_1},e^{j\omega_{20}}) + (1-x_0)B_j(e^{j\omega_1},e^{j\omega_{20}}) = 0 \quad (15)$$

then

$$\left.\frac{\partial \arg[B_i(e^{j\omega_1},e^{j\omega_2}) - B_j(e^{j\omega_1},e^{j\omega_2})]}{\partial \omega_2}\right|_{\omega_2=\omega_{20}}$$

$$= x_0 \left.\frac{\partial \arg[B_i(e^{j\omega_1},e^{j\omega_2})]}{\partial \omega_2}\right|_{\omega_2=\omega_{20}} + (1-x_0)\left.\frac{\partial \arg[B_j(e^{j\omega_1},e^{j\omega_2})]}{\partial \omega_2}\right|_{\omega_2=\omega_{20}} \quad (16)$$

Since for given $\omega_1 \in [0,2\pi]$, the 2-D polynomial $B(e^{j\omega_1},s_2)$ can be regarded as a 1-D polynomial with complex coefficients, the proof can refer to [9].

Lemma 5: For given $\omega_1 \in R$, an edge of a polytopes of bivariate polynomials defined by Eq.(10) is Hurwitz stable, if the following conditions hold,

(a) $B_i(j\omega_1,s_2)$ and $B_j(j\omega_1,s_2)$ are Hurwitz stable;

(b) for all $\omega_2 \in R$,

$$\frac{\partial \arg[B_i(j\omega_1,j\omega_2) - B_i(j\omega_1,j\omega_2)]}{\partial \omega_2} \geq 0 \quad (15)$$

Proof: Suppose Condition (a) holds, but there is an unstable 2-D polynomial on the edge polynomial, then there exist $\omega_{20} \in R$ and $x_0 \in [0,1]$, such that

$$x_0 B_i(j\omega_1,j\omega_{20}) + (1-x_0)B_j(j\omega_1,j\omega_{20}) = 0 \quad (16)$$

Following to Lemma 3, and replacing the right-hand side of Eq.(15) in Lemma 3 by the lower bound of Lemma 1, we have

$$\left.\frac{\partial \arg[B_i(j\omega_1,j\omega_2) - B_j(j\omega_1,j\omega_2)]}{\partial \omega_2}\right|_{\omega_2=\omega_{20}} < 0 \quad (17)$$

which contradicts (15), and this completes the proof. Q.E.D.

Lemma 6: For all $\omega_1 \in [0,2\pi]$, an edge of a polytopes of bivariate polynomials defined by Eq.(10) is Schur stable, if the following conditions hold,

(a) $B_i(e^{j\omega_1},s_2)$ and $B_j(e^{j\omega_1},s_2)$ are Schur stable;

(b) for all $\omega_2 \in [0,2\pi]$,

$$\frac{\partial \arg[B_i(e^{j\omega_1},e^{j\omega_2}) - B_j(e^{j\omega_1},e^{j\omega_2})]}{\partial \omega_2} \geq \frac{N}{2} \quad (18)$$

where N is the highest degree of variable s_2 in $B(e^{j\omega_1},s_2)$.

Proof: Suppose Condition (a) holds, but there is an unstable 2-D polynomial on the edge, then there exist $\omega_{20} \in [0,2\pi]$ and $x_0 \in [0,1]$, such that

$$x_0 B_i(e^{j\omega_1},e^{j\omega_{20}}) + (1-x_0)B_j(e^{j\omega_1},e^{j\omega_{20}}) = 0 \quad (19)$$

Following to Lemma 4, and replacing the right-hand side of Eq.(16) in Lemma 4 by the lower bound of Lemma 2, we have

$$\left.\frac{\partial \arg[B_i(e^{j\omega_1},e^{j\omega_{20}}) - B_j(e^{j\omega_1},e^{j\omega_{20}})]}{\partial \omega_2}\right|_{\omega_2=\omega_{20}} < \frac{N}{2} \quad \ldots(20)$$

which contradicts (18), and this completes the proof. Q.E.D.

Lemma 5 and Lemma 6 form the construction of reduced testing set for the root domain stability of polytopes of bivariate polynomials.

Theorem 3 The testing set for Hurwitz stability of a polytopes of bivariate polynomials includes the vertex polynomials and those edge polynomials that do not satisfy the conditions of Lemma 5.

Proof: It follows from Theorem 1, a polytopes of bivariate polynomials has a testing set, consisting the vertex polynomials and edge polynomials. If an edge polynomial satisfies conditions of Lemma 5, then the stability of vertex polynomials of the edge implies that of the edge polynomial. Then the edge is not crucial, and it can be excluded from the testing set of the polytopes of bivariate polynomials. Q.E.D.

Theorem 4 The testing set for Schur stability of a polytopes of bivariate polynomials includes the vertex polynomials and those edge polynomials that do not satisfy the conditions of Lemma 6.

The proof is nearly identical to that of Theorem 3.

Theorem 3 and Theorem 4 provide a approach to obtain test set of edge polynomials, we only test the edge polynomials which don't satisfy the Condition (b) of Theorem 3 or Theorem 4. For those edge polynomials that belong to the test set, we can use following theorems to check their domain stability.

Theorem 5: An edge $\mathrm{E}_{\mathrm{ij}}(s_1, s_2)$ in Definition 4 is Hurwitz stable, if and only if

(a) $B_i(s_1, s_2)$ and $B_j(s_1, s_2)$, the vertex polynomials of the polytope of bivariate polynomial, are Hurwitz stable;
(b) the frequency response plot of

$$B_i(j\omega_1, j\omega_2)/B_j(j\omega_1, j\omega_2)$$

does not cross $(-\infty, 0]$.

Proof: Necessity: $\mathrm{E}_{\mathrm{ij}}(s_1, s_2)$ is Schur stable implies Condition (a) and for all $\omega_1, \omega_2 \in R$,

$$xB_i(j\omega_1, j\omega_2) + (1-x)B_j(j\omega_1, j\omega_2) \neq 0, x \in (0,1] \quad ...(21)$$

Due to Condition (a), $B_i(s_1, s_2)$ and $B_j(s_1, s_2)$ are Hurwitz stable, then

$$B_i(j\omega_1, j\omega_2) \neq 0 \text{ and } B_j(j\omega_1, j\omega_2) \neq 0;$$

since $x \neq 0$, Eq. (21) is equivalent to

$$B_i(j\omega_1, j\omega_2)/B_j(j\omega_1, j\omega_2) + (1-x)/x \neq 0,$$

and $(1-x)/x \in [0, \infty)$, the frequency response plot of $B_i(j\omega_1, j\omega_2)/B_j(j\omega_1, j\omega_2)$ does not cross $(-\infty, 0]$.
Sufficiency: We show the sufficiency by contradiction. Suppose $B_i(s_1, s_2)$ and $B_j(s_1, s_2)$ are Hurwitz stable, else there will be contradiction to Condition (a). Suppose there exists some $x_0 \in (0,1]$ such that $\mathrm{E}_{\mathrm{ij}}(s_1, s_2)$ is not stable. Due to Theorem 1, we have

$$E_{ij}(j\omega_1, j\omega_2) = x_0 B_i(j\omega_1, j\omega_2) + (1-x_0)B_j(j\omega_1, j\omega_2) = 0$$

which is equivalent to

$$B_i(j\omega_1, j\omega_2)/B_j(j\omega_1, j\omega_2) + (1-x_0)/x_0 = 0.$$

However, it implies that the frequency response plot of $B_i(j\omega_1, j\omega_2)/B_j(j\omega_1, j\omega_2)$ crosses $(-\infty, 0]$, which contradicts Condition (b) of the theorem. Q.E.D.

Theorem 6: An edge $\mathrm{E}_{\mathrm{ij}}(z_1, z_2)$ in Definition 4 is Schur stable, if and only if

(a) $B_i(z_1, z_2)$ and $B_j(z_1, z_2)$, the vertex polynomials of the polytope of bivariate polynomial, are Schur stable;
(b) the frequency response plot of

$$B_i(e^{j\omega_1}, e^{j\omega_2})/B_j(e^{j\omega_1}, e^{j\omega_2})$$

does not cross $(-\infty, 0]$.
Proof: Necessity: $\mathrm{E}_{\mathrm{ij}}(z_1, z_2)$ is Schur stable implies Condition (a) and for all $\omega_1, \omega_2 \in [0, 2\pi]$,

$$xB_i(e^{j\omega_1}, e^{j\omega_2}) + (1-x)B_j(e^{j\omega_1}, e^{j\omega_2}) \neq 0, x \in (0,1] \quad ...(22)$$

Since $B_j(e^{j\omega_1}, e^{j\omega_2}) \neq 0, x \neq 0$, Eq. (22) is equivalent to

$$B_i(e^{j\omega_1}, e^{j\omega_2})/B_j(e^{j\omega_1}, e^{j\omega_2}) + (1-x)/x \neq 0,$$

and $(1-x)/x \in [0, \infty)$, the frequency response plot of $B_i(e^{j\omega_1}, e^{j\omega_2})/B_j(e^{j\omega_1}, e^{j\omega_2})$ does not cross $(-\infty, 0]$.
Sufficiency: We show the sufficiency by contradiction. Suppose $B_i(z_1, z_2)$ and $B_j(z_1, z_2)$ are Schur stable, else there will be contradiction to Condition (a). Suppose there exists some $x_0 \in (0,1]$ such that $\mathrm{E}_{\mathrm{ij}}(z_1, z_2)$ is not stable. Due to Theorem 2, we have

$$E_{ij}(e^{j\omega_1}, e^{j\omega_2}) = x_0 B_i(e^{j\omega_1}, e^{j\omega_2}) + (1-x_0)B_j(e^{j\omega_1}, e^{j\omega_2}) = 0,$$

which is equivalent to

$$B_i(e^{j\omega_1}, e^{j\omega_2})/B_j(e^{j\omega_1}, e^{j\omega_2}) + (1-x_0)/x_0 = 0.$$

However, it implies that the frequency response plot of

$$B_i(e^{j\omega_1}, e^{j\omega_2})/B_j(e^{j\omega_1}, e^{j\omega_2}) \text{ crosses } (-\infty, 0],$$

which contradicts Condition (b) of the theorem. Q.E.D.

3. The Test Algorithm and Application

From Theorem 3-6 we derived in Section 2, we can see that the theorem are not the simple extensions of 1-D presented results, their numerical implemented algorithms are much more complicated than those of 1-D tests. Generally, the algorithms are not finite ones, since they involve two-dimensional frequency grid test. To determine the domain stability of vertex polynomials of a given polytope of bivariate polynomial we need following two theorems [7,8].

Theorem 7: A fixed bivariate polynomial is Hurwitz stable, i.e.

$$B(s_1,s_2)=\sum_{n=0}^{N}\sum_{m=0}^{M} b_{mn} s_1^{M-m} s_2^{N-n} \neq 0,$$

for all s_1, s_2 with $\mathrm{Re}\ s_1 \geq 0$ and $\mathrm{Re}\ s_2 \geq 0$, where b_{mn} are fixed real numbers, if and only if

(a) $B(s_1,1) \neq 0$ for $\mathrm{Re}\, s_1 \geq 0$,

and

(b) $B(j\omega_1, s_2) \neq 0$ for all $\omega_1 \in R$ and $\mathrm{Re}\, s_2 \geq 0$.

Theorem 8: A fixed bivariate polynomial is Schur stable, i.e.

$$B(z_1, z_2) = \sum_{n=0}^{N}\sum_{m=0}^{M} b_{mn} z_1^{M-m} z_2^{N-n} \neq 0$$

for all z_1, z_2 with $|z_1| \geq 1$ and $|z_2| \geq 1$, where b_{mn} are fixed real numbers, if and only if

(a) $B(z_1,1) \neq 0$ for $|z_1| \geq 1$,

and

(b) $B(e^{j\omega_1}, z_2) \neq 0$ for all $\omega_1 \in [0,2\pi]$ and $|z_2| \geq 1$.

Now, we give a numerical implemented algorithm base on Theorems 3-8.

Step 1: According to Theorem 7 and Theorem 8, test the domain stability of vertex polynomials of a given polytope of bivariate polynomial; if find some vertex polynomial is not stable, end the test, we conclude that the polytope of bivariate polynomials is not stable; else enter next step;

Step 2: Construct the edge polynomials according to Definition 4;

Step 3: Apply Theorem 3 and Theorem 4 to determine test set; if we find no edge polynomial in the test set, we conclude that the polytope of bivariate polynomials is stable; else enter next step;

Step 4: Apply Theorem 5 and Theorem 6 to test stability of the edge polynomials in the test set, if we find edge polynomial is not stable, end the test, we conclude that the polytope of bivariate polynomials is not stable; else we conclude that the polytope of bivariate polynomials is stable.

Now, we illustrate the application of our results in previous sections by some example.

Example 1: Determine whether the following polytope of bivariate polynomials to be Schur stable

$$B(z_1,z_2,\mathbf{q}) = q_1 B_1(z_1,z_2) + q_2 B_2(z_1,z_2) + q_3 B_3(z_1,z_2) \qquad (23)$$

where

$$q_1 + q_2 + q_3 = 1, q_1 \geq 0, q_2 \geq 0, q_3 \geq 0,$$

and

$$B_i(z_1,z_2) = b_0^i(z_1) z_2^2 + b_1^i(z_1) z_2 + b_2^i(z_1), i = 1,2,3$$

where

$$b_0^1(z_1) = 1 - 1.2 z_1 + z_1^2$$
$$b_1^1(z_1) = -1.5 + 1.8 z_1 - 0.75 z_1^2,$$
$$b_2^1(z_1) = 0.6 - .72 z_1 + .272 z_1^2$$
$$b_0^2(z_1) = 1 - 1.2 z_1 + z_1^2$$
$$b_1^2(z_1) = -1.5 + 1.8 z_1 - 0.75 z_1^2,$$
$$b_2^2(z_1) = 0.62 - .72 z_1 + .29 z_1^2$$
$$b_0^3(z_1) = 1 - 1.2 z_1 + z_1^2$$
$$b_1^3(z_1) = -1.5 + 1.8 z_1 - 0.76 z_1^2.$$
$$b_2^3(z_1) = 0.6 - .72 z_1 + .29 z_1^2$$

Apply the above test algorithm, we find that vertex polynomials of the polytope (23) are all stable; since for $\omega_1, \omega_2 \in [0,2\pi]$,

$$\frac{\partial \arg[B_1(e^{j\omega_1}, e^{j\omega_2}) - B_2(e^{j\omega_1}, e^{j\omega_2})]}{\partial \omega_2} \in [1.0526, 20.0] \geq \frac{N}{2}, here\ N = 2$$

and

$$\frac{\partial \arg[B_2(e^{j\omega_1}, e^{j\omega_2}) - B_3(e^{j\omega_1}, e^{j\omega_2})]}{\partial \omega_2} \in [1.333, 4.0] \geq \frac{N}{2}, here\ N = 2$$

the edge bivariate polynomials $E_{12}(z_1, z_2)$ and $E_{23}(z_1, z_2)$ are Schur stable and they do not belong to the test set. Since

$$\frac{\partial \arg[B_1(e^{j\omega_1}, e^{j\omega_2}) - B_3(e^{j\omega_1}, e^{j\omega_2})]}{\partial \omega_2} = 0 < \frac{N}{2}, here\ N = 2$$

there is only the edge bivariate polynomial $E_{13}(z_1, z_2)$ in the test set and we test it by Theorem 6, we find that it is stable. Therefore, according to Theorem 2, the polytope of bivariate polynomials in Eq.(23) is Schur stable. The argument partial derivatives of $E_{12}(z_1, z_2)$, $E_{23}(z_1, z_2)$ and $E_{13}(z_1, z_2)$ are shown as Fig. 1, Fig.2, and Fig.3, respectively.

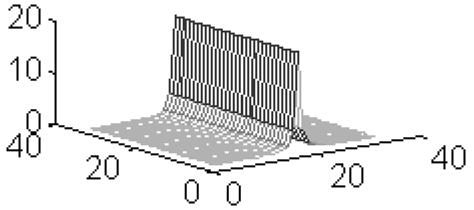

Fig.1 The argument partial derivatives of $E_{12}(z_1, z_2)$

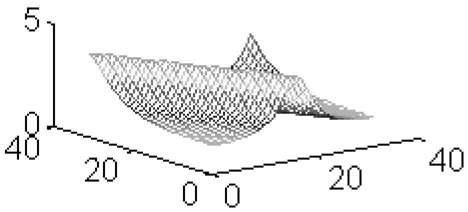

Fig.2 The argument partial derivatives of $E_{23}(z_1, z_2)$

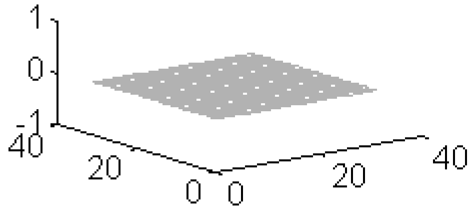

Fig.3 The argument partial derivatives of $E_{13}(z_1, z_2)$

4. Conclusions

In this paper, based on convex direction concept, we present an approach an approach to get the test set of domain stability of a polytope of bivariate polynomials, i.e. Theorem 3 and 4 Theorem. We employ Theorems 5 and 6 to test the stability of edges of the polytope, Theorems 7 and 8 to check the stability of vertex polynomials of the polytope. As a generation of the results, an algorithm is constructed by us to test domain stability of polytopes of bivariate polynomials. An example shows how the results to work.

References

[1] XIAO Yang and R. Unbehauen, New Stability Test Algorithm for Two-Dimensional Digital Filters, IEEE Transaction on Circuits and Systems, Vol. 45, No.7, July, 1998，pp.739-740.

[2] K. D. Kim, and N. K. Bose, "Invariance of the Strict Hurwitz Property for Bivariate Polynomials Under Coefficient Perturbations", IEEE Trans. Automatic Control, Vol.33, No.12, 1988, pp.1172-1174.

[3] B. T. Polyak and B. Shmulyian, "Frequency Domain Criteria for Robust Stability of Bivariate Polynomials", IEEE Trans. Circuits and Systems-I, Vol. 41, No.2, February 1994, pp.161-167.

[4] V. L. Kharitonov, J.A. Torres Munoz and M. I. Remires-Sosa M. "Stability and Robust Stability of Multivariate Polynomials", Proc. of the 36th Conference on Decision & Control, San Diego, California, USA, Dec. 1997, pp.3254-3259.

[5] XIAO Yang, DU Xiyu, and R. Unbehauen, Sufficient Conditions of Robust Uncertain 2-D Polynomials, Journal of Systems Science and Systems Engineering, Vol.8, No.3, Sept. 1999, pp.368-374.

[6] A. C. Bartlett, C. V. Hollot, H. Lin, "Root Locations of an Entire Polytope of Polynomials: It Suffices to Check the Edges", Math. Control Signals Systems, 1988, Vol. 1:61-71.

[7] Y. Xiao, R. Unbehauen and X. Du, Robust Hurwitz Stability Conditions of Polytopes of Bivariate Polynomials, Proc. of 38th IEEE Conference on Decision and Control, Phoenix, Arizona, USA, Dec. 7-10, 1999, pp.5030-5035.

[8] Y. Xiao, R. Unbehauen and X. Du, Schur Stability of Polytopes of Bivariate Polynomials, Proc. of IEEE ICECS'99. Pp.1269-1272.

[9] A. Rantzer, "Stability Conditions for Polytope of Polynomials", IEEE Trans. Automatic Control, Vol.37, No.1, 1992, pp.79-89.

[10] Y. Xiao, R. Unbehauen and X. Du, A Necessary Condition for Schur Stability of 2-D Polynomials, Proceedings of IEEE ISCAS'99, pp. III-439-442.

[11] T. S. Huang, "Stability of Two-Dimensional Recursive Filters", IEEE Trans. Audio Electroacoust., Vol. AU-20, June 1972, pp.158-163.

[12] Y. Xiao, R. Unbehauen and X. Du, “A Finite Test Algorithm for 2-D Schur Polynomials Based on Complex Lyapunov Equation,” Proceedings of IEEE International Symposium on Circuits and Systems, Orlando, Florida, May 30-June 2, 1999, Vol.3, pp.339-342.