Global Asymptotic Stability of a Class of Dynamic Neural Systems with Asymmetric Connection Weights

Youshen Xia and Jun Wang
Department of Automation and Computer-Aided Engineering
The Chinese University of Hong Kong
Shatin, New Territories, Hong Kong

Abstract— **Recently, a class of dynamic neural systems were presented and analyzed due to their good performance in optimization computation and low complexity for implementation. The global asymptotic stability of dynamic neural systems with symmetric weights was well studied. In this paper, we investigate the global asymptotical stability of a dynamic neural system with asymmetric weights. Since asymmetric weight cases are more general than symmetric ones, the new results are significant in both theory and applications.**

I. Introduction

In this paper, we are concerned with the following dynamic neural system

$$\frac{du}{dt} = -u + P_\Omega(Wu + \alpha q). \qquad (1)$$

where $q \in R^l$, α is a positive constant, $W = I - \alpha M$, M is an $l \times l$ matrix, $\Omega = \{u \in R^l \mid d_i \le u_i \le h_i, i \in I\}$, $I \subset L$, $L = \{1, ..., l\}$, and $P_\Omega : R^l \to \Omega$ is a projection operator which is defined by $P_\Omega(u) = [P_\Omega(u_1), ..., P_\Omega(u_l)]^T$ and for $i \in L - I$, $P_\Omega(u_i) = u_i$; for $i \in I$,

$$P_\Omega(u_i) = \begin{cases} d_i & u_i < d_i \\ u_i & d_i \le u_i \le h_i \\ h_i & u_i > h_i. \end{cases}$$

This dynamic system has two important properties. One is that its equilibrium point solves a linear variational inequality [4]: find $u^* \in \Omega$ such that

$$(u - u^*)^T (Mu^* - q) \ge 0, \quad \forall u \in \Omega. \qquad (2)$$

Thus, it can solve linear and convex quadratic programming problems, linear complementary problems, fixed point problems and bimatrix equilibrium points, and the analysis of piecewise linear resistive circuits. Another property is that this dynamic system is easy to be implemented by using a circuit with a single layer of neurons, where W is called a weight matrix, and thus is very amenable to parallel implementation. Hence, (1) can be viewed as a useful neural model. So, the stability of related neural networks has been investigated, and the global asymptotical stability of the neural system (1) with the symmetric connection weight matrix W has been studied in [1,2]. In this paper, by using the property of the gap function and projection technique [3] we explore the global asymptotical stability of the neural dynamic system with asymmetric connection weights. Specially, our new results can cover the asymptotic stability results of linear systems when $\Omega = R^l$.

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II. Main Results

Theorem 1. Assume that matrix M is positive definite but not necessarily symmetric, then for any $\alpha > 0$, the neural dynamic system (1) with $u_0 \in \Omega$ is globally asymptotically stable.

Theorem 2. Assume that matrix M is positive definite but not necessarily symmetric and $Mu^* - q = 0$, then for any $\alpha > 0$, dynamic system (1) with $u_0 \in \Omega$ is globally exponentially stable.

III. Illustrate Examples

Example 1. Consider the following neural system

$$\frac{dz}{dt} = -z + TP_\Omega(z) + q. \qquad (3)$$

where T is an $l \times l$ matrix. It is well known that this system belongs to Hopfield-type neural system. Let z^* be an equilibrium point. As a corollary of both Theorem 1 and Theorem 2, we have the following stability result of (3).

Corollary 1. Assume that the matrix T is nonsingular and matrix $-T+I$ is positive definite, then the neural system (3) with $u_0 \in \Omega$ is globally asymptotically stable. Moreover, when $(I-T)z^* - q = 0$, then the neural system (3) with $u_0 \in \Omega$ is globally exponentially stable.

Example 2. Consider a class of linear complementary problems (LCP) below: Find a vector $x \in R^l$ such that

$$\begin{aligned} x_i(Nx+c)_i &= 0, (Nx+c)_i \geq 0, x_i \geq 0 \quad \forall i \in I \\ (Nx+c)_j &= 0, \forall j \in L - I \end{aligned}$$

where $L = 1, ..., n, I \subset L, c \in R^n$, and $N \in R^{n \times n}$ is a positive semidefinite matrix. According to [4], this problem can written as (2), where $\Omega = \{x \in R^n | x_i \geq 0, \forall i \in I\}$ and $P_\Omega(\cdot)$ denotes the projection onto the set Ω. So the dynamic neural system for LCP is

$$\frac{dx}{dt} = P_\Omega((I - \alpha N)x - \alpha c) - x. \quad (4)$$

As an immediate corollary of both Theorem 1 and Theorem 2, we have the following stability result of (4).

Corollary 2. Assume that matrix N is positive definite but not necessarily symmetric, then for any $\alpha > 0$, the neural dynamic system (4) with $u_0 \in \Omega$ is globally asymptotically stable. Moreover, when $Nx^* + c = 0$, then the dynamic neural system (4) with $u_0 \in \Omega$ is globally exponentially stable.

Finally, we simulate by (4) a classical LCP, where N is an $n \times n$ upper triangular matrix

$$N = \begin{bmatrix} 2 & 2 & 2 & \dots & 2 \\ 0 & 2 & 2 & \dots & 2 \\ 0 & 0 & 2 & \dots & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 2 \end{bmatrix},$$

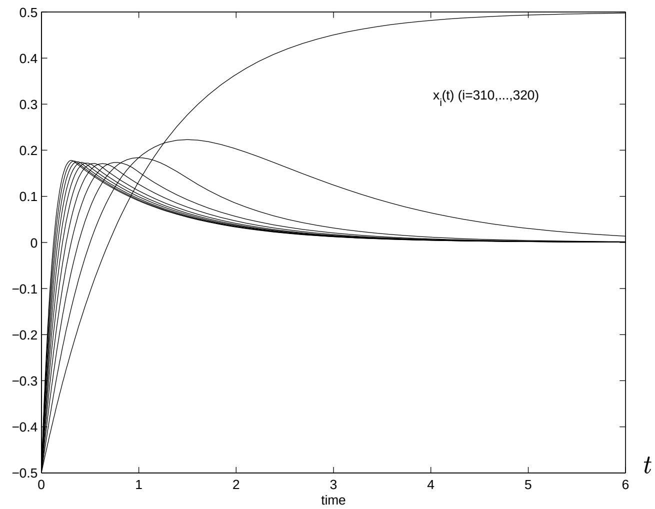

Fig. 1. Transient behavior of the system of (1) in Example 2

which is positive definite but asymmetric, and $c = (-1, ..., -1)^T \in R^n$. The solution of the problem is $u^* = (0, ..., 0, 0.5)^T \in R^n$. We use the neural system (1) to solve the above problem. Let $\alpha = 0.5$, then matrix $I - \alpha N$ is singular, however, all simulation results show that this system is always globally asymptotically stable to the solution u^*. For example, let $n = 320$ and the initial point be $-(1, ..., 1)^T \in R^{320}$. Figure 1 shows the transient behavior of this system.

References

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