Application of Nonlinear Filtering to Navigation System Design Using Nonlinear Passive Sensors *

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Abstract

This paper addresses the problem of navigation system design for autonomous aircraft landing. New nonlinear filter structures are introduced to estimate the position of an aircraft with respect to a possibly moving landing site, such as a Naval vessel, based on measurements provided by airborne vision and inertial sensors. Original results that address this problem are presented in [3]. In this paper these results are extended to include stereo vision. A detailed example that illustrates the use of the filters is also included.

1 Introduction

This paper paper extends the results reported in [3] to a stereo vision case. Furthermore, it includes a vastly expanded design example that reflects a realsitic scenario involved in shipboard landing The paper is organized as follows. Section 2 describes the class of integrated vision/inertial navigation systems that we consider and provides a rigorous mathematical formulation of the related filtering problems. Sectionstereo provides solution to the stereo vision problem. Section 4 presents a detailed example illustrating application of the proposed techniques to a shipboard landing problem. Finally, section 5 contains the main conclusions.

2 Problem Formulation.

This section describes the navigation problem that is the main focus of the paper and formulates it mathematically in terms of an equivalent filter design problem. For the sake of clarity we first introduce some required notation and review the kinematic relationships of an aircraft / ship carrier ensemble, where the former is equipped with a vision based system.

2.1 Notation

Consider Figure 1, which depicts an aircraft equipped with a vision camera operating in the vicinity of the ship. Let $\{\mathcal{I}\}$ denote an inertial reference frame, $\{\mathcal{B}\}$ a body-fixed frame that moves with the aircraft, and $\{\mathcal{C}\}$ a a camera-fixed frame. The symbol $\{\mathcal{S}\}$ denotes a ship-fixed body frame. The following symbols will be used:

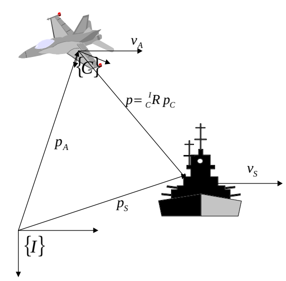

Figure 1: Coordinate Systems

- $\mathbf{p}_{\mathcal{B}} = [x_b\ y_b\ z_b]^T$ - position of the origin of $\{\mathcal{B}\}$ measured in $\{\mathcal{I}\}$ (i.e., inertial position of the aircraft).
- $\mathbf{p}_{\mathcal{S}} = [x_s\ y_s\ z_s]^T$ - inertial position of the ship.
- $\mathbf{p}_{\mathcal{SB}}$ (*abbv.* $\mathbf{p} = [x\ y\ z]^T$) - relative position of the ship with respect to the aircraft, resolved in $\{\mathcal{I}\}$.
- ${}^C\mathbf{p}_{SC}$ (*abbv.* $\mathbf{p}_C = [x_c\ y_c\ z_c]^T$) - relative position of the ship with respect to the aircraft, resolved in $\{\mathcal{C}\}$.
- $\mathbf{v}_{\mathcal{B}}$ - linear velocity of the origin of $\{\mathcal{B}\}$ measured in $\{\mathcal{I}\}$ (i.e., inertial velocity of the aircraft).
- $\mathbf{v}_{\mathcal{S}}$ - inertial velocity of the ship.
- ${}^B\mathbf{a}$ - linear acceleration of $\{\mathcal{B}\}$ with respect to $\{\mathcal{I}\}$, resolved in $\{\mathcal{B}\}$.
- Ω - angular velocity of $\{\mathcal{C}\}$ with respect to $\{\mathcal{I}\}$, resolved in $\{\mathcal{I}\}$.
- $\Lambda = [\phi\ \theta\ \psi]^T$ - vector of roll, pitch, and yaw angles that parameterize locally the orientation of frame $\{\mathcal{C}\}$ with respect to $\{\mathcal{I}\}$.

Given two frames $\{\mathcal{A}\}$ and $\{\mathcal{B}\}$, ${}^A_B\mathcal{R}$ denotes the rotation matrix from $\{\mathcal{B}\}$ to $\{\mathcal{A}\}$. In particular, ${}^I_C\mathcal{R}$ (abbreviated $\mathcal{R}$) is the rotation matrix from $\{\mathcal{C}\}$ to $\{\mathcal{I}\}$, parameterized locally by Λ, that is, $\mathcal{R} = \mathcal{R}(\Lambda)$.

2.2 Kinematic relations.

The rotation matrix $\mathcal{R}$ satisfies the orthonormality condition $\mathcal{R}^T\mathcal{R} = I$. Furthermore, [1]:

$$\dot{\mathcal{R}} = \mathcal{R}\mathcal{S}(\Omega), \quad (1)$$

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where

$$\mathcal{S}(\Omega) \quad := \quad \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{2}$$

is a skew symmetric matrix, that is, $\mathcal{S}^T = -\mathcal{S}$. The matrix $\mathcal{S}$ satisfies the relationship $\mathcal{S}(a)b = a \times b$, where a, b are arbitrary vectors and x denotes the cross product operation. Furthermore, $||\mathcal{S}(\Omega)|| = ||\Omega||$.

We introduce the following assumption. **A1** - *The ship's inertial velocity* $\mathbf{v}_S$ *is constant* Now, from the above definitions, it follows that

$$\begin{aligned} &\mathbf{p}_S = \mathbf{p}_B +^I_C \mathcal{R}\mathbf{p}_C \Rightarrow \\ &\frac{d^2}{dt^2} {}^I_C\mathcal{R}\mathbf{p}_C = \frac{d^2}{dt^2}\mathbf{p}_S - \frac{d^2}{dt^2}\mathbf{p}_B \end{aligned} \tag{3}$$

and since $\frac{d^2}{dt^2}\mathbf{p}_S = 0$ (assumption A1) we obtain

$$\frac{d^2}{dt^2}({}^I_C R\mathbf{p}_C) = -\frac{d^2}{dt^2}\mathbf{p}_B \tag{4}$$

Equation (4) shows that aside from a change in sign, the relative acceleration of the ship with respect to the aircraft resolved in $\{\mathcal{I}\}$ is equal to the aircraft's inertial acceleration resolved in $\{\mathcal{I}\}$. However, in the case of strapdown inertial navigation systems widely in use today [5] the aircraft's inertial acceleration is usually given in $\{\mathcal{B}\}$. Therefore, since

$$\frac{d^2}{dt^2}\mathbf{p}_B =^I_B R\ {}^B\mathbf{a}$$

it follows that

$$\frac{d^2}{dt^2}({}^I_C R\mathbf{p}_C) = -{}^I_B R\ {}^B\mathbf{a}. \tag{5}$$

2.3 Process Model

We assume the image of the origin of $\{\mathcal{S}\}$ acquired by a camera installed on-board the aircraft is obtained using a simple pinhole camera model of the form [2]

$$\begin{bmatrix} u \\ v \end{bmatrix} = \pi_f(x_c, y_c, z_c) = \frac{f}{x_c}\begin{bmatrix} y_c \\ z_c \end{bmatrix}, \tag{6}$$

where f is the focal length of the camera and $[u\ v]^T$ are the image coordinates of $\mathbf{p}_c = [x_c\ y_c\ z_c]^T$ in the camera's image plane. We also make the following assumption. **A2** - $x_c > 0$, that is, *the ship is always located in front of the camera's image plane* . We further assume that **A3** - the rotation matrices ${}^I_B R$ and ${}^I_C R$ are available from the on-board attitude measurement system. This assumption is quite reasonable, considering the sophistication achieved by such systems today.

Suppose the aircraft is equipped with a barometric-based sensor that provides a measurement of the altitude of the aircraft with respect to the mean sea level. Then, using the relation $\mathbf{p} =^I_C R\mathbf{p}_c$ and assuming that the aircraft is sufficiently away from the ship, so as to neglect the height h_s of the ship's deck above the mean sea surface we may assume that **A4:** $h_s = 0$. Thus we obtain that the altitude measurement equals

$$z = g(\mathbf{p}_c) = \sin\theta\ x_c - \cos\theta\sin\phi\ y_c + \cos\theta\cos\phi\ z_c.$$

In (2.3)ϕ and θ are the roll and pitch angles in the rotation matrix ${}^I_C\mathcal{R}$.

We now introduce the underlying design model that plays a fundamental role in this paper. Let $\mathbf{y} = [u\ v\ z]^T$. Then, the model that we consider can be written as

$$\mathcal{G} = \begin{cases} \frac{d}{dt}\mathbf{p} &= \mathbf{v} \\ \frac{d}{dt}\mathbf{v} &= -{}^I_B R({}^B\mathbf{a}_m + \mathbf{w}_a) \\ \mathbf{y}_m &= h_{\phi,\theta}(\mathbf{p}_C) + \mathbf{w}_y, \end{cases} \tag{7}$$

where $h_{\phi,\theta} : \mathcal{R}^3 \to \mathcal{R}^3$ is defined by

$$[u\ v\ z]^T = h_{\phi,\theta}(\mathbf{p}_C) = [\pi_f^T\ \ g(\mathbf{p}_c)]^T$$

and $\mathbf{a}_m$ and $\mathbf{y}_m$ denote the measured values of $\mathbf{a}$ and $\mathbf{y}$, respectively, the measurements being corrupted by the process noises $\mathbf{w}_a$ and $\mathbf{w}_y$. We assume that the matrix ${}^I_B R$ can be measured accurately. In the sequel we adopt the deterministic set-up of H_∞ filtering [4] and assume that $\mathbf{w}_a$ and $\mathbf{w}_y$ are arbitrary functions in L_2.

2.4 Problem Definition

The problem that we consider in this paper consists of determining the relative position and relative velocity of an aircraft with respect to a landing site using vision and other on-board passive sensors. For the sake of clarity, we first tackle the simplified problem of designing a filter with no measurement noise in the model. This exercise is simple, yet it captures some of the key ideas used in the development that follows.

The additional notation that is required is introduced next. In what follows, we let $\hat{\mathbf{p}}$ and $\hat{\mathbf{v}}$ denote estimates of $\mathbf{p}$ and $\mathbf{v}$, respectively. In the camera frame, they are denoted by $\hat{\mathbf{p}}_C$, $\hat{\mathbf{v}}_C$. We assume that the orientation of camera frame $\{\mathcal{C}\}$ with respect to $\{\mathcal{I}\}$ is restricted through the set

$$\Lambda_C = \left\{\Lambda = [\phi\ \theta\ \psi]^T : |\phi| \le \phi_{max},\ |\theta| \le \theta_{max},\ |\psi| \le \psi_{max}\right\}. \tag{8}$$

Notice, ψ_{max}, for example, should be set to π. We further assume that the vectors $\mathbf{p}_C$ lie in the compact set

$$\begin{aligned} &\mathcal{P}_C = \{\mathbf{p}_C = [x_C\ y_C\ z_C], x_{min} \le x_C \le x_{max}, \\ &y_{min} \le y_C \le y_{max}, z_{min} \le z_C \le z_{max}\}. \end{aligned} \tag{9}$$

where $x_{min}, ...z_{max}$ are determined from the geometry of the problem at hand. The set $\mathcal{P}_C$ can be determined as follows. First, compute $\mathcal{P}_C$ for a nominal orientation of the camera (usually inertial orientation). Determine the maximum range of camera orientation angles with respect to the nominal orientaion. Then compute $\mathcal{P}_C$ by allowing the angles to vary within these predetermined bounds.

Filter design will aim at ensuring that the estimates $\hat{\mathbf{p}}_C$ of $\mathbf{p}_C$ lie in a compact set

$$\begin{aligned} &\hat{\mathcal{P}}_C = \{\hat{\mathbf{p}}_C = [\hat{x}_C\ \hat{y}_C\ \hat{z}_C], |\hat{\mathbf{x}}_C - \mathbf{x}_C| \le dx, \\ &|\hat{\mathbf{y}}_C - \mathbf{y}_C| \le dx, |\hat{\mathbf{z}}_C - \mathbf{z}_C| \le dz\}. \end{aligned} \tag{10}$$

where dx, dy and dz are positive numbers, and $dx < x_{min}$. **F1: Regional Stability**. *Consider the process model (7) and assume that* $\mathbf{w}_a = \mathbf{w}_y = 0$. *For a given* $\hat{\mathcal{P}}_C$, *find a number* $\alpha > 0$, *and a dynamical system (filter)* $\mathcal{F}$ *that operates on* $\mathbf{y}_m$ *and* $\mathbf{a_m}$ *to produce estimates* $\hat{\mathbf{p}}$ *of* $\mathbf{p}$, *and* $\hat{\mathbf{v}}$ *of* $\mathbf{v}$, *such that*

- $\hat{\mathbf{p}}_C \in \hat{\mathcal{P}}_C$ *for any* $t > 0$,
- $\|\hat{\mathbf{p}} - \mathbf{p}\| + \|\hat{\mathbf{v}} - \mathbf{v}\| \to 0$ *as* $t \to \infty$.

provided that $||(\hat{p}_C(0) - p_C(0), \hat{v}(0) - v(0))^T|| < \alpha$.

Notice that the problem described aims at finding a filter that complements the information available from the vision system / barometric pressure sensor with that available from the inertial sensors.

The problem **F1** focuses on the stability of the filter. The second filtering problem addresses the scenario where the performance of the filter in the presence of disturbances is considered.

F2: Regional Stability and Performance . *Consider the process model (7) where* $\mathbf{w} = [\mathbf{w}_a \; \mathbf{w}_y]^T \in L_2$, $\|\mathbf{w}\|_2 \leq 1$ *and let the sets* $\mathcal{P}_C$ *and* $\hat{\mathcal{P}}_C$ *of allowable position vectors and allowable estimation vectors be given by (9) and (10), respectively. For given positive numbers* $\gamma > 0$ *and* $\alpha > 0$, *find a stable filter* $\mathcal{F}$ *that operates on* $\mathbf{y}_m$ *and* $\mathbf{a}_m$ *to obtain estimates* $\hat{\mathbf{p}}$ *of* $\mathbf{p}$, *and* $\hat{\mathbf{v}}$ *of* $\mathbf{v}$ *such that if* $\|[(\hat{\mathbf{p}}(0) - \mathbf{p}(0))^T \quad (\hat{\mathbf{v}}(0) - \mathbf{v}(0))^T]^T\| < \alpha$, *the filter satisfies the following conditions for all* $\|\mathbf{w}\|_2 \leq 1$

- $\hat{\mathbf{p}}_C(t) \in \hat{\mathcal{P}}_C$ *for all* $t \geq 0$.
- *If* $\mathbf{w} = 0$, *then* $\|\hat{\mathbf{p}}(t) - \mathbf{p}(t)\| + \|\hat{\mathbf{v}}(t) - \mathbf{v}(t)\| \to 0$ *as* $t \to \infty$.
- $\|T_{\mathbf{ew}}\|_{2,i} < \gamma$, *where* $\mathbf{e} := \hat{\mathbf{p}} - \mathbf{p}$ *is the estimation error and* $T_{\mathbf{ew}} : \mathbf{w} \to \mathbf{e}$.

Notice the technical requirement that an allowable set of position estimates $\hat{\mathcal{P}}_C$ be specified. As explained later, this requirement is essential to establishing the boundedness of a certain operator for all possible values of the estimates $\hat{\mathbf{p}}(t)$. In practice, the "size" of the allowable region $\hat{\mathcal{P}}$ plays the role of a design parameter.

3 Integration of Stereo Vision with Inertial Sensors

Solutions to Problems F1 and F2 were presented in [3]. In this section these results are extended to include stereo vision. When in the vicinity of the ship, using the barometric altimeter $g_{\phi,\theta}$ to determine the aircraft's altitude above the landing site may not be sufficient (i.e. assumption **A4** must be lifted). In this case, the idea is to replace the altimeter with another camera and use stereo vision together with the accelerometers to estimate the aircraft's position with respect to the landing site.

Let $\mathcal{G}$ denote the process model for this problem and suppose that $\{\mathcal{C}_1\}$ and $\{\mathcal{C}_2\}$ denote coordinate systems attached to cameras 1 and 2, respectively (see Figure 2). Then

$$\mathcal{G} = \left\{ \begin{array}{lcl} \frac{d}{dt}\mathbf{p} & = & \mathbf{v} \\ \frac{d}{dt}\mathbf{v} & = & -{}^I_B R({}^B\mathbf{a}_m + \mathbf{w}_a) \\ \mathbf{y}_m & = & \left[\begin{array}{c} \pi_f^1({}^{C_1}_I R\mathbf{p} - \mathbf{b}_1) \\ \pi_f^2({}^{C_2}_I R\mathbf{p} - \mathbf{b}_2) \end{array} \right] + \mathbf{w}_y, \end{array} \right. \tag{11}$$

where π_f^1 and π_f^2 define projections of the origin of the ship coordinate system $\mathcal{S}$ onto the image plane of each camera. Notice, in the case of stereo vision it is more convenient to define the vector $\mathbf{p}$ (relative position of the ship with respect to the aircraft resolved in $\mathcal{I}$) to be attached to an arbitrary point on the aircraft, rather than to the origin of one of the camera frames as was done before. Hence, the vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ define relative position of the origin of each camera frame with respect to the origin of $\mathbf{p}$. Then

$$H(\mathbf{p}) = \frac{dh}{d\mathbf{p}} = \left[\begin{array}{c} \frac{d\pi_f^1}{d\mathbf{p}_{C_1}} \, {}^{C_1}_I R \\ \frac{d\pi_f^2}{d\mathbf{p}_{C_2}} \, {}^{C_2}_I R \end{array} \right] \text{ and } \phi_1(\hat{\mathbf{p}}) = H^T(\mathbf{p})H(\mathbf{p}).$$

In this section, we use the following notation $H_i(\mathbf{p}_{C_i}) = \frac{d\pi_f^i}{d\mathbf{p}_{C_i}}$, $R_i = {}^I_{C_i}R$ where $i = 1, 2$.

In the sequel we employ the definitions of the sets $\mathcal{P}_{C_1}$ and $\hat{\mathcal{P}}_{C_1}$ proposed in Section 2.4. Since two cameras are involved in the process, the relative orientation of the two cameras plays an important role. The relative orientation (of camera 2 with respect to camera 1) is defined by the matrix $R_2^T R_1$. Assume that the relative orientation is restricted by a compact set $\mathcal{O}$ in $SO(3)$ containing the identity matrix. The sets $\mathcal{P}_{C_2}$ and $\hat{\mathcal{P}}_{C_2}$ are defined by

$$\begin{array}{l} \mathcal{P}_{C_2} = \{R_2^T R_1(\mathbf{p}_{C_1} + \mathbf{b}_1) - \mathbf{b}_2 : \; R_2^T R_1 \in \mathcal{O}, \; \mathbf{p}_{C_1} \in \mathcal{P}_{C_1}\} \\ \hat{\mathcal{P}}_{C_2} = \{R_2^T R_1(\hat{\mathbf{p}}_{C_1} + \mathbf{b}_1) - \mathbf{b}_2 : \; R_2^T R_1 \in \mathcal{O}, \; \hat{\mathbf{p}}_{C_1} \in \hat{\mathcal{P}}_{C_1}\} \end{array}$$

For $\hat{\mathcal{P}}_{C_2}$, we assume $|\hat{x}_{C_2} - x_{C_2}| < \min\{x_{C_2} : \hat{\mathbf{p}}_{C_2} \in \hat{\mathcal{P}}_{C_2}\}$. Define

$$r_x = \max_{\hat{\mathbf{p}}_{C_2} \in \hat{\mathcal{P}}_{C_2}, \mathbf{p}_{C_2} \in \mathcal{P}_{C_2}} \{\frac{x_{max} - x_{min} + dx}{x_{min}}, \frac{|\hat{x}_{C_2} - x_{C_2}|}{x_{C_2}}\}.$$

Theorem 3.1 *Given* $\mathcal{P}_{C_1}$, $\hat{\mathcal{P}}_{C_1}$ *and* $\mathcal{O}$, *let* r_x *be the number defined above. Let* α *be a positive number such that* $\alpha < \min(x_{max} - x_{min} + dx, y_{max} - y_{min} + dy, z_{max} - z_{min} + dz)$. *Define*

$$\begin{array}{rcl} \epsilon & = & \min_{\hat{\mathbf{p}}_{C_1} \in \hat{\mathcal{P}}_{C_1}, R_2^T R_1 \in \mathcal{O}} \lambda_{min}\{R_2^T R_1 H_1^T(\hat{\mathbf{p}}_{C_1}) H_1(\hat{\mathbf{p}}_{C_1}) R_1^T R_2 \\ & & + H_2^T(\hat{\mathbf{p}}_{C_2}) H_2(\hat{\mathbf{p}}_{C_2})\} \end{array}$$

Suppose assumptions $\mathbf{A1} - \mathbf{A3}, \mathbf{A5} - \mathbf{A6}$ *hold and* $r_x < 1$. *For a given gain* γ, *suppose there exists a matrix* $P = P^T \in R^{6 \times 6}$ *such that*

$$\begin{array}{l} P > 0 \\ \left[\begin{array}{c|c} F^T P + PF + \left[\begin{array}{cc} \frac{I}{\gamma^2} - (1 - r_x)^2 \epsilon I & 0 \\ 0 & 0 \end{array} \right] & PF^T \\ \hline FP & -I \end{array} \right] < 0, \\ P - \frac{4}{\delta} C^T C > 0, \\ \delta = \min(x_{max} - x_{min} + dx, \\ y_{max} - y_{min} + dy, z_{max} - z_{min} + dz) \\ \frac{1}{\alpha^2} - P > 0 \end{array}$$

Let

$$\left[\begin{array}{c} K_1 \\ K_2 \end{array} \right] = -P^{-1} \left[\begin{array}{c} (1 - r_x)I \\ 0 \end{array} \right].$$

The filter is defined by (see Figure 3)

$$\mathcal{F}_3 = \left\{ \begin{array}{lcl} \dot{\hat{\mathbf{p}}} & = & \hat{\mathbf{v}} + K_1 \, H^T(\hat{\mathbf{p}}_{C_1}, \hat{\mathbf{p}}_{C_2}) \, (h(\hat{\mathbf{p}}) - \mathbf{y}_m) \\ \dot{\hat{\mathbf{v}}} & = & -{}^I_B R \, {}^B\mathbf{a}_m + K_2 \, H^T(\hat{\mathbf{p}}_{C_1}, \hat{\mathbf{p}}_{C_2}) \, (h(\hat{\mathbf{p}}) - \mathbf{y}_m) \\ \hat{\mathbf{p}}_{C_i} & = & R_i^T \hat{\mathbf{p}} - \mathbf{b}_i, \qquad i = 1, 2. \end{array} \right. \tag{12}$$

Then $\mathcal{F}_3$ solves the filtering problem F2 using stereo vision and acceleration measurements if

$$\|[(\hat{\mathbf{p}}(0) - \mathbf{p}(0))^T \quad (\hat{\mathbf{v}}(0) - \mathbf{v}(0))^T]^T\| < \alpha.$$

4 Example

In this section we present design and simulation results for the filters $\mathcal{F}_1$-$\mathcal{F}_3$. The LMIs included in Theorems 3.1 and 3.2 of [3] and in Theorem 3.1 above were implemented in Matlab. The gains obtained by solving these LMIs were used to simulate response of a given filter to non-zero initial conditions and $\mathcal{L}_2$ measurement noise.

The simulation scenario involved a ship moving North at a constant speed of 10m/sec and the aircraft performing left turn from the course of -50 (NE) with altitude decrease and glideslope (Figure 4). The initial aircraft position is [-5000m;-2000m;-1000m] with respect to the ship's initial position, and its initial airspeed is 60m/sec. Initially, the aircraft is banking at an angle of 3.5 deg to capture a 3 deg glideslope. Glideslope is characterized by reduced aircraft speed of 40 m/sec and begins approximately within 1100m from the ship.

The onboard camera is fixed with -10 degree pitch angle with respect to aircraft's longitudinal axis. The projection of the ship's landing site onto the camera's image plane is shown in the Figure 5. The jump in the upper left corner is due to a simulated out-of-frame event discussed later in this section.

Filters $\mathcal{F}_1 - \mathcal{F}_3$ are employed sequentially as a function of range to the ship as shown in the Figure 6. Figure 7 summarizes the design parameters for each filter and shows time histories of the errors in position estimation for each filter in response to large initial errors and L_2 sensor noise (In Figure 6 the largest error corresponds to errors in x component of the position vector, while the errors in y and z are the same).

Figure 8 illustrates the influence of the size of the cube $\mathcal{P}_c$ and the parameter dx on the value of ϵ. It turns out, due to the simple geometry, that ϵ reaches a minimum value, at the corner x_{max}, z_{min} of the cube $\mathcal{P}_c$ (see Figure 9). This dependence can be approximated by the following expression

$$\epsilon = (\frac{z_{min} - dx}{x_{max}^2})^2, \qquad (13)$$

which fits experimental data with mean-squared value of at least 0.9997. Figure 8 also includes the graph of the best achievable value of ϵ obtained by solving the LMI's in Theorem 3.1. The intersection of the two lines provides the designer with size of the cube $\mathcal{P}_c$ where the filter $\mathcal{F}_2$ is guaranteed to have asymptotic stability and given H_∞ performance characterized by γ.

The analytic expression (13) that shows dependence of ϵ on the size of the cube $\mathcal{P}_c$ can also be used to determine an estimate of the best achievable value of γ. As was shown in [3] the lower bound γ_0 on the best achievable value of γ is $\gamma_0 = \frac{1}{(1-r_x)\sqrt{\epsilon}}$. This expression combined with (13) can be used to show dependence of γ_0 on the geometry of the problem as shown in Figure 10.

Finally, Figure 11 shows an example of recovery from an out-of frame event by the Filter $\mathcal{F}_1$. As can be seen in Figure 5 (left top corner) after the elimination of initial errors a disturbance in pith causes an out-of frame event which lasts approximately 1 sec. At this point the feedback loops are turned off (see Figure 12). Figure 11 shows the resulting increase in the position error. As can be seen in Figure 11 once the image is recaptured the accumulated errors are eliminated rapidly.

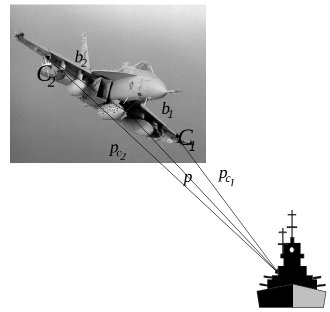

Figure 2: Geometry of the Stereo Vision Process Model

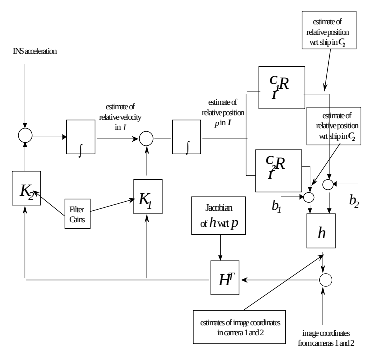

Figure 3: Filtering Structure: Filter $\mathcal{F}_3$

5 Conclusions

This paper addressed the problem of estimating the relative position and velocity of an aircraft with respect to a moving landing site such as a Naval vessel using stereo vision. The problem was cast in the LPV framework and solved using tools that borrow from the theory of Linear Matrix Inequalities. This approach resulted in nonlinear filtering structures that integrate vision with inertial measurements and have regional stability and performance guarantees. Employing inertial sensors has an additional benefit of offering robustness with respect to out-of-camera events and occlusions, whereby these sensors can be used by the filter to provide the estimates of the image coordinates of the ship to the image processing algorithms.

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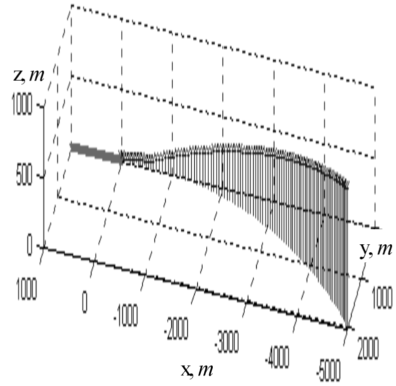

Figure 4: 3D representation of the simulation scenario

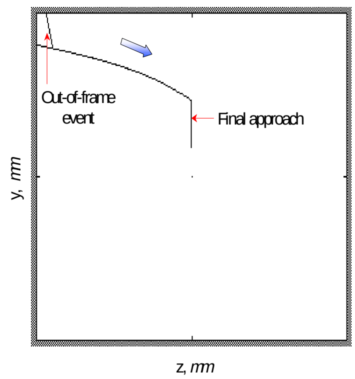

Figure 5: Ship's smokestack projection in the camera image plane during landing

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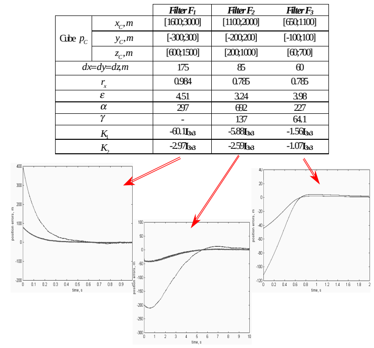

		Filter F_1	Filter F_2	Filter F_3
Cube p_C	x_C, m	[1600;3000]	[1100;2000]	[650;1100]
	y_C, m	[-300;300]	[-200;200]	[-100;100]
	z_C, m	[600;1500]	[200;1000]	[60;700]
$dx=dy=dz, m$		175	85	60
r_x		0.984	0.785	0.785
ε		4.51	3.24	3.98
α		297	692	227
γ		-	137	64.1
K_1		$-60.1\mathbf{I}_{3x3}$	$-5.88\mathbf{I}_{3x3}$	$-1.56\mathbf{I}_{3x3}$
K_2		$-2.97\mathbf{I}_{3x3}$	$-2.59\mathbf{I}_{3x3}$	$-1.07\mathbf{I}_{3x3}$

Figure 6: Numerical results for Filters $\mathcal{F}_1 - \mathcal{F}_3$

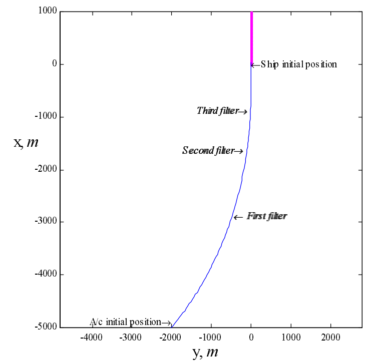

Figure 7: Switching times between filters during landing

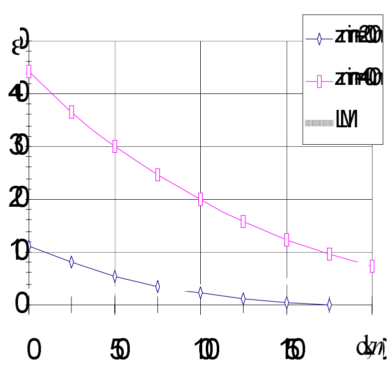

Figure 8: Dependence of ϵ on the value of dx for the Filter $\mathcal{F}_2$

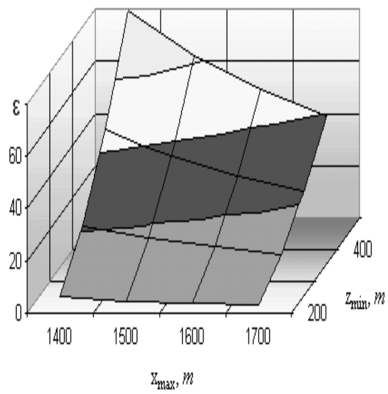

Figure 9: Dependence of ϵ on the values of x_{max} and z_{min} for the Filter $\mathcal{F}_2$

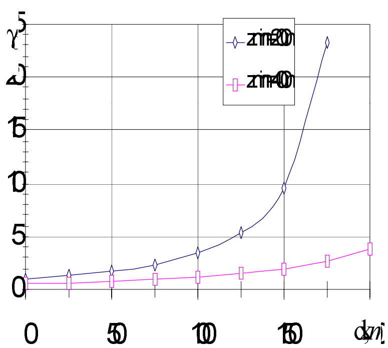

Figure 10: Dependence of γ on the values on dx for the Filter $\mathcal{F}_2$

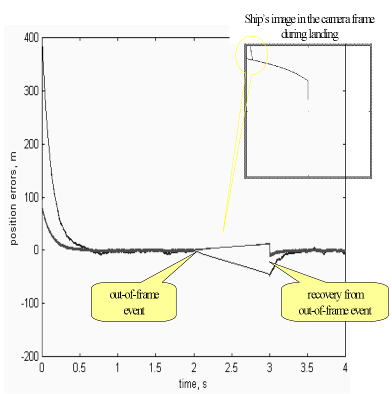

Figure 11: Time history of $\mathcal{F}_1$ position errors during an out-of-frame event

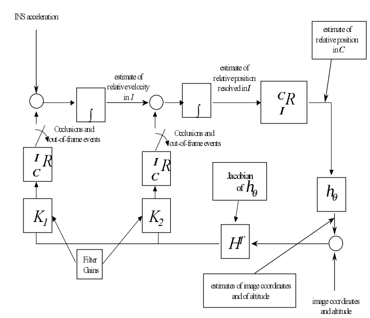

Figure 12: The feedback loops in Filters $\mathcal{F}_1$ and $\mathcal{F}_2$ are turned off due to an out-of-frame event or occlusion