On the dynamic behavior of a hydraulic cylinder

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Abstract

In this paper we are interested in the stability properties of a hydraulic cylinder when no control is applied. We give a nonlinear model that take into account the leakages and show that the system is Lyapunov stable.
Key words: center manifold, hydraulic cylinder, nonlinear systems, servovalve leakages, stability.

1 Introduction and Description

The system under consideration is composed of a four-way electrohydraulic servovalve driven a symmetrical linear cylinder. The linear actuator, which has a stroke equal to $2l$ and a useful area S carries a mass M under a viscous friction $-bv$. The scheme of figure 1 represents this system.

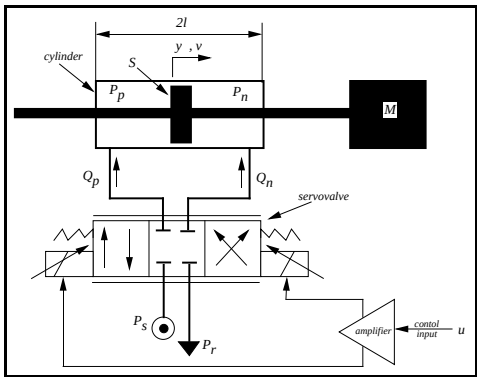

Figure 1: *Scheme of the system.*

The experiment show that this system is stable when no input is applied. In this preliminary paper, we shall prove this fact by using a nonlinear model that take into account the flow leakages. The linearization of the system arround an equilibrium point is critical so it does not allow to say anything about the stability. So we use the center manifold theory to prove the stability of the free system ($u = 0$). In a forthcoming we use this proporty with the results of [1] in order to construct a feedback control that achieves the stabilization of our system, i.e, such that the closed-loop system is asymptotically stable.

2 The assumptions and the model of the system

In order to obtain the system model the following main assumtions are made: (i) the servovalve is symmetrical and its dynamic is negligeable, (ii) the pressures are homogeneous in each cylinder chambers, (iii) the cylinder has no leakage flows and no dry friction, (iv) the temperature is constant. Under the previous assumptions, the system model is deduced from the three following laws: (i) the pressure evolution law written for each cylinder chamber, (ii) the volumetric flow rates $Q_p(P_p, u)$ and $Q_n(P_n, u)$ provided by the servovalve ports, (iii) the mechanical equation of the moving part. Consider a chamber of volume $V(t)$ filled with a mass m of fluid and density ρ. The continuity equation leads to $\dot{m} = \rho\dot{V} + V\dot{\rho} = \rho_{in}Q_{in} - \rho Q_{out}$ where Q_{in} and Q_{out} represent respectively the volumetric flow rate of the fluid entering and leaving the chamber and ρ_{in} is the density of the fluid entering into the chamber. Using the the bulk modulus definition $B \stackrel{def}{=} \rho\dfrac{\partial P}{\partial \rho}$, which is assumed to be constant, it is easy to obtain $\dfrac{dP}{dt} = \dfrac{B}{V}\left(\dfrac{\rho_{in}}{\rho}Q_{in} - Q_{out} - \dfrac{dV}{dt}\right)$. Now, by integrating the bulk modulus definition, it follows $\dfrac{\rho_{in}}{\rho} = \exp\left(\dfrac{P_{in} - P}{B}\right)$. If numerical values are considered $P_{in} - P \approx 100\,bar$, $B \approx 14000\,bar$, it is clear that $\rho \approx \rho_{in}$. Then, the well known law of the pressure evolution into a variable volume chamber [3], [5], [6] can be written:

$$\frac{dP}{dt} = \frac{B}{V}\left(Q_{in} - Q_{out} - \frac{dV}{dt}\right) \qquad (1)$$

In this article, the dynamic behavior of the drift is studied, *i.e.* the dynamic behavior corresponding to $u = 0$. For $u = 0$, the volumetric flow rates provided by the servovalve are only leakage flows which are laminar flows. Then, they are proportionnal to the pressure drop across a restriction [3], [5], [6]. From the scheme of figure 2 which represents the servovalve restrictions, and using the symmetrical assumption, the volumetric

flows are given by [4]: $Q_p(P_p,0) = \alpha(P_s + P_r - 2P_p)$ and $Q_n(P_n,0) = \alpha(P_s + P_r - 2P_n)$, where α is a positive constant.

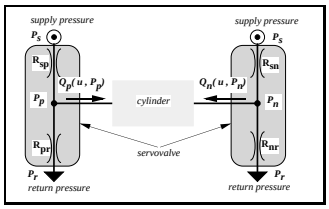

Figure 2: *Servovalve restrictions.*

Finally, written relation (1) for each cylinder chamber and the mechanical equation, the following non-linear state model is obtained:

$$\left\{\begin{array}{lcl} \dot{P}_p &=& \frac{B}{V_0+Sy}\left(\alpha(P_s + P_r - 2P_p) - Sv\right) \\ \dot{P}_n &=& \frac{B}{V_0-Sy}\left(\alpha(P_s + P_r - 2P_n) + Sv\right) \\ \dot{v} &=& \frac{1}{M}\left(S(P_p - P_n) - bv\right) \\ \dot{y} &=& v \end{array}\right. \tag{2}$$

where the state vector components are respectively: chamber "p" pressure, chamber "n" pressure, velocity and position and V_0 is the chamber volume corresponding to the middle stroke of the piston.

3 Stability analysis

The equilibriums are: $\left(\frac{P_s+P_r}{2}\ ,\ \frac{P_s+P_r}{2}\ ,\ 0\ ,\ y^e\right)$ with $y^e \in]-l_0, l_0[$. Let y^e be a fixed position. We are interested in the stability proporties of the above system (2) arround its equilibrium. Thanks to the change of coordinates $z_1 = P_1 - \frac{P_s+P_r}{2}$, $z_2 = P_2 - \frac{P_s+P_r}{2}$, $z_3 = v$, $z_4 = y - y^e$, the equilibrium point becomes $(0,0,0,0)$ and system (2) can be written:

$$\left\{\begin{array}{lcl} \dot{z_1} &=& \frac{B}{V_0 + Sy^e + Sz_4}\left(-2\alpha z_1 - Sz_3\right) \\ \dot{z_2} &=& \frac{B}{V_0 - Sy^e - Sz_4}\left(-2\alpha z_2 - Sz_3\right) \\ \dot{z_3} &=& \frac{1}{M}\left(Sz_1 - Sz_2 - bz_3\right) \\ \dot{z_4} &=& z_3 \end{array}\right. \tag{3}$$

By computing the linearisation of the above nonlinear system at the origin we can write:

$$\dot{z} = Az + f(z). \tag{4}$$

Where:

$$A = \begin{pmatrix} \frac{-2\alpha B}{V_0 + Sy^e} & 0 & \frac{-SB}{V_0 + Sy^e} & 0 \\ 0 & \frac{-2\alpha B}{V_0 - Sy^e} & \frac{SB}{V_0 + Sy^e} & 0 \\ \frac{S}{M} & -\frac{S}{M} & -\frac{b}{M} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and $f(z) = (f_1(z), f_2(z), 0, 0)$. A simple computation shows that the matrix A has zero as an eigenvalue and three eigenvalues with negative real part. So the linear approximation of system (3) is critical and so it does not allow to conclude about the local stability of the considered system (3). To study the stability, we shall use the center manifold theorem [2]. To this end we first use a linear change of coordinates $x = T^{-1}z$.

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{MS(V_0 + Sy^e)}{2B(b\alpha + S^2)} & \frac{MS(V_0 - Sy^e)}{2B(b\alpha + S^2)} & -\frac{M\alpha}{b\alpha + S^2} & 1 \end{pmatrix}$$

With the new coordinates, system (3) is governed by the following equations

$$\left\{\begin{array}{l} \dot{\tilde{x}} = \tilde{A}\tilde{x} + g(\tilde{x}, x_4) \\ \dot{x}_4 = g_4(\tilde{x}, x_4) \end{array}\right. \tag{5}$$

Where: $\tilde{x} = (x_1, x_2, x_3)$. The functions g and g_4 satisfy: $g(0, x_4) = 0$, $g_4(0, x_4) = 0$.

$$\tilde{A} = \begin{pmatrix} -\frac{2\alpha B}{V_0 + Sy^e} & 0 & -\frac{SB}{V_0 + Sy^e} \\ 0 & -\frac{2\alpha B}{V_0 - Sy^e} & \frac{SB}{V_0 - Sy^e} \\ \frac{S}{M} & -\frac{S}{M} & -\frac{b}{M} \end{pmatrix}$$

All the eigenvalues of $\tilde{A}$ have negative real parts and $\frac{\partial g_4}{\partial x}(0,0) = 0$. The set $\tilde{x} = 0$ is a center manifold for system (5) since $g(0, x_4) = 0$. Thanks to ([2] Theorem 2), the stability of the zero solution of (5) is equivalent to the stability of the zero solution of

$$\dot{u} = g_4(h(u), u) = g_4(0, u) = 0. \tag{6}$$

The zero solution is clearly stable and not attractive for (6). We then deduce that the origin is Lyapunov stable for (5) and so, the considered equilibrium $(P_p^e,\ P_n^e,\ v^e,\ y^e) = \left(\frac{P_s+P_r}{2}\ ,\ \frac{P_s+P_r}{2}\ ,\ 0\ ,\ y^e\right)$ is Lyapunov stable for the original system (2).

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