Control of Waveforms and Detection Thresholds for Optimal Target Tracking in Clutter

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Abstract

An optimization problem is formulated to obtain the combined sequence of waveform parameters (pulse amplitudes and lengths, and FM sweep rates) and detection thresholds for optimal target tracking in clutter. The optimal combined sequence minimizes a tracking performance index that is a function of the probability of track loss and the estimation accuracy. A measurement model is also developed based on the resolution cell in the delay-Doppler plane for a Gaussian pulse.

1 Introduction

With the advent of flexible digital waveform generation techniques, many active radar and sonar systems are capable of adaptively generating waveforms for their transmitted pulses. The transmitted waveform has a significant effect on measurement accuracy and consequently on tracking performance. Recently, attention has been focused on the question of optimal waveform parameter design as an integral part of the overall tracking system design process to achieve optimum performance for target tracking [1, 2, 3, 4]. In this paper, we consider an optimization problem to obtain the optimal combined sequence of waveform parameters and detection thresholds that minimize a performance index under a set of parameter constraints. Specifically, we consider the case of a Gaussian pulse and develop a measurement model based on the resolution cell in the delay-Doppler plane. We assume that the probabilistic data association (PDA) filter [5] is employed for tracking. Numerical experiments were performed to solve the optimization problem for several examples.

2 Sensor and Measurement Modeling

Assuming a return from a Swerling I target, we can obtain the relationship between the probability of detection (P_D), and the false alarm probability (P_F) such that $P_D(\tau,\omega) = P_F^{\frac{1}{1+SNR\cdot X(-\tau,\omega)}}$. Here, *SNR* denotes the signal-to-noise ratio of the received signal, and $X(-\tau,\omega)$ denotes the ambiguity function [3] for the actual return with time delay τ and Doppler shift ω when a receiver filter is matched to a zero delay and a zero Doppler shift. To be more specific, we will consider the case where the envelope function $s(t)$ is a linear frequency modulated (LFM) pulse with Gaussian amplitude modulation, that is, $s(t) = (\frac{1}{\pi\eta^2})^{1/4}\exp(-(\frac{1}{2\eta^2} - jb)t^2)$ where η is the pulse length parameter and b is the linear frequency sweep rate parameter. Often the resolution cell in the delay-Doppler (τ-ω) plane is defined in a tessellate shape, and it contains a region of P_D that is higher than a certain level [3, 4]. The region can be expressed in terms of the ambiguity function and a corresponding parameter γ^2 as $R_C = \{(\tau,\omega) : X(-\tau,\omega) \geq \exp(-\frac{1}{2}\gamma^2)\}$. Clearly the region R_C is an ellipse in the case of an LFM Gaussian pulse. For the sake of simplicity, we approximate our resolution cell by the rectangle which encloses the ellipse R_C and is tangent to the sides. Using this approximation we develop a simple but reasonably accurate model of target detection and measurement. We denote the rectangular resolution cell by C and its size by $|C|$.

Assuming that the target location is uniformly distributed over the resolution cell C and the *SNR* is sufficiently high, an approximate expression for the expected value of P_D in the cell C, denoted by $\bar{P}_D$, is given by

$$\bar{P}_D \approx P_{D0}\, a_1^2 \tag{1}$$

where $P_{D0} = P_F^{\frac{1}{1+SNR}}$ and $a_1 = \sum_{k=0}^{\infty} \frac{1}{(2k+1)\cdot k!}(\frac{1}{2}\gamma^2 \ln P_{D0})^k$. Furthermore, for a measurement vector $[\tau,\omega]^T$ which is assumed to be obtained by using the "strongest-neighbor" approach [4], the measurement error covariance matrix is

$$R = \gamma^2 \frac{a_2}{a_1}\begin{bmatrix} \eta^2 & 2b\eta^2 \\ 2b\eta^2 & \frac{1}{\eta^2} + 4b^2\eta^2 \end{bmatrix} \tag{2}$$

where $a_2 = \sum_{k=0}^{\infty} \frac{1}{(2k+3)\cdot k!}(\frac{1}{2}\gamma^2 \ln P_{D0})^k$. Note that $\bar{P}_D$ is a function of *SNR* and P_F, and that R is a function of η and b as well as the *SNR* and P_F.

3 Tracker Characterization and Waveform Optimization

Among various algorithms for tracking in clutter, we chose to use the PDA filter. For our PDA tracker characterization, we modified the hybrid conditional average (HYCA) algorithm [5] in order to obtain an approximate propagation of the covariance under the condition that the number of the validated measurements is maintained to be smaller than a certain integer M. The integer M is a tolerable limit on the number of validated measurements for stable tracking. As a result, our modified HYCA algorithm provides a hybrid approximation of the covariance of the state estimation error under a stable tracking condition.

Based on the measurement model and the tracker characterization, we formulate an optimization problem to minimize a performance index for PDA tracking. The sequence of the combined parameter set $\theta(k) = \{A(k), \eta(k), b(k), P_F(k)\}$ represents the parameters to be optimized. We define an index function for our optimization problem

$$J = c_1 P_{TL}(K) + c_2 \sum_{k=1}^{K} f(\bar{P}(k|k)), \tag{3}$$

where c_1 and c_2 are weighting factors, $f(\bar{P}(k|k))$ is a function of the approximate covariance $\bar{P}(k|k)$, and $P_{TL}(K)$ is the cumulative

probability of track loss at time K. The approximate covariance $\bar{P}(k|k)$ and the probability $P_{TL}(K)$ are evaluated by the modified HYCA algorithm.

4 Numerical Experiments and Discussions

The optimization problem was solved numerically for various examples to examine the behavior of the optimum parameter set $\{A(k), \eta(k), b(k), P_F(k)\}$. Limitations on the peak amplitude, length parameter, FM sweep rate, and energy of transmitted pulses are imposed on the optimization as parameter constraints. We consider the two cases: the process noise power spectral density σ_q^2 and the spatial density of clutter measurements ρ are 1.0 m^2/sec^3 and 5×10^{-3} $(rad)^{-1}$, respectively (Case I); and $\sigma_q^2 = 10$ m^2/sec^3 and $\rho = 0$ (Case II). Let us assume that, for simplicity, $2\sigma_b^2/(N_o/2) = 2/\sqrt{\pi}$. This gives $SNR = (2/\sqrt{\pi})E_T = A^2\eta$. Under this assumption we set the limits on the energy (or SNR), amplitude, and length parameters of transmitted (Gaussian) pulses as follows: $SNR = A^2\eta \leq 63.1(18\text{dB})$, $1.5 \times 10^{-3} \leq \eta \leq 2.5 \times 10^{-2}$ in sec, and $A \leq 100$. The FM sweep rate is limited to an interval $|b| \leq 5\pi \times 10^3$ rad/sec^2. In our experiments, we set $T = 2$ sec, $\omega_c = 5\pi \times 10^4$ rad/sec, $c_o = 1500$ m/sec, $g = 4$ (with $P_G = 0.9997$ for 4-sigma gate on the two dimensional space), $\gamma^2 = 1/2$ (with $|C| = 2$), and $K = 20$. M was set to 8. For the index function $f(\bar{P}(k|k))$ in (3), we used $W\bar{P}(k|k)W^T$ where W is a weighting row vector $[1\ T]$. The function $W\bar{P}(k|k)W^T$ equals $\bar{P}_{11}(k|k) + 2T\bar{P}_{12}(k|k) + T^2\bar{P}_{22}(k|k)$, where $\bar{P}_{ij}(k|k)$ denotes the (i,j)-th element of $\bar{P}(k|k)$. This function represents a measure of the position estimation accuracy projected one-step-ahead in time. The weighting factors c_1 and c_2 were set to 1 and 10^{-3}, respectively.

The optimal solution for Case I shows that the constraint on SNR is active on all scans, and indicates that relatively short pulses with the maximum energy are optimum for this particular example where most scans are in an initial transient period. The pulse length is close to 6.31×10^{-3} sec that is the minimum length required to maintain SNR with its maximum limit value (18dB). The short pulses imply that the system puts more emphasis on accurate range measurements in order to minimize the index function. The optimal solution also shows that the optimal sequence begins with LFM pulses with a negative sweep rate and then changes to pulses with a positive sweep rate; see Fig. 1. We can observe that the change occurs when PDA tracking becomes stable. This result is in contrast with the results in [3, 4], where FM with a negative sweep rate is not recommended on the basis of steady-state estimation performance.

We also performed simulations of 50,000 Monte Carlo runs for the constant parameter set: $A = 100$, $\eta = 6.31 \times 10^{-3}$ sec, and $P_F = 10^{-3}$. The FM sweep rate was chosen in four different ways: $b = 5\pi \times 10^3$ (FM$^+$), $b = -5\pi \times 10^3$ (FM$^-$), $b = 0$ (CW), and switching (SW) between FM$^-$ and FM$^+$ (FM$^-$, if $\lambda(k)V_G(k) > 0.5$, and FM$^+$, otherwise, where $V_G(k)$ is evaluated for $b = 0$ in order to decide either FM$^-$ or FM$^+$ consistently). Fig. 2 shows the resulting sequence of $P_{TL}(k)$ for the four set of parameters. Even though the parameter set with FM$^-$ was most effective in reducing $P_{TL}(k)$, SW was the best in terms of the overall performance (3). The optimal solution of Case II was similar to the one of Case I, but the optimal sweep rate was FM$^+$ in Case II. We performed Monte Carlo simulations as in Case I, and confirmed that, though the advantage was not significant in this particular case, FM$^+$ was the best in terms of both the index (3) and the second term of (3). Through the numerical experiments, we demonstrated that the optimal solution provides insight into the selection of a near-optimal parameter set that is more appropriate for real-time implementation.

Acknowledgments

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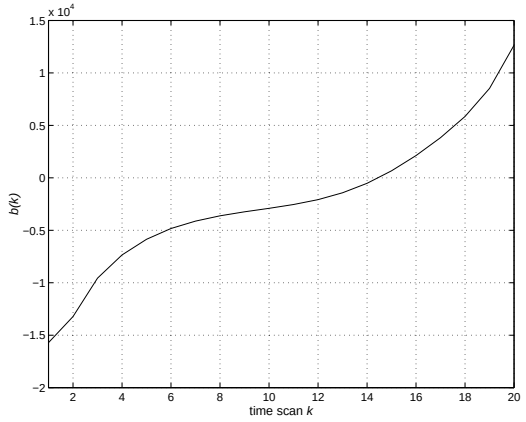

Fig. 1. Optimal FM sweep rate $b(k)$.

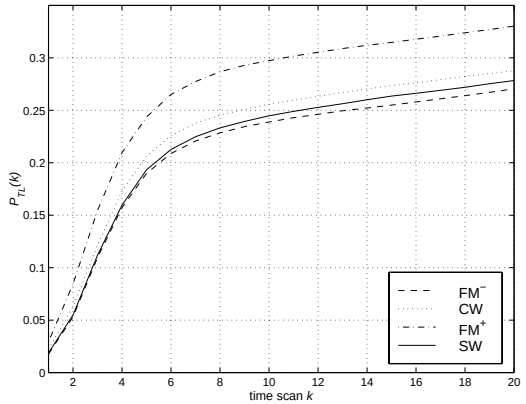

Fig. 2. $P_{TL}(k)$ obtained from Monte Carlo simulations.