Fault-Tolerant Control About Integrity for Descriptor Systems

YuePeng Chen, Qingling Zhang
College of Science, Northeastern University, Shenyang, Liaoning 110006, P.R. of China

Wanquan Liu
School of Computing,Curtin University of Technology, Australia

Abstract

This short paper discusses the design of integrity controller for linear time-invariant discrete descriptor system. Some conditions for integrity of the system are obtained by Lyapunov method.

1 INTRODUCTION

The problem of stabilization of normal system is of both theoretical and practical importance and attracts the attention of many researchers, a number of significant results on this issue have been reported and different approaches have been proposed in the earlier literature. The interested readers, for instance, are refer to [2]. But multivariable feedback control system may become unstable when the feedback signals are switched off by a failure in the actuator or the sensor. We say that the system possesses integrity if it still remains keeping stable in the presence of such failures.

The studies of descriptor systems begin at the end of 1970's, although they are first mentioned in 1973 [1]. Descriptor systems capture the dynamic behavior of many natural phenomena, and have applications in many fields, such as network theory and robotics, and so on. It is a pity that few articles appear to discuss the problem about descriptor systems fault-tolerant control so far. Faults in descriptor systems are difficult to avoid in real applications due to unexpected factors in operating conditions. we should consider them when a practical control system is designed. For descriptor multivariable control systems, integrity is one of fault-tolerant characters.

The problem of stabilization about descriptor systems is of importance both practical and theoretical. A descriptor multivariable feedback control system may become unstable when the feedback signals are broken by a failure in the actuator or the sensor. We call that the descriptor system owns integrity if it still remains stable in the presence of such failures. Integrity of descriptor control systems is thus a type of fault-tolerance with respect to stability in the multivariable feedback control system.

This short paper is concerned with the discrete descriptor linear fault-tolerant control. We carry on the study that when the system actuator appear failure, it still stable. i. e. we design controller and made the closed-loop discrete descriptor systems integritiable.

2 SYSTEMS DESCRIPTOR AND PROBLEM STATEMENT

Consider the linear time-invariant discrete descriptor system of the form

$$Ex(t+1) = Ax(t) + Bu(t),\ t = 0, 1, 2, \cdots \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the input; E, A and B are the matrices of appropriate size, respectively. Usually, $rankE < n$, assume that system (1) is regular. The state feedback is

$$u(t) = -Fx(t), \tag{2}$$

where $F \in \mathbb{R}^{m\times n}$, then the closed loop system is

$$Ex(t+1) = (A - BF)\,x(t). \tag{3}$$

We introduce the matrix L to describe actuator failure, and define the matrix L as:

$$L = diag(l_1, l_2, \cdots, l_m),\ l_i \in [0, 1]. \tag{4}$$

System actuators are said to be in order when l_i is equal to 1. $l_i \in (0, 1)$ means that system actuators have fault in different extent, and $l_i = 0$ denotes that system actuators are complete invalid . In these cases the closed loop system is

$$Ex(t+1) = (A - BLF)x(t). \tag{5}$$

Remark 1 *The problem formulation given above (4) and (5) depicting actuator failure is different from [5]. In this article, we consider that* l_i *can continue to change from* 0 *to*1*, and the problem model of being debated is also different from [5].*

Definition 1 *We say that the closed loop system (5) possesses integrity if it still remains stable when the actuator failures exist.*

The problem is to design controller (2) to make the closed loop system (5) stable, when actuator failure happen, *i.e.* the closed-loop system (5) has integrity.

3 MAIN RESULTS

Lemma 1 *[3] If all the matrices J, F and $(J+KFM)$ are nonsingular, then*

$$(J+KFM)^{-1} = J^{-1} - J^{-1}K(F^{-1}+MJ^{-1}K)^{-1}MJ^{-1}$$

hold.

Constructing a generalized discrete Lyapunov function

$$V(E(t)) = x^T(t)E^TQEx(t), \qquad (6)$$

where $Q=Q^T>0$, of appropriate size. Obviously ,$V(Ex(t))>0$, if $Ex(t)\neq 0$.

From (6), we get a new type Lyapunov matrix equation :

$$A^TQA - E^TQE = -E^TPE. \qquad (7)$$

where $P\in\mathbb{R}^{n\times n}$ is an symmetric matrix. Equation (7) is called a generalized discrete Lyapunov equation (GDLE) of system (1).

Let $\lambda(E,A)$ denote the set of eigenvalues of system (1) and $D(0,1)$ denote the open disk centered at the origin with radius1.

Lemma 2 *[4]Consider system (1) and corresponding GDLE (7), if $\lambda(E,A)\subset D(0,1)$ and $degdet(sE-A)=rankE$. Then for arbitrary positive definite symmetric matrix P, Lyapunov equation (7) has semi-positive definite solution Q.*

Theorem 1 *For system (1), if the conditions of lemma 2 are satisfied and the state feedback law is*

$$u(t) = -Fx(t) = -r(R+B^TQB)^{-1}B^TQAx(t), \qquad (8)$$

where $0<r<1$ and

$$\rho\ \left(B(R+B^TQB)^{-1}B^TQ\right) < \frac{1}{r}. \qquad (9)$$

where $R\in\mathbb{R}^{n\times n}$ is positive definite matrix, and ρ denotes the maximum radius of a matrix spectrum, then the closed loop system (3) is stable.

Theorem 2 *For system (1) and Lyapunov equation (7), when the matrix $(I-rB^TQB)$ is nonsingular. Then the feedback law*

$$u = -rB^TQAx(t),\ \ (r>0). \qquad (10)$$

makes the closed loop system

$$Ex(t+1) = (A - rBB^TQA)x(t). \qquad (11)$$

stable.where the matrix $Q\in\mathbb{R}^{n\times n}$ is positive semi-definite solution of GDLE (7).

When some actuators of system (1) are failure, we have the following result about its integrity.

Theorem 3 *Consider system (1), when $\rho\left(BLB^TQ\right) < \frac{1}{r}$, some actuators are failure and feedback law is*

$$u = -rLB^TQAx(t),$$

where the matrix $Q\in\mathbb{R}^{n\times n}$ is positive semi-definite solution of GDLE (7). Then the system (5) has integrity.

Remark 2 *:When actuators or sensors are fault in a single input and single output system, its output is zero, then system is open loop, its stability is determined by itself. In a multi-input and multi-output system, every actuator and sensor only take on part control task. If some actuators or sensors are fault , system can still finish some function by making use of the rest normal actuators and sensors replacing for fault components acting some functions,* e.g. *stability.*

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