FIRING SPEEDS ESTIMATION FOR CONTINUOUS PETRI NETS

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Abstract: Production frequencies provide useful information for analysis and faults diagnosis of manufacturing systems. However, such frequencies are not always measurable. In this case they can be estimated from the observation of other variables. This work deals with the production frequencies estimation for systems that are modelled by continuous Petri nets. The buffers contents are measured on-line and the production frequencies estimation results from the inversion of the Petri net model. The exact and approximated solutions of the estimation problem are described. Accuracy of the estimation is related to the measurement error and to the Petri net structure. When the estimation provides several solutions, results are proposed to complete the Petri net such that a unique solution is obtained.

Keywords: continuous Petri nets, manufacturing systems, estimation.

1 Introduction

The analysis and monitoring of manufacturing systems are difficult problems because such systems often present a combination of non linear behaviours and complex dynamics (Cassandras 1993, Cao *et al.* 1990). Among the existing models, the continuous Petri nets are well adapted to represent manufacturing workshops composed of buffers and machines (David *et al.* 1992).

This work is concerned with the production frequencies estimation for such workshops based on the sampled observation of the buffers contents. The problem is solved with the inversion of the evolution equation of the continuous Petri net model. This problem is complementary to the Petri nets state estimation (Giua 1997, Kailath 1980), where transitions firings are observed and marking vector is estimated. Our method provides interesting results for the performances analysis (Pagnoni 1986, Ramchandani, 1973) and the faults diagnosis (Isermann 1984, Knapp *et al.* 1992, Wang *et al.* 1993, Zeng *et al.* 1991) of manufacturing systems.

The paper is organised as follows. Section two is about the continuous Petri nets models. In section three, the firing speeds estimation problem is solved with a linear system inversion (Gantmacher 1966, Rotella *et al.* 1995). Solutions are obtained with the help of the Moore Penrose inverse (Ben-Israel *et al.* 1974, Campbell *et al.* 1979, Rotella *et al.* 1995) of the incidence matrix. Results are also proposed to reduce the number of solutions and to evaluate the estimation error. The last section is an illustrative example.

2 Continuous Petri nets models

A Petri net (PN) with n places and p transitions is defined as < P, T, Pre, Post, M_0 > where $P=\{P_i\}_{i=1,\ldots,n}$ is a not empty finite set of places, $T=\{T_j\}_{j=1,\ldots,p}$ is a not empty finite set of transitions, such that $P \cap T = \emptyset$. IN is defined as the set of integer numbers and IR^+ as the set of non negative real numbers. Pre: $P \times T \rightarrow IN$ is the pre-incidence application : Pre (P_i, T_j) is the weight of the arc from place P_i to transition T_j and $W_{PR} = (w^{PR}_{ij})_{i=1,\ldots,n,\ j=1,\ldots,p} \in IN^{n \times p}$ with w^{PR}_{ij} = Pre (P_i, T_j) is the pre-incidence matrix. Post: $P \times T \rightarrow IN$ is the post-incidence application : Post (P_i, T_j) is the weight of the bond from transition T_j to place P_i and $W_{PO} = (w^{PO}_{ij})_{i=1,\ldots,n,\ j=1,\ldots,p} \in IN^{n \times p}$ with w^{PO}_{ij} = Post (P_i, T_j) is the post-incidence matrix. The PN incidence matrix W is defined as $W = W_{PO} - W_{PR} \in IN^{n \times p}$. ${}^{\circ}T_j$ stands for the preset places of T_j.

A continuous PN with n places and p transitions is defined as < PN, V_{max} > where PN is a Petri net and $V_{max} = (v_{max\ j})_{j=1,\ldots,p} \in IR^{+\ p}$ is the vector of transitions maximal firing speeds. Continuous PN have been deduced from timed PN (David *et al.* 1992). The marking $m_i(t)$ of each place P_i, $i = 1,\ldots,n$, has an integer value in timed PN and has a real value in continuous PN. Let us define $M(t) = (m_i(t))_{i=1,\ldots,n} \in IR^{\ n}$ as the marking vector at time t and $M_0 \in IR^{\ n}$ as the initial marking vector. Each transition firing occurs at accurate time in timed PN and becomes a continuous crossing in continuous PN. Let us define $V(t) = (v_j(t))_{j=1,\ldots,p} \in IR^{+\ p}$ as the firing speeds vector at time t. The marking evolution is given by the following differential system:

$$\frac{dM(t)}{dt} = W.V(t)\ . \qquad (1)$$

Among the existing models of continuous PN, the continuous PN with variable speeds (VCPN) (David *et al.* 1992) were proved to give the best continuous approximation of discrete manufacturing systems. The components of the firing speeds vector $V(t)$ of VCPN depend continuously on the marking of the preset places:

$$v_j(t) = v_{max\ j}\ .\ \min_{P_i \in {}^{\circ}T_j} (1, m_i(t))\ , \qquad (2)$$

Considering a sampling period Δt, $V_k = (v^k_j)_{j=1,\ldots,p} \in IR^{+\ p}$ is defined as the firing speeds vector at time $t = k.\Delta t$ and $(dM/dt)_k = ((dm_i/dt)_k)_{i=1,\ldots,n} \in IR^{\ n}$ is defined as the derivative of the marking vector at time $t = k.\Delta t$. Equation (1) results in:

$$\left(\frac{dM}{dt}\right)_k = W.V_k\,, \quad k > 0. \qquad (3)$$

3 Estimations of the firing speeds

3.1 *Problem formulation*

The manufacturing systems that are considered in this work are serial lines or assembly workshops that process big sets of tokens. Such systems correspond to weak uncertainty workshops with strong complexity (Gousty *et al.* 1988). They are numerous in semi - conductors and car factories. A workshop is composed of p machines A_j, $j = 1,.., p$ and of n buffers S_i, $i = 1,..,n$, (figure 1).

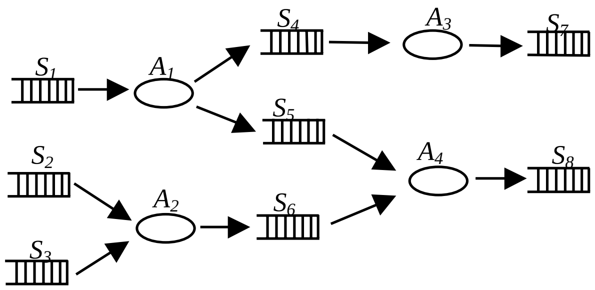

Figure 1: Assembly workshop

With VCPN models, buffers are represented by places and machines are represented by transitions. The marking vector stands for buffers content, and the firing speeds stand for the production frequencies. The buffers contents are measured on-line. Let us define $\hat{M}_k = (\hat{m}^k_i)_{i=1,..,n} \in IR^n$, $\Delta\hat{M}_k = \hat{M}_k - \hat{M}_{k-1} \in IR^n$ and $(\Delta\hat{M}/\Delta t)_k \in IR^n$ as the measured values of respectively $M(k.\Delta t)$, ΔM_k and $(dM/dt)_k$. The production frequencies are estimated by inversion of the VCPN evolution equation. Let us define $\hat{V}_k = (\hat{v}^k_j)_{j=1,...,p} \in IR^p$ as the estimated value V_k. Equation (3) results in (4):

$$\left(\frac{\Delta\hat{M}}{\Delta t}\right)_k = W.\hat{V}_k\,, \quad k > 0. \qquad (4)$$

To estimate the firing speeds, equation (4) must be solved. This equation is considered as a set of n linear relations with the unknown vector $\hat{V}_k$ of dimension p.

3.2 *Set of solutions*

Let us call r the rank of matrix W. Let us also define h as the rank of matrix $(W, (\Delta\hat{M}/\Delta t)_k) \in IR^{n \times (p+1)}$, where matrix $(W, (\Delta\hat{M}/\Delta t)_k)$ stands for the aggregation of matrix W and vector $(\Delta\hat{M}/\Delta t)_k$. Equation (4) may have none, one or several exact solutions. When equation (4) has no exact solution, it may have one or several approximated solutions (Gantmacher 1966, Rotella *et al.* 1995). The vector $\hat{V}_k$ is called an approximated solution for equation (4) if it minimises the difference $\| (\Delta\hat{M}/\Delta t)_k - W.\hat{V}_k \|$ where $\| . \|$ stands for the Euclidean norm. The set of solutions for equation (4) is defined either as the set of exact solutions when such solutions exist, either as the set of approximated solutions when equation (4) has no exact solution. Let us define $(u_i)_{i=1...p}$ as the column vectors of matrix W and Vect(W) = Vect$\{u_1,...u_p\}$ as the vector space defined by the vectors u_1 to u_p. When $r = h$, $(\Delta\hat{M}/\Delta t)_k \in$ Vect$\{u_1,...u_p\}$ and equation (4) has at least one exact solution. In this case the system is said to be compatible. When $r < h$, $(\Delta\hat{M}/\Delta t)_k \notin$ Vect$\{u_1,...u_p\}$ and equation (4) has no exact solution. But (4) has one or several approximated solutions. In this case, the system is said to be not compatible.

The set of solutions for equation (4) can be expressed with the Moore Penrose inverse of matrix W (Ben-Israel and *al* 1974, Campbell and *al* 1979, Rotella and *al* 1995). The Moore Penrose inverse of $W \in IR^{n \times p}$ is the unique matrix $W^+ \in IR^{p \times n}$, that verifies the properties $W.\, W^+.W = W$, $W^+.W.\, W^+ = W^+$, $(W.W^+)^T = W.\, W^+$ and $(W^+.W)^T = W^+.W$. Theorem 1 provides the set of solutions G^V_k for equation (4) at time $t = k.\Delta t$.

Theorem 1: *The set of solutions G^V_k for equation (4) is given by:*

$$G^V_k = \left\{ \hat{V}_k \;\middle/\; \hat{V}_k = W^+.(\Delta\hat{M}/\Delta t)_k + (I_p - W^+.W)z \right\}, \quad k > 0, \qquad (5)$$

where z stands for any vector of IR^p and $I_p \in IR^{p \times p}$ stands for the p-identity matrix

For the sake of brevity, the proof of theorem 1 is omitted, because it is the direct application of a well known result in matrix theory (Campbell *et al.* 1979, Rotella *et al.* 1995). The Moore Penrose inverse of matrix W can be worked out with a help of maximal rank factorisation of W or by application of the Greville constructive algorithm (Rotella *et al.* 1995).

3.3 *Unicity of the solution*

There exist two regular matrices $P \in IR^{n \times n}$ and $Q \in IR^{p \times p}$ such that :

$$W = P.\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix} Q\,.$$

$P^{-1}.\, W.Q^{-1}$ is known as the Smith form of matrix W. Let us define the vectors $\hat{Y}_{k1} \in IR^r$, $\hat{Y}_{2k} \in IR^{n-r}$ and $\hat{V}_{1k} \in IR^r$, $\hat{V}_{2k} \in IR^{p-r}$ such that :

$$\begin{pmatrix} \hat{Y}_{1k} \\ \hat{Y}_{2k} \end{pmatrix} = P^{-1}.(\Delta\hat{M}/\Delta t)_k\,, \qquad \begin{pmatrix} \hat{V}_{1k} \\ \hat{V}_{2k} \end{pmatrix} = Q.\hat{V}_k\,.$$

Equation (4) results in equation (6):

$$\begin{cases} \hat{Y}_{1k} = \hat{V}_{1k} \\ \hat{Y}_{2k} = 0 \end{cases} \qquad (6)$$

According equation (6), theorem 2 gives the necessary and sufficient condition such that (4) has a unique solution.

Theorem 2: *The set G^V_k has a unique solution if and only if $p = r$. When $p < r$, G^V_k is of dimension $p - r$.*

Proof : From equation (5), one can write that the dimension of G^V_k equals the dimension of Vect(I_p - $W^+.W$). Let us show that Vect(I_p - $W^+.W$) = $\Omega(W)$, where $\Omega(W)$ stands for the kernel of (W). Let us assume that $x \in$ Vect(I_p - $W^+.W$). There exists $z \in IR^p$, such that $x = (I_p - W^+.W).z$. Thus, according the definition of the Moore-Penrose inverse of W, $W.x = (W - W.W^+.W).z = 0$ and $x \in \Omega(W)$. Reciprocally, let us assume that $x \in \Omega(W)$. Thus $W.x = 0$. Theorem 1 can be applied to this equation and the set of solutions is given by : $x = W^+.0 + (I_p - W^+.W).z = (I_p - W^+.W).z$, with $z \in IR^p$ and $x \in$ Vect(I_p - $W^+.W$). Thus, Vect(I_p - $W^+.W$) = $\Omega(W)$ and the dimension of G^V_k equals the dimension of $\Omega(W)$, that is to say $p - r$ (dim Vect(W) + dim $\Omega(W) = p$).
If $p = r$, the dimension of Vect(I_p - $W^+.W$) equals 0 and equation (5) results in:

$$G^V_k = \left\{\hat{V}_k \;\Big/\; \hat{V}_k = W^+.(\Delta\hat{M} / \Delta t)_k \right\}, \quad k > 0.$$

In this case, the set G^V_k has a unique element. The other cases are summarised in the table in table 1.

h	***n/p***	***r***	W^+	**Solution**
$h=r$	$n=p$	$r=n$	W^{-1}	Exact/Unique
		$r<n$	Max. rank fact.	Exact/Several
	$p<n$	$r=p$	$(W^T W)^{-1} W^T$	Exact/Unique
		$r<p$	Max. rank fact.	Exact/Several
	$n<p$	$r=n$	$W^T (W.W^T)^{-1}$	Exact/Several
		$r<n$	Max. rank fact.	Exact/Several
$h>r$	$n=p$	$r<n$	Max. rank fact.	Approx./Several
	$p<n$	$r=p$	$(W^T W)^{-1} W^T$	Approx./Several
		$r<p$	Max. rank fact.	Approx./Several
	$n<p$	$r<n$	Max. rank fact.	Approx./Several

Table 1: Set of solutions in function of *n, p, r and h*

When $r < p$, the vector $\hat{V}_{1k}$ is uniquely determined, but $\hat{V}_{2k}$ can not be determined. In this case, the proposed method only provides a unique solution for r linear combinations of components of vector $\hat{V}_k$ (vector $\hat{V}_{1k}$). Nevertheless, theorems 3 and 4 provide results to obtain a unique solution $\hat{V}_k$.
When $r = n$, $\hat{Y}_{1k} = P^{-1}.(\Delta\hat{M} / \Delta t)_k$. All components of vector $(\Delta\hat{M} / \Delta t)_k$ are necessary to determine $\hat{V}_{1k}$. When $r < n$, only the vector $\hat{Y}_{1k}$ is required to determine $\hat{V}_{1k}$. In this case, only a partial observation of vector $(\Delta\hat{M} / \Delta t)_k$ is required to obtain the set of solutions. But, the complete observation of vector $(\Delta\hat{M} / \Delta t)_k$ is useful to obtain redundancy relations between the observed variables and to take the sensors failures into account. Theorem 5 relates the estimation errors to the measurement accuracy.

3.4 Reduction of the number of solutions

When G^V_k contains several elements, the marking vector $\hat{M}_k$ does not contain enough information to work out a unique estimation of the firing speeds. Two results (theorems 3 and 4) are proposed to decrease the number of solutions.
In one hand, new places can be added to the VCPN such that the rank of matrix W increase to p.

Theorem 3: *When equation (4) has several solutions, a unique estimation $\hat{V}_k$ of vector V_k. is obtained with the addition of $p - r$ complementary places located such that the rank of matrix W increases to p.*

Proof (constructive): Let us assume that matrix W is such that $r < p$ and let us define w_i as the i^{th} row of matrix W. There exists $w_{n+1}^T \in IR^p$ such that rank $(w_1^T, \dots, w_n^T, w_{n+1}^T) = r + 1$. The row w_{n+1}^T is linearly independent with $w_1^T, \dots, w_n^T$. Repeating the same operation$p - r$ times, the rank of the augmented incidence matrix increases to p. Each row defines the location of a new place on the PN. Thus the estimation of the vector V_k requires the addition of $p - r$ places.

Example: Let consider the ordinary VCPN given on figure 2 as an example.

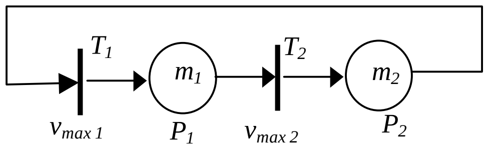

Figure 2: Closed line

The evolution of the marking vector is given by (7):

$$\begin{pmatrix} \dot{m}_1(t) \\ \dot{m}_2(t) \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix}, \quad t > 0. \qquad (7)$$

The rank of matrix W equals 1. According to theorem 2, the set of solutions G^V_k is of dimension 1. Let us now suppose, that the flow coming from transition T_2 is directly measurable. This assumption is equivalent to add a post-set place P_3 to the transition T_2 (figure 3). The modified system has an augmented incidence matrix of full column rank 2:

$$\begin{pmatrix} \dot{m}_1(t) \\ \dot{m}_2(t) \\ \dot{m}_3(t) \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix}, \quad t > 0.$$

The unique solution is given by (8):

$$\hat{V}_k = \frac{1}{2} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 0 & 2 \end{pmatrix} \left(\frac{\Delta\hat{M}}{\Delta t} \right)_k, \quad k > 0. \qquad (8)$$

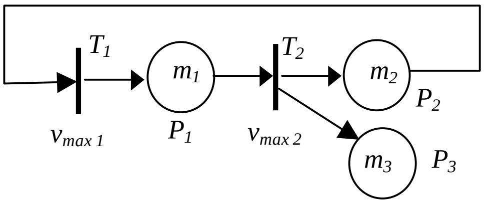

Figure 3: VCPN complement

In the other hand, one can add an *a priori* knowledge about the firing speeds to equation (1) without modifying the structure of the net.

Theorem 4: *If two transitions T_i and T_j have the same preset places $° T_i = ° T_j$ and if the relation between the maximal firing speeds $v_{max\,i}$ and $v_{max\,j}$ is known :*

$$v_{max\,i} = \alpha_{ij}\, v_{max\ j}, \tag{9}$$

then relation (9) can be added to equation (1) in order to increase the rank of matrix W.

Proof (constructive): Let us assume that both transitions T_i and T_j have the same preset places $°T_i = °T_j = °T$, the firing speeds are given by the relations :

$$\begin{aligned} v_i(t) &= v_{max\,i} \,.\, \min_{P_k \in °T}(1, m_k(t)), \\ v_j(t) &= v_{max\,j} \,.\, \min_{P_k \in °T}(1, m_k(t)). \end{aligned} \tag{10}$$

If $v_{max\ i} = \alpha_{ij}.v_{max\ \ j,}$ the firing speeds equations (10a) and (10b) are also proportional: $v_i(t) = \alpha_{ij}\, v_j(t)$. Considering the vector $w^T_{n+1} = (0,\ldots,0,\ 1_i,\ 0,\ldots,-(\alpha_{ij})_j,0,\ldots,0)$, where 1_i stands for 1 placed at i^{th} position and $-(\alpha_{ij})_j$ stands for $-\alpha_{ij}$ placed at j^{th} position, equation (11) results from (1):

$$\begin{pmatrix} I_n & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{M}(t) \\ \dot{m}_{n+1}(t) \end{pmatrix} = \begin{pmatrix} W \\ w_{n+1} \end{pmatrix} V(t), \quad t > 0. \tag{11}$$

If w^T_{n+1} is linearly independent with $(w_i)^T_{i=1\ldots n}$, the rank of W increases to $r + 1$. Repeating the same operation $p - r$ times, the rank of matrix W increases to p.

Example : Let us consider the VCPN given in figure 4 as an example. W is of rank 1 and the dimension of G^V_k equals 2.

$$\begin{pmatrix} \dot{m}_1(t) \\ \dot{m}_2(t) \end{pmatrix} = \begin{pmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix}, \quad t > 0. \tag{12}$$

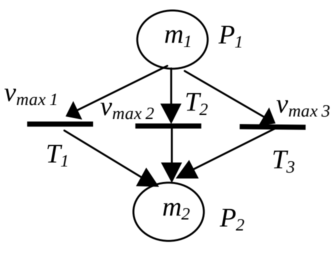

Figure 4 : Multi servers station

The flow coming from the place P_1 can always be separated in 3 tokens to feed the 3 transitions T_1, T_2 and T_3. The firing of the 3 transitions is achieved continuously and simultaneously and the repartition of the flow can be related to the maximal firing speed of the transitions. For this reason, the relations between the maximal speeds can be added to equation (12). For example, let us assume that $v_{max\ 1} = 2.v_{max\ 2} = 2.v_{max\ 3}$. Two complementary relations can be added to equation (12):

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{m}_1(t) \\ \dot{m}_2(t) \\ \dot{m}_3(t) \\ \dot{m}4(t) \end{pmatrix} = \begin{pmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix}, \quad t > 0, \tag{13}$$

that results in:

$$\hat{V}_k = \begin{pmatrix} -1/4 & 1/4 \\ -1/8 & 1/8 \\ -1/8 & 1/8 \end{pmatrix}\left(\frac{\Delta\hat{M}}{\Delta t}\right)_k, \quad k > 0. \tag{14}$$

3.5 Evaluation of the estimation error

Let us assume that equation (4) has a unique solution (if equation (4) has several solutions, the VCPN is completed according the results given by theorem 3 or 4, such that a unique solution fit to the estimation problem). From a theoretic point of view, equation (3) is always compatible. The vector $(dM/dt)_k$ result from vector V_k and belongs to Vect(W). From a numerical point of view, the compatibility of equation (4) is not warranted. Only an approximation $(\Delta\hat{M}/\Delta t)_k$ of vector $(dM/dt)_k$ is measured. This measured value is not exact and includes a measurement error $\delta(\Delta\hat{M}/\Delta t)_k$ such that :

$$(dM/dt)_k = (\Delta\hat{M}/\Delta t)_k + \delta(\Delta\hat{M}/\Delta t)_k . \tag{15}$$

Theorem 5: *The estimation $\hat{V}_k$ of V_k, includes an error $\delta\hat{V}_k$ such that:*

$$\left\|\delta\hat{V}_k\right\| \le \sqrt{\sigma_{W^+}}\,.\left\|\delta\left(\frac{\Delta\hat{M}}{\Delta t}\right)_k\right\|, \quad k > 0, \tag{16}$$

where σ_{W+} stands for the spectral radius of matrix $(W^+)^T.W$ (σ_{W+} is the maximal module of the eigenvalues of matrix $(W^+)^T.W^+$).

Proof: Equation (15) results in:

$$V_k = \hat{V}_k + W^+.\delta(\Delta\hat{M}/\Delta t)_k , \tag{17}$$

Thus $\delta\hat{V}_k = W^+.\delta(\Delta\hat{M}/\Delta t)_k$ represents the influence of the measurement error $\delta(\Delta\hat{M}/\Delta t)_k$ on the estimation $\hat{V}_k$ of V_k. Let us mention that the Euclidean norm is a multiplicative norm, thus:

$$\left\|\delta\hat{V}_k\right\| \le \left\|W^+\right\|.\left\|\delta(\Delta\hat{M}/\Delta t)_k\right\| .$$

The Euclidean norm of matrix W results from the vectorial Euclidean norm:

$$\left\| W^{+} \right\| = \max_{\|X\|=1} \left\{ \left\| W^{+}.X \right\| \right\} = \sqrt{\sigma_{W^{+}}} \ . \qquad (18)$$

Thus equation (16) holds.

The norm of the estimation error $\delta \hat{V}_k$ depends on $\sqrt{\sigma_{W^{+}}}$ that is to say on the structure of the VCPN and on $\left\| \delta(\Delta \hat{M} / \Delta t)_k \right\|$ that is to say on the accuracy of the measurement $(\Delta \hat{M} / \Delta t)_k$ of vector $(dM / dt)_k$.

Let us notice that theorem 5 is not suitable for numerical applications because $\delta(\Delta \hat{M} / \Delta t)_k$ is difficult to evaluate. When $(dM / dt)_k$ is measured as the difference between two consecutive values of the marking vector, we have $(\Delta \hat{M} / \Delta t)_k = \Delta \hat{M}_k / \Delta t$. For VCPN, one can notice that components of the marking vector are either linear functions, either exponential ones (David *et al.* 1992). If the marking functions are linear, we can write:

$$\left\| \delta \left(\frac{\Delta \hat{M}}{\Delta t} \right)_k \right\| = \left\| \frac{\Delta M_k}{\Delta t} - \frac{\Delta \hat{M}_k}{\Delta t} \right\| \leq \frac{2}{\Delta t} . \left\| \delta \hat{M}_k \right\| . \qquad (19)$$

If marking functions are exponential, for small values of τ, we have:

$$M(k.\Delta t + \tau) = M_k + \left(\frac{dM}{dt} \right)_k . \tau + \left(\frac{d^2 M}{dt^2} \right)_k . \frac{\tau^2}{2} + o(\tau^2) .$$

Considering $\tau = -\Delta t$:

$$M_{k-1} = M_k - \left(\frac{dM}{dt} \right)_k . \Delta t + \left(\frac{d^2 M}{dt^2} \right)_k . \frac{\Delta t^2}{2} + o(\Delta t^2) .$$

Thus:

$$\left(\frac{dM}{dt} \right)_k = \frac{\Delta M_k}{\Delta t} + \left(\frac{d^2 M}{dt^2} \right)_k . \frac{\Delta t}{2} + o(\Delta t) , \qquad (20)$$

and:

$$\left\| \delta \left(\frac{\Delta \hat{M}}{\Delta t} \right)_k \right\| \leq \frac{\Delta t}{2} . \left\| \left(\frac{d^2 M}{dt^2} \right)_k \right\| + \frac{2}{\Delta t} . \left\| \delta \hat{M}_k \right\| . \qquad (21)$$

An usual approximation of $(d^2M / dt^2)_k$ is given by:

$$\left(\frac{d^2 M}{dt^2} \right)_k \approx \frac{1}{\Delta t^2} . (\Delta \hat{M}_k - \Delta \hat{M}_{k-1}) . \qquad (22)$$

Thus:

$$\left\| \delta \left(\frac{\Delta \hat{M}_k}{\Delta t} \right) \right\| \approx \frac{1}{2.\Delta t} . \left\| \Delta \hat{M}_k - \Delta \hat{M}_{k-1} \right\| + \frac{2}{\Delta t} . \left\| \delta \hat{M}_k \right\| . \qquad (23)$$

Relation (23) provides an evaluation of $\delta(\Delta \hat{M}_k / \Delta t)$ in function of the marking measurement during the exponential phases of the marking evolution.

4 Example

Let us consider, as an example, the ordinary VCPN of maximal firing speeds $v_{max\,1} = 5$, $v_{max\,2} = 3$, $v_{max\,3} = 4$, and $v_{max\,4} = 6$. (figure 5). The initial marking vector is given by $M_0 = (10\ 20\ 0\ 0)^T$ and the marking evolution is given by equation (24) :

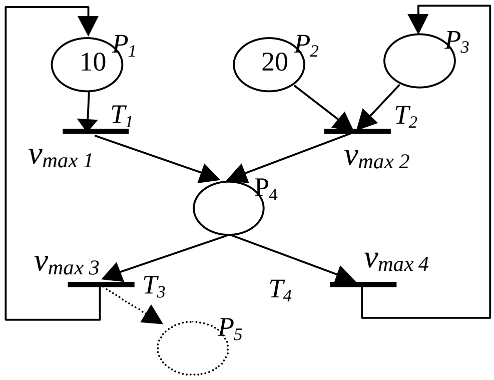

Figure: 5: Example of VCPN

$$\dot{M}(t) = \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix} V(t) , \quad t > 0. \qquad (24)$$

The incidence matrix $W \in IR^{4 \times 4}$ is of rank 3. Thus the dimension of G_k^V equals 1 and the estimation $\hat{V}_k$ of the firing speeds vector is not unique. Thus, the VCPN must be completed with a additional place P_5 such that the rank of the augmented incidence matrices increases to 4 (this solution is represented with dashed points in figure 5). The firing speeds estimation is given by equation (25):

$$\hat{V}_k = \frac{1}{3} \begin{pmatrix} -2 & 0 & 1 & 1 & 3 \\ 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \\ -1 & -3 & 2 & -1 & 0 \end{pmatrix} \left(\frac{\Delta \hat{M}}{\Delta t} \right)_k , \quad k > 0. \qquad (25)$$

The figure 6 represents the marking evolution and the measurement points (Δ) for a sampling period Δt = 0.5 time units (TU).

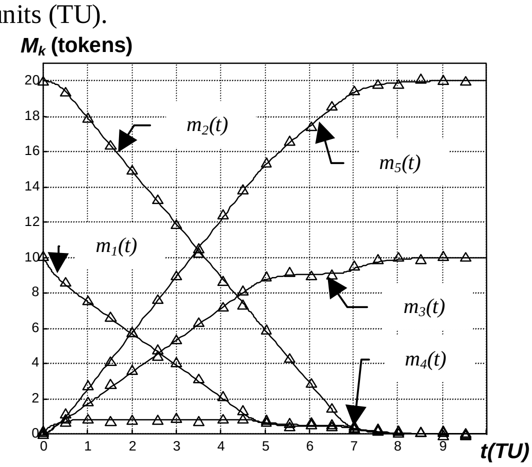

Figure 6: VCPN marking evolution

The set of non null eigenvalues of $(W^+)^T. W^+$ is given by {0.1529, 0.3820, 2.1805, 2.6180}. The vector $(\Delta\hat{M} / \Delta t)_k$ is assumed to be directly measurable with a given accuracy. Considering a local measurement error of maximal value $\alpha = 0.1$ for each component of $(dM / dt)_k$, we have :

$$\left\| \delta\hat{V}_k \right\| < 0.1.\sqrt{5}.\sqrt{2.6180} \approx 0.36, \quad k > 0. \qquad (26)$$

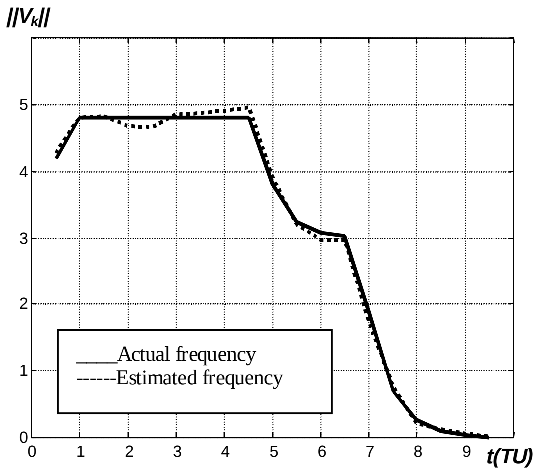

Figure 7: Estimation of the firing speeds for T_4

Estimation results for transition T_4 are given in figure 7. Estimated firing speeds are also compared with actual ones. The norm of the firing speeds estimation error is represented in figure 8. This error depends only on the accuracy of the marking vector derivative. The sampling period has no influence on the accuracy of the firing speeds estimation as represented in figure 8.

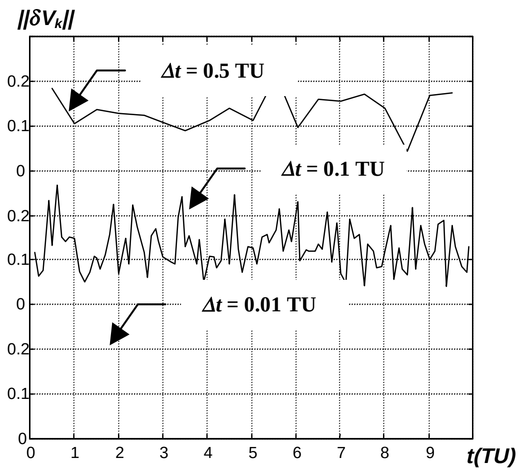

Figure 8: Estimation error

5 Conclusions

This paper has proposed an estimation method for the production frequencies of manufacturing systems modelled by VCPN. The estimation problem was solved with the inversion of the PN incidence matrix. The set of solutions was described (theorems 1 and 2). When several solutions exist, the Petri net was completed with additional relations in order to provide a unique solution (theorems 3 and 4). The accuracy of the estimation was related to the measurement error and to the PN structure (theorem 5). Our further work is to adapt the method to estimate not only the production frequencies but also the maximal production frequencies. In comparison with production frequencies, maximal production frequencies are characteristic parameters that do not depend on the marking vector. Estimation of the maximal production frequencies will provide useful information for the identification of Petri nets. Another perspective is to apply the production frequencies estimation to develop model-based methods for faults diagnosis of manufacturing systems.

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