# On Global Output Feedback Tracking Control of Robot Manipulators*

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## Abstract

*In this paper, we revisit the global, output feedback (OFB), tracking control problem for rigid-link robot manipulators subject to parametric uncertainty. Motivated by misunderstandings in the literature concerning our previous result, we propose a new global, OFB, adaptive controller, which in contrast to our previous work, eliminates the need for a post-stability analysis transformation to derive a velocity-independent control strategy. The structure of the new controller along with a new Lyapunov function are used to illustrate global asymptotic link position tracking. Experimental results are included to demonstrate the controller performance.*

## 1. Introduction

The problem of output feedback (OFB), position tracking control of robot manipulators has been a topic of considerable interest over the past several years. This interest can be credited to the fact that the robot dynamics can be used to model a very general class of nonlinear mechanical systems (*e.g.*, rotors, spacecraft, *etc.*), and hence, be used as a benchmark for the design of nonlinear control algorithms. A limitation that exists in almost all of the proposed OFB link position tracking controllers is the semi-global[1] nature of the stability results. In contrast, global[2] solutions to the OFB link position setpoint control problem have been presented by several researchers. For example, model-based OFB controllers, composed of a linear feedback loop plus feedforward gravity compensation, were proposed in [2, 6, 15] to globally asymptotically stabilize the manipulator dynamics. In [1], Arimoto *et al.* also presented a model-based, global regulating OFB controller; however, the gravity compensation term was dependent on the desired link position setpoint as opposed to the actual link position. With the intent of overcoming the requirement of exact model knowledge, Ortega *et al.* [22] designed a OFB regulator which compensated for uncertain gravity effects; however, the stability result was semi-global asymptotic. In [11], Colbaugh *et al.* proposed a global regulating OFB controller that compensates for uncertain gravity effects; however, the control strategy requires the use of two different control laws (*i.e.*, one control law is used to drive the setpoint error to a small value, then another control law is used to drive the setpoint error to zero).

With respect to the more general problem of OFB link position tracking control, semi-global results have dominated the scenario. For example, in [2, 19], a model-based observer/ controller yielded semi-global exponential link position tracking while in [21] a semi-global asymptotic tracking result was achieved. Robust, filter-based control schemes were designed in [3, 24, 27] to compensate for parametric uncertainty while producing semi-global, uniform ultimate bounded link position tracking. In [8, 9, 10], variable structure OFB controllers were designed to compensate for uncertainty and the lack of link velocity measurements. For other OFB tracking control work, the reader is referred to [17]. Finally, in [4, 5, 14, 28], adaptive OFB controllers were presented which yielded semi-global asymptotic link position tracking.

To the best of our knowledge, the only previous results which are targeted at the global, OFB, tracking control problem are given in [20, 7, 29]. Specifically, in [20], Loria developed a model-based controller that yields global uniform asymptotic stability of the closed-loop system; however, the control was designed for a nonlinear, one degree-of-freedom (DOF) system. In [7], Burkov used a singular perturbation analysis to show that a model-based controller, used in conjunction with a linear observer, can force the link position to asymptotically track a trajectory from any initial condition; however, as pointed out in [20], no explicit bound on the singular perturbation parameter was given. More recently, Zhang *et al.* [29] proposed the first global, OFB, adaptive tracking controller for uncertain, $n$ DOF robot manipulators. In particular, while the structure of the controller of [29] resembles that of [20] in certain aspects, the use of a desired compensation adaptation law [26], a different filter structure, and a different error system development/analysis allowed the extension of the controller of [20] to the more general, multivariable case with parametric uncertainty. The solution of [29] to the global, OFB, tracking control problem has been subject to some misunderstandings in the literature (see [23], page 111). Motivated by this fact, we revisit the solution originally proposed in [29] from a different design and analysis perspective. Namely, the controller, velocity-surrogate filter, and closed-loop stability analysis in [29] were first developed as though link velocity were available. After the stability analysis was performed, a transformation was utilized to illustrate how the control strategy could be implemented without link velocity measurements..

In this paper, we propose a new global, OFB, adaptive controller, which in contrast to the work presented in [29], eliminates the need for using a post-stability analysis transformation to derive a link velocity-independent control strategy. Specifically, the form of the new controller generates a different structure for the closed-loop error system. The new structure of the closed-loop error system is then fused with a different Lyapunov function to illustrate global asymptotic tracking. We complement the control design and analysis with an experimental implementation of the OFB adaptive controller on a two-link, robot testbed. The paper is organized as follows. Section 2. presents the robot model along with its properties. The control design, error system development,

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[1]In a *semi-global* stability result, a control gain often has to be adjusted according to the "size" of the initial conditions.

[2]In accordance with the terminology used in the robotics literature, here, *global* position tracking means that the controller must drive the link position tracking error to zero for any finite, initial position and velocity tracking errors, with no conditions on the size of the initial tracking errors.

and stability analysis are presented in Section 3.. Section 4. contains the experimental results while the conclusions are drawn in Section 5..

## 2. Robot Model

The mathematical model for an $n$ DOF, revolute joint, direct drive, robot manipulator is assumed to have the following form [18]

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} = \tau \tag{1}$$

where $q(t), \dot{q}(t), \ddot{q}(t) \in \mathbb{R}^n$ denote the link position, velocity, and acceleration, respectively, $M(q) \in \mathbb{R}^{n\times n}$ represents the positive-definite, symmetric inertia matrix, $V_m(q,\dot{q}) \in \mathbb{R}^{n\times n}$ represents the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ is the gravitational vector, $F_d \in \mathbb{R}^{n\times n}$ denotes the constant, diagonal, positive-definite, viscous friction matrix, and $\tau(t) \in \mathbb{R}^n$ represents the torque input control vector. We will assume that the left-hand side of (1) is first-order differentiable.

The dynamic system given by (1) exhibits the following properties that are utilized in the subsequent control development and stability analysis.

**Property 1:** The inertia matrix can be upper and lower bounded by the following inequalities [18]

$$m_1 \|\xi\|^2 \leq \xi^T M(q)\xi \leq m_2 \|\xi\|^2 \qquad \forall\, \xi \in \mathbb{R}^n \tag{2}$$

where $m_1$ and $m_2$ are positive constants, and $\|\cdot\|$ denotes the Euclidean norm.

**Property 2:** The inertia and the centripetal-Coriolis matrices satisfy the following relationship [18]

$$\xi^T\left(\frac{1}{2}\dot{M}(q) - V_m(q,\dot{q})\right)\xi = 0 \qquad \forall\, \xi \in \mathbb{R}^n \tag{3}$$

where $\dot{M}(q)$ represents the time derivative of the inertia matrix.

**Property 3** The centripetal-Coriolis matrix satisfies the following relationship [21]

$$V_m(q,\nu)\xi = V_m(q,\xi)\nu \qquad \forall\, \xi,\nu \in \mathbb{R}^n. \tag{4}$$

**Property 4:** The norm of the centripetal-Coriolis and friction matrices can be upper bounded as follows [18]

$$\|V_m(q,\dot{q})\| \leq \zeta_{c1}\|\dot{q}\|, \qquad \|F_d\| \leq \zeta_f \tag{5}$$

where $\zeta_{c1}$ and $\zeta_f$ are positive constants.

**Property 5:** The robot dynamics given in (1) can be linearly parameterized as follows [18]

$$Y(q,\dot{q},\ddot{q})\theta = M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} \tag{6}$$

where $\theta \in \mathbb{R}^p$ contains the constant system parameters, and $Y(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times p}$ denotes the regression matrix that is a function only of $q(t)$, $\dot{q}(t)$, and $\ddot{q}(t)$. The formulation of (6) can also written in terms of the desired trajectory in the following manner

$$Y_d(q_d,\dot{q}_d,\ddot{q}_d)\theta = M(q_d)\ddot{q}_d + V_m(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + F_d\dot{q}_d \tag{7}$$

where the desired regression matrix $Y_d(q_d,\dot{q}_d,\ddot{q}_d) \in \mathbb{R}^{n\times p}$ is a function of the desired link position, velocity, and acceleration vectors denoted by $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t) \in \mathbb{R}^n$, respectively.

To aid the subsequent control design and analysis, we define the vector function $Tanh(\cdot) \in \mathbb{R}^n$ and the matrix function $Cosh(\cdot) \in \mathbb{R}^{n\times n}$ as follows

$$Tanh\,(\xi) \triangleq [\tanh(\xi_1),\ \ldots,\ \tanh(\xi_n)]^T \tag{8}$$

and

$$Cosh(\xi) \triangleq diag\{\cosh(\xi_1), \cdots, \cosh(\xi_n)\} \tag{9}$$

where $\xi = [\xi_1,\ \ldots,\ \xi_n]^T \in \mathbb{R}^n$, and $diag\{\cdot\}$ denotes a diagonal matrix. Based on the definition of (8), it can easily be shown that the following inequalities hold

$$\begin{gathered}\frac{1}{2}\tanh^2(\|\xi\|) \leq \ln(\cosh(\|\xi\|)) \leq \sum_{i=1}^{n} \ln(\cosh(\xi_i)) \leq \|\xi\|^2, \\ \tanh^2(\|\xi\|) \leq \|Tanh\,(\xi)\|^2 = Tanh^T(\xi)\,Tanh(\xi).\end{gathered} \tag{10}$$

**Assumption 1:** The following bounds for the inertia, centripetal-Coriolis, and gravity terms of (1) are assumed to exist for $\forall \xi, \nu \in \mathbb{R}^n$

$$\begin{aligned}\|M(\xi) - M(\nu)\|_{i\infty} &\leq \zeta_m \|Tanh\,(\xi-\nu)\|, \\ \|G(\xi) - G(\nu)\| &\leq \zeta_g \|Tanh\,(\xi-\nu)\|, \\ \|V_m(\xi,\dot{q}) - V_m(\nu,\dot{q})\|_{i\infty} &\leq \zeta_{c2}\|\dot{q}\|\,\|Tanh\,(\xi-\nu)\|\end{aligned} \tag{11}$$

where $\zeta_m$, $\zeta_g$, and $\zeta_{c2}$ are positive constants. In [29], we illustrated how the bounds given in (11) hold for the six DOF Puma robot. In a similar manner, it can be shown that the bounds given in (11) hold for other revolute joint robots. Hence, from a practical point of view, Assumption 1 resembles a property more than an assumption.

## 3. Control Design and Analysis

The control objective is to design a global link position tracking controller for the robot manipulator model given by (1) under the constraints that only the link position variable $q(t)$ is available for measurement and that the parameter vector $\theta$ defined in (5) is an unknown constant vector. We will quantify the control objective by defining the link position tracking error $e(t) \in \mathbb{R}^n$ as follows

$$e \triangleq q_d - q \tag{12}$$

where we assume that $q_d(t)$, defined in Property 3, and its first three time derivatives are bounded functions of time. In addition, we define the difference between the actual and estimated parameters as follows

$$\tilde{\theta} \triangleq \theta - \hat{\theta} \tag{13}$$

where $\tilde{\theta}(t) \in \mathbb{R}^p$ represents the parameter estimation error vector, and $\hat{\theta}(t) \in \mathbb{R}^p$ represents a dynamic estimate of $\theta$ defined in (5).

### 3.1. Adaptive Output Feedback Tracking Control Law

Based on the subsequent error system development and stability analysis, we propose the following torque input control

$$\tau = Y_d\hat{\theta} - kT^{-1}y + Tanh(e) \tag{14}$$

where $y(t) \in \mathbb{R}^n$ is an auxiliary tracking error signal defined below, $k$ is a positive, scalar control gain, $T(y) \in \mathbb{R}^{n\times n}$ is defined as the following diagonal matrix

$$T \triangleq diag\left\{\ \left(1-y_1^2\right)^2,\quad \left(1-y_2^2\right)^2,\quad \ldots,\quad \left(1-y_n^2\right)^2\ \right\}, \tag{15}$$

the parameter estimate vector $\hat{\theta}(t)$ is generated according to

$$\begin{aligned}\hat{\theta} = \ & \Gamma\int_0^t \Big[Y_d^T(q_d(\sigma),\dot{q}_d(\sigma),\ddot{q}_d(\sigma))\,(Tanh(e(\sigma)) + y(\sigma)) \\ & \qquad - \dot{Y}_d^T(q_d(\sigma),\dot{q}_d(\sigma),\ddot{q}_d(\sigma))e(\sigma)\Big]\,d\sigma \\ & + \Gamma Y_d^T e\end{aligned} \tag{16}$$

and $\Gamma \in \mathbb{R}^{p\times p}$ is a constant, diagonal, positive-definite, adaptation gain matrix. The elements of velocity error surrogate signal $y(t)$, defined in (14), are calculated via the following filter

$$\left\{\begin{aligned}\dot{p}_i &= -\left(1-(p_i - ke_i)^2\right)^2 (p_i - ke_i - \tanh(e_i)) \\ &\qquad - k(\tanh(e_i) + p_i - ke_i) \\ y_i &= p_i - ke_i\end{aligned}\right. \tag{17}$$

where $p_i(t) \in \mathbb{R}$ is an auxiliary variable with initial condition satisfying

$$-\frac{1}{\sqrt{n}} + ke_i(0) < p_i(0) < \frac{1}{\sqrt{n}} + ke_i(0). \tag{18}$$

### 3.2. Error System Development

We begin the formulation of the error systems by manipulating the two equations of (17) to produce the following filter dynamics

$$\dot{y}_i = -(1-y_i^2)^2(y_i - \tanh(e_i)) - k\eta_i,\ |y_i(0)| < \frac{1}{\sqrt{n}} \tag{19}$$

where (17) and (18) were used, and the filter tracking error variable $\eta(t) \in \mathbb{R}^n$ is defined as follows

$$\eta \triangleq \dot{e} + Tanh\,(e) + y. \tag{20}$$

To obtain the dynamics of $\eta(t)$, we differentiate (20) and premultiply both sides of the resulting equation by $M(q)$ to produce

$$\begin{aligned} M(q)\dot{\eta} = \;& M(q)\ddot{q}_d + V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} \\ & -\tau + M(q)Cosh^{-2}(e)\dot{e} + M(q)\dot{y} \end{aligned} \tag{21}$$

where (1) was utilized. After adding and subtracting $Y_d\theta$ of (7) to the right-hand side of (21), we can utilize (4), (12), (20), and (19) to rewrite the open-loop dynamics for $\eta(t)$ as follows

$$M(q)\dot{\eta} = -V_m(q,\dot{q})\eta + Y_d\theta - \tau - kM(q)\eta + \tilde{Y} + \chi \tag{22}$$

where the disturbance terms $\tilde{Y}(e,y,\eta,t)$, $\chi(e,y,\eta,t) \in \mathbb{R}^n$ are defined as follows

$$\tilde{Y} \triangleq M(q)\ddot{q}_d + V_m(q,q_d)\dot{q}_d + F_d\dot{q} + G(q) - Y_d\theta \tag{23}$$

$$\begin{aligned} \chi \triangleq \;& M(q)Cosh^{-2}(e)(\eta - Tanh(e) - y) \\ & -M(q)T\,(y - Tanh(e)) \\ & +V_m(q,\dot{q}_d + Tanh(e) + y)\,(Tanh(e) + y) \\ & +V_m(q,\dot{q}_d)\,(Tanh(e) + y) \\ & -V_m(q,\eta)\,(\dot{q}_d + Tanh(e) + y)\,. \end{aligned} \tag{24}$$

After substituting the control law (14) into (22), we obtain the following closed-loop dynamics for $\eta(t)$

$$\begin{aligned} M(q)\dot{\eta} = \;& -V_m(q,\dot{q})\eta + Y_d\tilde{\theta} + kT^{-1}y \\ & - Tanh(e) - kM(q)\eta + \tilde{Y} + \chi \end{aligned} \tag{25}$$

where (13) was used.

**Remark 1** *As illustrated in Appendix A, we can exploit the boundedness properties of the desired trajectory, (2), (5), and the properties of hyperbolic functions to show that $\chi(\cdot)$ of (24) can be upper bounded as follows*

$$\begin{aligned} \|\chi\| \le \;& \zeta_1\|x\| + \zeta_2\|y\|^2 + \zeta_3\|y\|^3 \\ & +\zeta_4\|y\|^4 + \zeta_5\|y\|^5 + \zeta_6\|\eta\|\|y\| \end{aligned} \tag{26}$$

*where $\zeta_i$, $i = 1,...,6$ are some positive constants that depend on the mechanical parameters and desired trajectory, and the composite state vector $x(t) \in \mathbb{R}^{3n}$ is defined as*

$$x \triangleq \left[\begin{array}{ccc} \eta^T & y^T & Tanh^T(e) \end{array}\right]^T. \tag{27}$$

*Furthermore, by utilizing (11), it can be shown that $\tilde{Y}(e,e_f,\eta,t)$ of (23) can be upper bounded as follows*

$$\left\|\tilde{Y}\right\| \le \zeta_7\|x\| \tag{28}$$

*where $\zeta_7$ is some positive constant that depends on the mechanical parameters and desired trajectory and $x(t)$ was defined in (27).*

Now, note that by differentiating (13) and then (16) with respect to time, we can form the following dynamics for the parameter estimation error

$$\dot{\tilde{\theta}} = -\Gamma Y_d^T\eta. \tag{29}$$

Finally, (20) can be rearranged to obtain the dynamics for $\dot{e}(t)$ as follows

$$\dot{e} = -Tanh\,(e) - y + \eta. \tag{30}$$

### 3.3. Stability Analysis

**Theorem 1** *Given the robot dynamics of (1), the adaptive controller of (14), (16), and (17) ensures the stability of the equilibrium $\left(\eta = 0, \tilde{\theta} = 0, y = 0, e = 0\right)$ and convergence to the set $\{\eta = 0, y = 0, e = 0\}$ in the region*

$$\left\{\left(\eta,\tilde{\theta},y,e\right) \in \mathbb{R}^n \times \mathbb{R}^p \times \left(-\frac{1}{\sqrt{n}},\frac{1}{\sqrt{n}}\right)^n \times \mathbb{R}^n\right\}, \tag{31}$$

*provided the control gain $k$ introduced in (14) and (17) is selected as follows*

$$\begin{aligned} k = \;& \tfrac{1}{m_1}\left(1 + k_n(\zeta_1+\zeta_7)^2 + 16\zeta_2^2\right. \\ & \left. +8\zeta_3^2 + 4\zeta_4^2 + 16\zeta_5^2 + \zeta_6\right) \end{aligned} \tag{32}$$

*where $m_1$ was defined in (2), $\zeta_i$, $i = 1,...,7$ were defined in (26) and (28), and $k_n$ is an additional control gain selected to satisfy the following sufficient condition*

$$k_n > 2. \tag{33}$$

*This means, in particular, that global (see Footnote 2 for definition of "global") asymptotic link position tracking is achieved.*

**Proof.** We start by defining the function

$$\begin{aligned} V(\eta,\tilde{\theta},y,e) \triangleq \;& \frac{1}{2}\eta^T M(q)\eta + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \\ & +\tfrac{1}{2}\textstyle\sum_{i=1}^n \frac{y_i^2}{1-y_i^2} + \sum_{i=1}^n \ln(\cosh(e_i))\,. \end{aligned} \tag{34}$$

With regard to the structure of (34), we note that $\ln(\cosh(e_i))$ is positive-definite and radially unbounded, and that the function $\frac{y_i^2}{1-y_i^2}$ is positive-definite and radially unbounded on the interval $[-1,1]$; hence, $V(t)$ of (34) is a positive-definite, radially unbounded function in the set

$$S \triangleq \left\{\left(\eta,\tilde{\theta},y,e\right) \in \mathbb{R}^n \times \mathbb{R}^p \times [-1,1]^n \times \mathbb{R}^n\right\}. \tag{35}$$

After taking the time derivative of (34) along (25), (19), (29), and (30), we obtain the following expression for $\dot{V}(t)$

$$\dot{V} = \eta^T\left(-kM(q)\eta + \tilde{Y} + \chi\right) - \sum_{i=1}^n y_i^2 - \sum_{i=1}^n \tanh^2(e_i) \tag{36}$$

where (3) was utilized. After substituting (2), (26), (28), and (32) into (36), we obtain

$$\begin{aligned} \dot{V} \le \;& -\|\eta\|^2 - \|y\|^2 - \|Tanh(e)\|^2 \\ & + \left[(\zeta_1+\zeta_7)\|x\|\|\eta\| - k_n(\zeta_1+\zeta_7)^2\|\eta\|^2\right] \\ & + \left[\zeta_2\|y\|^2\|\eta\| - 16\zeta_2^2\|\eta\|^2\right] \\ & + \left[\zeta_3\|y\|^3\|\eta\| - 8\zeta_3^2\|\eta\|^2\right] \\ & + \left[\zeta_4\|y\|^4\|\eta\| - 4\zeta_4^2\|\eta\|^2\right] \\ & + \left[\zeta_5\|y\|^5\|\eta\| - 16\zeta_5^2\|\eta\|^2\right] \\ & +\zeta_6(\|y\|-1)\|\eta\|^2\,. \end{aligned}$$

After completing the squares on the bracketed terms of (??) and making use of (27), we can place the following new upper bound on $\dot{V}(t)$

$$\begin{aligned} \dot{V} \;\le\; & -\frac{1}{2}\|x\|^2 + \frac{1}{k_n}\|x\|^2 \\ & +\frac{1}{2}\|y\|^2\left[-1 + \frac{1}{8}\|y\|^2 + \frac{1}{4}\|y\|^4 + \frac{1}{2}\|y\|^6 + \frac{1}{8}\|y\|^8\right] \\ & +\zeta_6\left[\|y\|-1\right]\|\eta\|^2\,. \end{aligned} \tag{37}$$

If $k_n$ is selected according to (33), then we can use (37) to state the following fact

$$\dot{V} \le -\beta\|x\|^2 \quad \text{if} \quad \|y(t)\| < 1\ \forall t \ge 0 \tag{38}$$

where $\beta$ is some positive constant which satisfies $0 < \beta < \frac{1}{2}$. After noting that $\|y\|^2 \leq n \max_i |y_i|^2$, we define the set

$$S_1 \triangleq \left\{ \left(\eta, \tilde{\theta}, y, e\right) \in \mathbb{R}^n \times \mathbb{R}^p \times \left(-\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)^n \times \mathbb{R}^n \right\}. \quad (39)$$

It now follows from (38) and (39) that

$$\dot{V} \leq -\beta \|x\|^2 \quad \text{if} \quad \left(\eta, \tilde{\theta}, y, e\right) \in S_1. \quad (40)$$

Since $S_1 \subset S$ where $S$ was defined in (35), the region of attraction will contain the largest level set of $V(t)$ inside the set $S_1$. Since all level sets of $V(t)$ are contained inside $S$, then $S_1$ is invariant and an estimate of the stability region. Hence, for initial conditions inside $S_1$, it is now easy to show that $x(t) \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and $\dot{x}(t) \in \mathcal{L}_\infty$. Application of Barbalat's lemma [16] now yields

$$\lim_{t \to \infty} x(t) = 0, \quad (41)$$

from which global asymptotic link position tracking (*i.e.*, $\lim_{t\to\infty} e(t) = 0$) directly follows due to the definition of (27) and the properties of the hyperbolic tangent function. □

**Remark 2** *Note that despite the fact that the initial conditions for the auxiliary signal $y(t)$ have been initialized to zero (i.e., see (17) and (19)), the stability result is still global for the link position tracking error signal $e(t)$ since no restrictions are placed on the size of $\|e(0)\|$ (note that no restriction are placed on the size of $\|\eta(0)\|$, $\left\|\tilde{\theta}(0)\right\|$, and $\|\dot{e}(0)\|$ as well).*

## 4. Experimental Results

The proposed, OFB adaptive controller was implemented on a two-link, direct-drive, planar robot manipulator built by Integrated Motion Inc. [13]. This robot is a torque-controlled system which utilizes special purpose electronic hardware to enable the implementation of torque input control algorithms. Specifically, the robot features links that are directly actuated by switched-reluctance motors which are controlled through Nippon Seiko K.K. drives. A Pentium 266 MHz PC running QNX (a real-time, micro-kernel based operating system) hosted the control algorithm. The control environment *Qmotor 3.0* [12, 25] was used to develop and tune the controllers. The Quanser MultiQ I/O board provided for data transfer between the computer subsystem and the robot.

As provided by the manufacturer, the two-link robot has the following dynamic model [13]

$$\begin{aligned} &\begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} \\ &+ \begin{bmatrix} -p_3s_2\dot{q}_2 & -p_3s_2(\dot{q}_1 + \dot{q}_2) \\ p_3s_2\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \\ &+ \begin{bmatrix} f_{d1} & 0 \\ 0 & f_{d2} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \end{aligned} \quad (42)$$

where $p_1 = 3.473$ [kg-m$^2$], $p_2 = 0.193$ [kg-m$^2$], $p_3 = 0.242$ [kg-m$^2$], $f_{d1} = 5.3$ [Nm-sec], $f_{d2} = 1.1$ [Nm-sec], $c_2 \triangleq \cos(q_2)$, and $s_2 \triangleq \sin(q_2)$. Based on (6) and (42), the unknown parameter vector $\theta$ can be constructed as

$$\theta = \begin{bmatrix} p_1 & p_2 & p_3 & f_{d1} & f_{d2} \end{bmatrix}^T. \quad (43)$$

The experiment was performed using the following desired position trajectory

$$q_d(t) = \begin{bmatrix} 57.30 \sin(t) \left(1 - \exp\left(-0.3t^3\right)\right) \\ 45.84 \sin(t) \left(1 - \exp\left(-0.3t^3\right)\right) \end{bmatrix} \text{[deg]} \quad (44)$$

where the exponential term was included to ensure that $\dot{q}_d(0) = \ddot{q}_d(0) = \dddot{q}_d(0) = 0$. To improve the controller performance during the implementation, some additional gains were introduced in the control law of (14), (16), and (17) resulting in the following controller[3]

$$\tau = Y_d\hat{\theta} - kT^{-1}y + \alpha_4 \, Tanh(e), \quad (45)$$

$$\hat{\theta} = \Gamma \int_0^t \left[ Y_d^T \left(\alpha_1 \, Tanh(e) + \alpha_2 y\right) - \dot{Y}_d^T e \right] d\sigma + \Gamma Y_d^T e, \quad (46)$$

and

$$\left\{ \begin{aligned} \dot{p}_i &= -\left(1 - (p_i - ke_i)^2\right)^2 \left(\alpha_3 \left(p_i - ke_i\right) - \alpha_2\alpha_4 \tanh\left(e_i\right) \right. \\ &\qquad \left. -k\left(\alpha_1 \tanh\left(e_i\right) + \alpha_2 \left(p_i - ke_i\right)\right)\right) \\ y_i &= p_i - ke_i \end{aligned} \right. \quad (47)$$

where $p(0) = ke_i(0)$ and $\alpha_i$, $i = 1, ..., 4$ are positive, scalar control gains. The control and adaptation gains were set to the following values[4]

$$\begin{aligned} &k = diag\{260, \ 84.64\}, \\ &\Gamma = diag\{12.6, \ 4.86, \ 2.26, \ 86.26, \ 12.4\} \\ &\alpha_1 = 36.126, \qquad \alpha_2 = 1.12, \\ &\alpha_3 = 78.64, \qquad \alpha_4 = 260. \end{aligned} \quad (48)$$

We note that all the above gains were tuned by trial-and-error until the best link position tracking performance was achieved. The parameter estimate $\hat{\theta}(t)$ was initialized to zero, and the experiment was run with a sampling frequency of 2 kHz.

The experimental results are shown in Figures 1-3. The link position tracking errors are depicted in Figure 1. Note that the steady-state error for links 1 and 2 stay within approximately 0.07 [deg] and 0.1 [deg], respectively. The parameters estimates are depicted in Figure 2 while the control torques are shown in Figure 3.

## 5. Conclusion

In this paper, we have presented a new global, output feedback, adaptive tracking controller for rigid-link robot manipulators. Through the new design, the proposed adaptive controller overcomes some potential drawbacks associated with the stability analysis used in [29]. That is, the closed-loop error system development and Lyapunov stability analysis originate from a velocity-independent version of the control strategy. As in [29], the proposed control scheme guarantees asymptotic link position tracking with no restrictions on the size of the initial position/velocity tracking error. Experimental results for a two-link, robot testbed were presented to illustrate the controller performance.

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[3] The modified controller of (45)-(47) will lead to some minor changes to the definition of $\eta(t)$ in (20) and to the filter dynamics of (19). However, the stability result delineated in Subsection 3.3. will still be valid.

[4] The stability analysis required that the gain $k$ be defined as a scalar; however, notice that $k$ was defined as a matrix during the control implementation. Although we cannot theoretically justify this modification, we have verified from experience that it usually improves the tracking performance in numerical simulations and real-time implementations.

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## Appendix A. Proof of Bounds for $\chi$ and $\tilde{Y}$

In this appendix, we illustrate that the bounds given by (26) and (28) are valid. We start by using the expression given in (24) to upper bound $\|\chi\|$ as follows

$$\begin{aligned}\|\chi\| \leq\ & m_2\left(\|\eta\|+\|Tanh(e)\|+\|y\|\right)\\ &+m_2\|T\|\left(\|y\|+\|Tanh(e)\|\right)\\ &+\zeta_{c1}\left(\|\dot{q}_d\|+\|Tanh(e)\|+\|y\|\right)\left(\|y\|+\|Tanh(e)\|\right)\\ &+\zeta_{c1}\|\dot{q}_d\|\left(\|y\|+\|Tanh(e)\|\right)\\ &+\zeta_{c1}\|\eta\|\left(\|\dot{q}_d\|+\|Tanh(e)\|+\|y\|\right)\end{aligned} \tag{49}$$

where (2) and (5) were utilized. From the definition of $T(\cdot)$ given in (15), we note that

$$\|T\| \leq \left(1+\|y\|^2\right)^2. \tag{50}$$

After substituting (50) into (49), using the fact that $|\tanh(x)| \leq 1$, $\forall x \in \mathbb{R}$, and making use of (27), it is not difficult to reach the result given in (26).

To prove (28), we substitute (7) into (23) to produce

$$\begin{aligned}\tilde{Y} =\ & \left(M(q)-M(q_d)\right)\ddot{q}_d+\left(V_m(q,\dot{q}_d)-V_m(q_d,\dot{q}_d)\right)\dot{q}_d\\ &+F_d(q-q_d)+G(q)-G(q_d).\end{aligned} \tag{51}$$

After applying (5) and (11) to (51), an upper bound can be placed on $\left\|\tilde{Y}\right\|$ as follows

$$\begin{aligned}\left\|\tilde{Y}\right\| \leq\ & \|\ddot{q}_d\|\left(\zeta_m\|\mathrm{Tanh}\,(e)\|\right)+\zeta_{c2}\|\dot{q}_d\|\|\mathrm{Tanh}\,(e)\|\\ &+\zeta_f\left(\|\eta\|+\|Tanh(e)\|+\|y\|\right)+\zeta_g\|\mathrm{Tanh}\,(e)\|.\end{aligned} \tag{52}$$

The result of (28) now follows directly from (52) due to the definition of (27) and the assumptions on the boundedness of $\dot{q}_d(t)$ and $\ddot{q}_d(t)$.

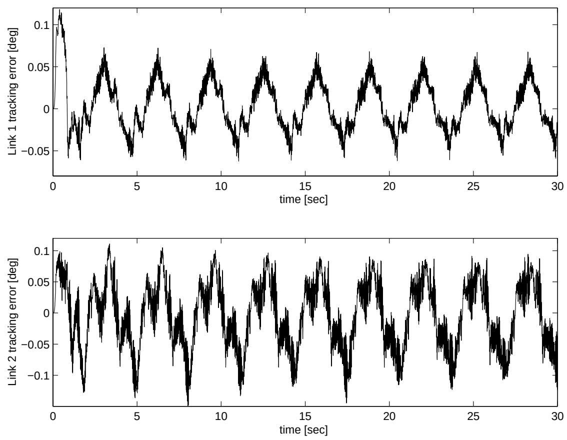


Figure 1 – Experimental results: link position tracking error $e(t)$.

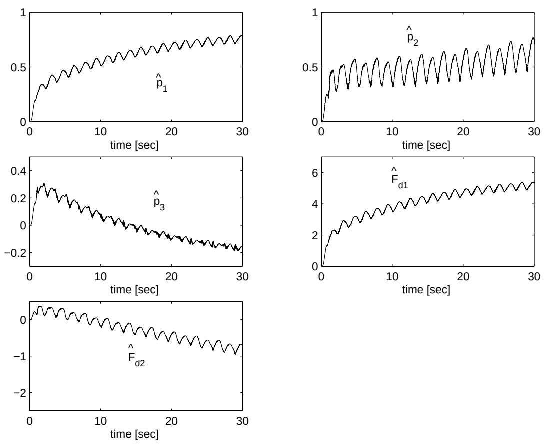


Figure 2 – Experimental results: parameter estimate $\hat{\theta}(t)$.

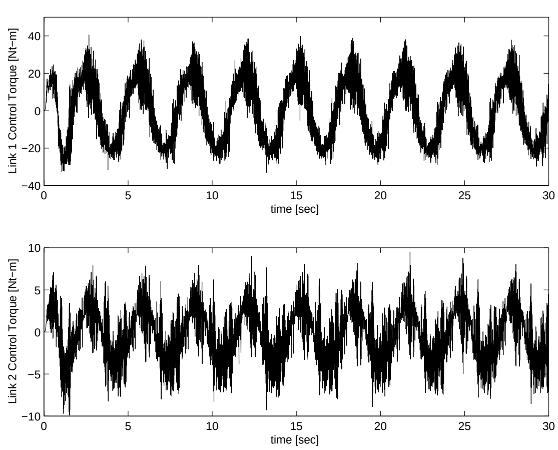


Figure 3 – Experimental results: torque input control $\tau(t)$.