Least-Squares Identification of Dynamic Systems in Closed Loop

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Abstract

The bias-eliminated least-squares (BELS) methods have been recently proposed as the indirect approach to perform unbiased parameter estimation of closed-loop systems subject to colored noise. This paper introduces a direct approach version of the BELS algorithm for identification of dynamic systems with an ARMAX model structure operating under linear feedback. Built upon linear regression and with no need to estimate parameters of the noise model, the developed algorithm is very attractive computationally while being able to yield open-loop plant parameter estimates with good accuracy. The performance of the developed BELS algorithm is corroborated with simulation results.

1 Introduction

The problem of identification of dynamic systems in closed-loop operation is of fundamental significance in many practical applications such as industrial, biological, and economical systems [2], [3]. Moreover, the interplay between closed-loop identification and control systems design has received considerable attention [4]. A recent study in [1] has shown that the direct closed-loop identification approach typically gives better accuracy than the indirect approach. However, use of the instrumental variable (IV) methods and the output error (OE) method may produce error-prone results due to the inherent correlation, caused by the feedback, between the process noise and the plant input. To circumvent this problem, the prediction error (PE) methods may be applied to consistently identify the plant with closed-loop data, provided that the noise model can describe the true noise properties. For instance, if the open-loop plant is represented by an ARMAX model, the PE (ARMAX) method (i.e. the PE method with an ARMAX model structure) is applicable. A drawback with the PE (ARMAX) method, however, is that it involves implementation of a numerically costly iterative nonlinear optimization scheme and estimation of (somehow uninteresting) parameters of the noise model. The objective of this paper is to develop a direct approach version of the bias-eliminated least-squares (BELS) algorithm (see [5]) for identification of ARMAX systems in closed-loop operation.

2 System Description

Assume that the open-loop plant to be identified is described by an ARMAX model

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + C(q^{-1})w(t) \qquad (1)$$

where $y(t)$ stands for the plant output, $u(t)$ the plant input, and d (≥ 1) the plant time delay. Moreover,

$$e(t) = C(q^{-1})w(t) \qquad (2)$$

represents the stochastic disturbance exerted on the plant, where the source of the disturbance, $w(t)$, is assumed to be a zero-mean white noise sequence. In (1), $\deg A(q^{-1}) = n_a$, $\deg B(q^{-1}) = n_b$ and $\deg C(q^{-1}) = n_c$. The plant is further assumed to be stabilized by a regulator. Another assumption is that

$$d \geq nc + 1. \qquad (3)$$

Note that this assumption is not restrictive, and it is closely related to the issue of identifiability. In fact, it is shown in [3] that when the PE method is applied to ARMAX systems operating under general linear feedback, identifiability is secured only if the time delay and/or the order of the regulator are large enough. Define the plant parameter vector and the regression vector respectively as

$$\boldsymbol{\theta}^\top = [\boldsymbol{a}^\top;\ \boldsymbol{b}^\top] = [a_1\ \dots\ a_{na};\ b_0\ b_1\ \dots\ b_{nb}] \qquad (4)$$

$$\begin{aligned}\boldsymbol{\phi}_t^\top = [\boldsymbol{y}_t^\top; \boldsymbol{u}_t^\top] = [&y(t-1)\ \dots\ y(t-na);\\ &u(t-d)\ u(t-d-1)\ \dots\ u(t-d-nb)]. \end{aligned}\qquad (5)$$

3 A Bias Correction Algorithm

Step 1. Design a nath-order stable prefilter $F(q^{-1})$, which gives the relation

$$\overline{\boldsymbol{\theta}} = \boldsymbol{M}\boldsymbol{\theta} \qquad (6)$$

where

$$\overline{nb} = nb + na \qquad (7)$$

$$\overline{\boldsymbol{\theta}}^\top = [\boldsymbol{a}^\top; \overline{\boldsymbol{b}}^\top] = [a_1\ \dots\ a_{na};\ \overline{b}_0\ \overline{b}_1\ \dots\ \overline{b}_{\overline{nb}}] \qquad (8)$$

$$\boldsymbol{M} = \mathrm{diag}\{\boldsymbol{I}_{na}; \boldsymbol{F}\} \in \mathbb{R}^{(na+\overline{nb}+1)\times(na+nb+1)} \quad (9)$$

and $\boldsymbol{F} \in \mathbb{R}^{(\overline{nb}+1)\times(nb+1)}$ is a Sylvester matrix spanned by the coefficients of $F(q^{-1})$. Then evaluate the $(na + \overline{nb} + 1) \times na$ full-column-rank matrix $\boldsymbol{H}$, which consists of the first na linearly independent columns of

$$\boldsymbol{W} = \boldsymbol{I}_{na+\overline{nb}+1} - \boldsymbol{M}(\boldsymbol{M}^\top \boldsymbol{M})^{-1}\boldsymbol{M}^\top. \quad (10)$$

Step 2. Compute the augmented open-loop plant parameter LS estimate:

$$\hat{\overline{\boldsymbol{\theta}}}_{LS}(N) = \hat{\boldsymbol{R}}_{\overline{\phi}\overline{\phi}}^{-1}(N)\hat{\boldsymbol{R}}_{\overline{\phi}y}(N) \quad (11)$$

where

$$\hat{\boldsymbol{R}}_{\overline{\phi}\overline{\phi}}(N) = \frac{1}{N}\sum_{t=1}^{N} \overline{\boldsymbol{\phi}}_t \overline{\boldsymbol{\phi}}_t^\top, \ \ \hat{\boldsymbol{R}}_{\overline{\phi}y}(N) = \frac{1}{N}\sum_{t=1}^{N} \overline{\boldsymbol{\phi}}_t y(t) \quad (12)$$

$$\overline{\boldsymbol{\phi}}_t^\top = [\boldsymbol{y}_t^\top; \overline{\boldsymbol{u}}_t^\top] = [y(t-1) \ \dots \ y(t-na); \ \overline{u}(t-d) \ \overline{u}(t-d-1) \ \dots \ \overline{u}(t-d-\overline{nb})]. \quad (13)$$

Step 3. Calculate the noise covariance estimate:

$$\hat{\boldsymbol{R}}_{ye}(N) = (\boldsymbol{H}^\top \hat{\boldsymbol{R}}_{\overline{\phi}\overline{\phi}}^{-1}(N)\overline{\boldsymbol{D}})^{-1}\boldsymbol{H}^\top \hat{\overline{\boldsymbol{\theta}}}_{LS}(N). \quad (14)$$

Step 4. Compute the augmented open-loop plant parameter BELS estimate:

$$\hat{\overline{\boldsymbol{\theta}}}_{BELS}(N) = \hat{\overline{\boldsymbol{\theta}}}_{LS}(N) - \hat{\boldsymbol{R}}_{\overline{\phi}\overline{\phi}}^{-1}(N)\overline{\boldsymbol{D}}\hat{\boldsymbol{R}}_{ye}(N). \quad (15)$$

Step 5. Evaluate the underlying open-loop plant parameter BELS estimate:

$$\hat{\boldsymbol{\theta}}_{BELS}(N) = (\boldsymbol{M}^\top \boldsymbol{M})^{-1}\boldsymbol{M}^\top \hat{\overline{\boldsymbol{\theta}}}_{BELS}(N). \quad (16)$$

4 Simulations and Concluding Remarks

For simulation, the ARMAX model for an open-loop unstable plant is given by

$$y(t) - 1.4y(t-1) = u(t-2) + 0.5u(t-3) + w(t) - 1.1w(t-1) \quad (17)$$

where the white noise $w(t)$ is with variance 4.46. With the reference input $v(t)$ being a zero-mean white noise sequence of unit variance, this open-loop unstable plant is then stabilized by a constant feedback $u(t) = v(t) - 0.35y(t)$, which results in a closed-loop system having its poles at $z = 0.8185 \pm 0.2610i$ and $z = -0.2371$. The signal-to-noise ratio (SNR) at the plant output is about 5dB, so the plant is corrupted by high noise.

For comparison, application of the following four algorithms in direct closed-loop identification of the above open-loop plant is considered: the conventional LS method, the OE method, the PE (ARMAX) method, and the developed BELS algorithm that employs a prefilter designed as $F(q^{-1}) = 1 - 0.9q^{-1}$. Note that since in the ARMAX model (17) $d = 2$ and $nc = 1$, the assumption (3) is satisfied for the use of the developed BELS algorithm. The simulations are conducted in the MATLAB environment, where the MATLAB codes `oe` and `armax` are used to implement the OE method and the PE (ARMAX) method, respectively. The performance of these four identification algorithms is compared in terms of the arithmetic means and standard deviations of the estimates, and the number of `flops`. Table 1 summarizes the simulation results obtained with 1000 sampled data points and 500 Monte-Carlo tests.

Table 1: Simulation Results
($N = 1000$, SNR = 5dB, 500 Monte-Carlo tests)

Method	LS	OE	PE	BELS
$a_1 = 1.4$	1.0654	0.8223	1.4002	1.3997
	± 0.0267	± 0.0519	± 0.0083	± 0.0441
$b_0 = 1.0$	0.3496	-1.7134	1.0013	0.9981
	± 0.0769	± 0.0978	± 0.0538	± 0.1308
$b_1 = 0.5$	0.4674	1.6505	0.4994	0.5002
	± 0.0563	± 0.1016	± 0.0365	± 0.0567
Gain	-17.0405	-0.2459	-3.7499	-3.8249
$= -3.75$	± 27.2914	± 0.4882	± 0.0252	± 0.5319
flops # per test	18156	805246	1044749	28723

One can clearly see that the developed BELS algorithm exhibits the superior performance. Its estimation accuracy is highly comparable to that of the PE (ARMAX) method, but its computational burden is only a tiny fraction (less than 3%) of that of the PE (ARMAX) method. Since the developed BELS algorithm can produce parameter estimates with good accuracy at a low numerical cost, it is reasonable to expect that this efficient algorithm will have great potential in practical applications where dynamic systems operating under feedback control are identified via the direct approach. Future work will consider the relation of the developed BELS algorithm to the IV methods in the framework of direct closed-loop identification.

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