Globally Exponential Stabilization of Switched Nonlinear Systems with Arbitrary Switchings

Z. G. Li, W. X. Xie, C. Y. Wen and Y. C. Soh
School of EEE, Nanyang Technological University
Nanyang Avenue, Singapore 639798
ECYWEN@ntu.edu.sg

Abstract

Switched nonlinear systems (SNS) are control systems that involve both continuous and discrete dynamics. The design of switched nonlinear systems is mainly composed of two parts. One is to design a smooth controller for each continuous subsystem such that each subsystem satisfies certain given requirements, the other is to define a switching law for the continuous controllers such that the overall switched nonlinear system achieves some given specifications. The continuous controllers, together with the corresponding switching law, form a switched controller. In this paper, we consider the problem of globally exponential stabilization of switched nonlinear systems with arbitrary and infinite number of "switchings". We handle both the case where the switchings of the system coincide exactly with the switchings of the controllers and the case where the switchings of the system do not coincide exactly with the switchings of controllers. An SNS in the first case and in the second case is called a synchronous SNS and an asynchronous SNS, respectively. Methods are presented to design switched controllers in both cases. Some sufficient conditions are derived to globally exponentially stabilize the system in both cases. However, emphasis is placed on the second case.

1 Introduction

A switched nonlinear system (SNS) is a collection of continuous variable dynamic subsystems (CVDSs) along with some maps for "switchings" among them [1]. Typical examples of such systems include computer disk drives [2], transmission and stepper motors [3], constrained robotic systems [4], intelligent vehicle/highway systems [5], continuous caster mills [6], and so on. These "switchings" occur whenever the states of some CVDSs satisfy certain conditions which are described by their membership in the specified subset of the state space. The "switchings" perform a reset to the subsystem of the system and change the continuous states of the CVDSs.

The stabilization of switched linear system has been investigated by many researchers. Morse et al. in [7, 8] and Li et al. in [9] studied the stabilization problems of switched linear systems for both the case where the switchings of the controllers coincide exactly with the switchings of the system, i.e. the case of synchronization, and the case where the switchings of the controllers do not coincide exactly with the switchings of the system, i.e. the case of asynchronization. However, the stabilization of switched nonlinear systems has not been well studied, especially in the later case. As pointed out in [10], it is difficult to to design a switched controller for a switched nonlinear system because of possibility of finite escape time. That is, if a wrong controller is used for a specified amount of time, the solution to the system might escape to infinity before a correct controller is switched into action [10].

In this paper, we present an approach to overcome the problem mentioned above and achieve globally exponential stabilization of switched nonlinear systems by designing a switched controller. The design is mainly composed of two parts. One is to design a smooth controller for each subsystem such that it is globally exponentially stable. The other is to define a switching law of the controllers such that the overall SNS is also globally exponentially stable. Since the first part has been well studied [11], we will only consider the second part. Both a synchronous SNS and an asynchronous SNS are considered but we shall emphasize on the later one. In practice, we may not know the initial subsystem and its subsequent subsystems of the system in advance. So, we do not know which controller should be initially used, and which controller and when it should be switched into action. To solve these problems, we propose a scheme to identify the initial subsystem and the subsequent subsystems, and define a switching law for the controllers. The results of this paper show that any arbitrary switching scheme would yield a stable system, and at the same time achieves some specified decay rate. This is very important because we can get a handle on the types of "overshoot" behavior.

The rest of the paper is organized as follows. In the following section, the problem formulation is given. A synchronous SNS is considered in section 3. Section 4 handles an asynchronous SNS. A numerical example is given in section 5 to illustrate the applications of the main results. Finally, concluding remarks are given in section 6.

2 Problem Formulation

In this paper, we consider the globally exponential stabilization of the following switched nonlinear system.

$$\dot{X}(t) = f(X(t), u(t), m(t)) \tag{1}$$

where $X(t) \in R^r$ is the continuous state, $u(t) \in R^q$ is the input, $m(t) \in \bar{M} = \{1, \cdots, n\}$ is the discrete state and is left continuous with each i corresponding to a vector field $f(\cdot,\cdot,i)$ and the switchings of $m(t)$ are arbitrary. $f : R^r \times R \times \bar{M} \to R^r$ are continuously differentiable vector fields satisfying $f(0,0,i) = 0$. This implies that $X_e = 0$ is an equilibrium of the closed-loop switched nonlinear system.

We shall first introduce three basic definitions.

Definition 1. *A switched nonlinear system is locally exponentially stable if there exist $k > 0, \theta > 0$ and $\gamma > 0$, such that when $\|X(t_0)\| \le \theta$, we have*

$$\|X(t)\| \le ke^{-\gamma(t-t_0)}\|X(t_0)\| \ ; \ \forall t \ge t_0$$

If $\theta = \infty$, then the system is globally exponentially stable. □

Definition 2. *A switched nonlinear system is said to be locally (globally) exponentially stabilizable, if there exist a feedback law $u(t) = u^*(X(t), m(t))$ and a corresponding switching law, such that in the corresponding closed–loop system, the equilibrium $X = 0$ is locally (globally) exponentially stable.* □

Definition 3. *A logical path in switched nonlinear system (1) is a sequence of discrete states $m(t^s_{j_1})$,$m(t^s_{j_1+1})$, $\cdots$, $m(t^s_{j_1+k})$ with t^s_k being the kth switching instant of the system. A finite logical path $m(t^s_{j_1}), m(t^s_{j_1+1}), \cdots$, $m(t^s_{j_1+k})$ is closed if $m(t^s_{j_1}) = m(t^s_{j_1+k})$. A closed logical path $LC = m(t^s_{j_1}), m(t^s_{j_1+1}), \cdots, m(t^s_{j_1+k})$ in which no discrete state appears more than once except for the first and the last one is a cycle.*

Here, we consider the class of SNSs satisfying the following assumptions:

Assumption 1. *Each subsystem can be globally exponentially stabilized. That is, there exist a smooth controller $u^*(X(t), i)$ with $u^*(0, i) = 0$ and a positive definite and proper function $V(X(t), i)$ such that [12, 11]*

$$c_1(i)\|X(t)\|^2 \le V(X(t), i) \le c_2(i)\|X(t)\|^2$$
$$\frac{\partial V(X(t), i)}{\partial X(t)} f(X(t), u^*(X(t), i), i) \le -c_3(i)\|X(t)\|^2$$

where $c_1(i), c_2(i)$ and $c_3(i)$ are positive numbers.

Assumption 2. *The continuous state $X(t)$ is available for measurement.*

Assumption 3. *For each subsystem i, let $t^{0,j}_i$ and $t^{f,j}_i$ denote respectively the jth starting time and the jth ending time of subsystem i. Suppose that*

$$\inf_j \{t^{f,j}_i - t^{0,j}_i\} = \Delta T_i > 0. \tag{2}$$

Further, $\Delta T_i (1 \le i \le n)$ are large enough such that there exist $0 < \tilde{\lambda}(i) < \frac{c_3(i)}{c_2(i)} (1 \le i \le n)$ satisfying that for $j = 1, 2, \cdots, n, l = 1, 2, \cdots, n, j \ne l$

$$\frac{c_2(j)}{c_1(j)} e^{-2\tilde{\lambda}(j)\Delta T_j} \frac{c_2(l)}{c_1(l)} e^{-2\tilde{\lambda}(l)\Delta T_l} < 1. \tag{3}$$

Assumption 4. *For any $t > 0$, any $\Delta T' > 0$ and any input $u(t)$, there exists only one subsystem $i \in \bar{M}$ such that*

$$X(\tau) = X_{s_i}(\tau) \ ; \ \tau \in [t, t + \Delta T'] \tag{4}$$

where $X(t)$ is the state of system (1) under the control of $u(t)$, and $X_{s_i}(t)$ is the state of subsystem i (which runs off-line on a parallel computer) under the control of $u(t)$. The original system and each subsystem are started from the same initial state.

Remark 1. Since the switching of the subsystems are arbitrary and the number of switchings is infinite, a simple case of cycle, j, l, j ($j \in \{1, 2, \cdots, n\}, l \in \{1, 2, \cdots, n\}, j \ne l$), can arise. Based on Assumption 3, the Lyapunov functions are non-increasing along cycle j, l, j for a synchronous SNS. Under this assumption, the Lyapunov function is nonincreasing along any type of cycle possibly arise in a synchronous SNS. Thus, it is globally exponentially stable with respect $X_e = 0$ in the sense of Lyapunov. However, the switching law of the controllers for an asynchronous SNS is required to be designed such that the asynchronous SNS is also globally exponentially stable with respect $X_e = 0$ in the sense of Lyapunov. This requirement was also used in [13, 10] and it is quite a common assumption. □

Remark 2. Note that $u^*(0, i) = 0$ holds for all i. If there exists a $t \ge t_0$ such that $X(t) = 0$, then we no longer switch the controllers after t. Furthermore, from then on, the state $X(t)$ will always stay at $X_e = 0$. Obviously, the whole system is globally exponentially stabilized. Thus, without loss of generality, we suppose that $X(t) \ne 0$ holds for any $t \ge t_0$ in the rest of this paper. □

The objective: The objective is to design a switched controller $u(t) = u^*(X(t), i)$, which is composed of n continuous controllers $u^*(X(t), i)(1 \le i \le n)$ and the corresponding switching law of the controllers, such that system (1) is globally exponentially stable under the control of the switched controller.

In the following section, we shall consider a synchronous SNS. The design of an asynchronous SNS will be studied in Section 4.

3 Synchronous Switched Nonlinear System

In order to obtain fast and accurate responses of the overall system, appropriate controllers should be designed and stored for different subsystems. At any time instant, the controller corresponding to the active subsystem should be used. Therefore, if the active subsystem can be known exactly at any time, the switchings of the controllers can coincide exactly with those of the system. Let t_k^s and t_k^c denote the kth switching instants of system (1) and the controllers, respectively.

For this ideal case, we have the following result.

Theorem 1. *A synchronous SNS (1) satisfying Assumptions 1-3 is globally exponentially stable and the state trajectory satisfies*

$$\|X(t)\|^2 \le \max_{1\le i\le n} \frac{c_3^2(i)}{c_2^2(i)} e^{-2\hat{\lambda}(t-t_0)} \|X(t_0)\|^2 \; ; \; \forall t \ge t_0 \quad (5)$$

where

$$\hat{\lambda} = \min_{1\le i\le n} (\frac{c_3(i)}{c_2(i)} - \tilde{\lambda}(i))$$

4 Asynchronous Switched Nonlinear Systems

In practice, the switchings of the controllers may not coincide exactly with those of the system since we cannot know the initial subsystem and the subsequent subsystems of the system in advance. In other words, we may not know the active subsystem exactly at any moment. Thus, it is necessary to identify the initial subsystem and its subsequent subsystems and estimate the switching instances of the system such that we can know which controller should be initially used, which subsequent controller and when it should be switched into action.

In this case, certain amount of delay is imposed on the switching instances of the controllers. Such delay is necessary for identifying the initial subsystem and the next subsystem of the system and determining the corresponding controller.

Before presenting the method in details, we shall define two design parameters as follows.

$$\Delta T_0 = \min\{ \min_{1\le i\le n} \{\Delta T_i/3\}, \min_{1\le i,j\le n} \{ \frac{2\tilde{\lambda}(i)\Delta T_i + 2\tilde{\lambda}(j)\Delta T_j - \ln(\frac{c_3(i)}{c_2(i)}) - \ln(\frac{c_3(j)}{c_2(j)})}{10 \max_{1\le i\le n} \tilde{\lambda}(i)} \}\} \quad (6)$$

$$\gamma = \min_{1\le i,j\le n} e^{(\frac{c_3(i)}{c_2(i)} - \tilde{\lambda}(i))(\Delta T_i - 2\Delta T_0) + (\frac{c_3(j)}{c_2(j)} - \tilde{\lambda}(j))(\Delta T_j - 2\Delta T_0)} \quad (7)$$

They have the following property.

Property 1.

$$\gamma > 1 \quad (8)$$

$$\Delta T_0 > 0 \quad (9)$$

$$\frac{c_3(i)}{c_2(i)} e^{-2\tilde{\lambda}(i)(\Delta T_i - 2\Delta T_0)} \frac{c_3(j)}{c_2(j)} e^{-2\tilde{\lambda}(j)(\Delta T_j - 2\Delta T_0)} \le 1, \quad \forall i,j \quad (10)$$

$$\gamma e^{-(\frac{c_3(i)}{c_2(i)} - \tilde{\lambda}(i))(\Delta T_i - 2\Delta T_0) - (\frac{c_3(j)}{c_2(j)} - \tilde{\lambda}(j))(\Delta T_j - 2\Delta T_0)} \le 1, \quad \forall i,j \quad (11)$$

$$\Delta T_i - 2\Delta T_0 > 0, \forall i \quad (12)$$

Now, we present a method to identify the initial subsystem and the subsequent subsystems and determine the switching instances of the controllers. The method is mainly composed of three steps:

Step 1: Identify the initial subsystem.
Step 2: Estimate each switching instant of the system.
Step 3: Impose some delays on the switchings of the controller such that the next subsystem can be identified by using the knowledge within the period of delay.

They are given in detail as follows.

Step 1 Identify the initial subsystem.

Since we do not know the initial subsystem, we cannot decide which controller can be used to stabilize the system. Thus, we do not apply any input to system (1) when identifying the initial subsystem. This idea has also been used in the design of linear systems with communication constraints [14].

Let t_i^{se} denote the estimation of the ith switching instant of the system. Since the initial time t_0 of the system is known, we let $t_0^{se} = t_0$. Note that all subsystems of the SNS are known, so they can run off-line in a parallel computer. Moreover, all subsystems can parallel with the original SNS. Apply control $u(t) = 0$ to switching nonlinear system (1) and each subsystem i, and let the original system and all its subsystems be started from the same initial state, i.e. $X_{s_i}(t_0) = X(t_0) \ne 0 (1 \le i \le n)$. Denote

$$\bar{t}_0 = \sup_t \{ t_0^{se} \le t \le t_0^{se} + \Delta T_0 | \, \|X(t)\| \le \sqrt{\gamma}\|X(t_0)\| \} \quad (13)$$

Note that $\bar{t}_0$ has the following property.

Property 2. *$\bar{t}_0$ is well defined and $t_1^s > \bar{t}_0 > t_0^{se}$.*

From Assumption 4, we know that there exists only one subsystem whose state can match the state of system (1) exactly within the interval $[t_0, \bar{t}_0]$. This subsystem is the initial subsystem. Then, the corresponding controller is used as the initial controller from $\bar{t}_0$. In other words, $t_0^c = \bar{t}_0$.

Steps 2 and 3: These two steps will be carried out together alternatively.

We shall first estimate the first switching instant of the system.

Similar to identifying the initial subsystem, $u(t) = u^*(X(t), m(t_0^c))$ is applied to system (1) and each subsystem. Moreover, the starting state of all its subsystems is the same as the state of the original system, $X(t_0^c)$. This is possible from Assumption 2. Using Assumption 4, we have

$$X(t) = X_{s_{m(t_0^c)}}(t); \quad t \in [t_0^c, t_1^{se}]$$

where t_1^{se} is the estimate of the first switching instant of the system, and

$$\left.\begin{aligned} t_1^{se} = \sup_t \Big\{ & t > t_0^c | X(t) = X_{s_{m(t_0^c)}}(t) \ \ and \\ \|X(t)\|^2 \le & \frac{c_2(m(t_0^c))}{c_1(m(t_0^c))} e^{-2\frac{c_3(m(t_0^c))}{c_2(m(t_0^c))}(t-t_0^c)} \|X(t_0^c)\|^2 \end{aligned}\right\} \quad (14)$$

t_1^{se} has the following property.

Property 3.

$$t_1^s \le t_1^{se} < t_1^s + \Delta T_0$$

We shall now identify the second subsystem. Let

$$\left.\begin{aligned} \bar{t}_1 = \sup_t \{ & t_1^{se} \le t \le t_1^{se} + \Delta T_0 | \, |X(t)| \le \\ & \sqrt{\gamma} \frac{c_2(m(t_0^c)}{c_1(m(t_0^c))} e^{-2\frac{c_3(m(t_0^c))}{c_2(m(t_0^c))}(t-t_0^c)} \|X(t_0^c)\|^2 \end{aligned}\right\} \quad (15)$$

It can be shown that $\bar{t}_1$ has the following property.

Property 4. *$\bar{t}_1$ is well defined and $t_2^s > \bar{t}_1 > t_1^{se}$.*

Let $t_1^c = \bar{t}_1$. From Assumption 4, the subsystem in $[t_1^s, t_2^s]$ can be determined by using the state knowledge of the system (1) and each subsystem within $[t_1^{se}, t_1^c]$. Then, the controller used in $[t_1^c, t_2^c]$ is the controller corresponding to the subsystem in $[t_1^s, t_2^s]$.

Repeating the above procedures, the estimated switching instant of the system t_k^{se} and switching instant of the controller $\bar{t}_k$ are given to be

$$\begin{aligned} & t_k^{se} = \sup_t \{ t > t_{k-1}^c | X(t) = X_{s_{m(t_{k-1}^c)}}(t); \|X(t)\|^2 \le \\ & \frac{c_2(m(t_{k-1}^c))}{c_1(m(t_{k-1}^c))} e^{-2\frac{c_3(m(t_{k-1}^c))}{c_2(m(t_{k-1}^c))}(t-t_{k-1}^c)} \|X(t_{k-1}^c)\|^2 \}, \quad (16) \\ & \bar{t}_k = t_k^c = \sup_t \{ t_k^{se} + \Delta T_0 \ge t \ge t_k^{se} | \|X(t)\|^2 \le \\ & \sqrt{\gamma} \frac{c_2(m(t_{k-1}^c))}{c_1(m(t_{k-1}^c))} e^{-2\frac{c_3(m(t_{k-1}^c))}{c_2(m(t_{k-1}^c))}(t-t_{k-1}^c)} \|X(t_{k-1}^c)\|^2 \}, (17) \end{aligned}$$

where $k = 1, 2, \cdots$.

Now, we can summarize the process on how to obtain the switching law of the controllers as follows.

1. From (6) and (7), we define an ΔT_0 and a γ.
2. Identify the initial subsystem of the system and let $t_0^c = \bar{t}_0$. Set $k = 1$.
3. From (16), we get t_k^{se}.
4. From (17), we further obtain $\bar{t}_k$.
5. The subsystem in $[t_k^s, t_{k+1}^s]$ is determined by using the state knowledge of the original SNS and each subsystem within $[t_k^{se}, \bar{t}_k]$.
6. Let $t_k^c = \bar{t}_k$ and the controller used in $[t_k^c, t_{k+1}^c]$ be the controller corresponding to the subsystem determined as above.
7. Let $k = k + 1$ and go back to 3.

We shall now analyze the proposed switching law.

Lemma 1. *$\bar{t}_k (k = 1, 2, \cdots)$ are well defined and*

$$t_k^{se} < \bar{t}_k < t_{k+1}^s \ ; \ \forall k \quad (18)$$

$$t_k^s \le t_k^{se} \le t_k^s + \Delta T_0 \ ; \ \forall k \quad (19)$$

In other words, the proposed switching law is well defined.

Then, the main result of this paper can be presented as follows.

Theorem 2. *Consider an asynchronous SNS satisfying Assumptions 1–4 and the switching law of the controllers given in (16) and (17). Then, the system is globally exponentially stable in the sense that $\forall t \ge t_0$*

$$\|X(t)\|^2 \le \gamma^{3/2} e^{\Delta T_0 \hat{\lambda}} \max_{1 \le i \le n} \frac{c_3^2(i)}{c_2^2(i)} e^{-\hat{\lambda}(t-t_0)} \|X(t_0)\|^2. \quad (20)$$

Remark 3. A subsystem together with a smooth controller can be regarded as a mode of an asynchronous SNS. Note that some modes within $[t_k^s, t_k^c](k = 0, 1, 2, \cdots)$ can be unstable. This implies that an asynchronous SNS can be composed of some stable and unstable modes. However, almost all existing results [13, 1, 15, 16, 17] can only be used to study the stability of SNSs totally composed of stable modes and they required the Lyapunov function to be always non-increasing. Thus comparing with the results in this paper , all these results are conservative. □

5 A Numerical Example

In this section, we consider a SNS composed of three subsystems given as follows.

Subsystem 1:

$$\begin{aligned} \dot{X}_1(t) = & -X_1(t) + X_1(t)X_2(t)/240 + \\ & X_1(t)\sin(X_1(t)) + X_1^2(t) + X_1(t)u(t) \\ \dot{X}_2(t) = & -X_2(t) + \frac{X_2(t)\cos(X_1(t))}{2} - X_1^2(t)/240 + \\ & X_2(t)\sin(X_1(t)) + X_1(t)X_2(t) + X_2(t)u(t) \end{aligned}$$

Subsystem 2:

$$\dot{X}_1(t) = -X_1(t) + \frac{X_1(t)\sin(X_2(t))}{2} + X_2^2(t)/100 + (\cos(X_2(t)) - 1)\sin(2X_1(t)) + \sin(2X_1(t))u(t)$$
$$\dot{X}_2(t) = -X_2(t) - X_1(t)X_2(t)/50 + X_1(t)(\cos(X_2(t)) - 1) + X_1(t)u(t)$$

Subsystem 3:

$$\dot{X}_1(t) = -X_1(t)/2 + X_1(t)X_2^2(t) + X_1(t)X_2(t)u(t)$$
$$\dot{X}_2(t) = X_2^3(t) + (X_2^2(t) + 1/2)u(t)$$

and the duration time of each CVDS is given as follows.

$$\Delta T_1 = \Delta T_2 = \Delta T_3 = 0.5$$

Choose

$$\begin{aligned} u^*(X(t),1) &= -\sin(X_1(t)) - X_1(t) \\ V(X(t),1) &= X_1^2(t) + X_2^2(t) \\ u^*(X(t),2) &= 1 - \cos(X_2(t)) \\ V(X(t),2) &= 2X_1^2(t) + X_2^2(t) \\ U^*(X(t),3) &= -X_2(t) \\ V(X(t),3) &= X_1^2(t) + X_2^2(t) \end{aligned}$$

It can be shown that Assumption 1 holds with $c_1(1) = c_2(1) = c_3(1) = 1$, $c_1(2) = 1$, $c_3(2) = 2$, $c_3(2) = 2$ and $c_1(3) = c_2(3) = c_3(3) = 1$.

Choose

$$\tilde{\lambda}(1) = \tilde{\lambda}(2) = \tilde{\lambda}(3) = 0.4,$$

then, we have

$$\min\{\frac{c_3(1)}{c_2(1)} - \tilde{\lambda}(1), \frac{c_3(2)}{c_2(2)} - \tilde{\lambda}(2), \frac{c_3(3)}{c_2(3)} - \tilde{\lambda}(3)\} = 0.6$$

Note that

$$\begin{aligned} \frac{c_1(1)}{c_2(1)}e^{-\tilde{\lambda}(1)}\frac{c_1(2)}{c_2(2)}e^{-\tilde{\lambda}(2)} &< 1 \\ \frac{c_1(1)}{c_2(1)}e^{-\tilde{\lambda}(1)}\frac{c_1(3)}{c_2(3)}e^{-\tilde{\lambda}(3)} &< 1 \\ \frac{c_1(2)}{c_2(2)}e^{-\tilde{\lambda}(2)}\frac{c_1(3)}{c_2(3)}e^{-\tilde{\lambda}(3)} &< 1 \end{aligned}$$

This implies that Assumption 3 holds.

We shall first consider a synchronous SNS. Using Theorem 1, we know that the system is globally exponentially stable and the state satisfies (5) with $\hat{\lambda} = 3/10$. This can be illustrated as Figures 1 and 2.

We shall now consider an asynchronous SNS and calculate ΔT_0 and γ as follows.

$$\Delta T_0 = \min\{\frac{2\tilde{\lambda}(1)\Delta T_1 + 2\tilde{\lambda}(2)\Delta T_2 - \ln(\frac{c_2(1)}{c_1(1)}) - \ln(\frac{c_2(2)}{c_1(2)})}{4}, \frac{2\tilde{\lambda}(2)\Delta T_2 + 2\tilde{\lambda}(3)\Delta T_3 - \ln(\frac{c_2(2)}{c_1(2)}) - \ln(\frac{c_2(3)}{c_1(3)})}{4}, \frac{2\tilde{\lambda}(1)\Delta T_1 + 2\tilde{\lambda}(3)\Delta T_3 - \ln(\frac{c_2(1)}{c_1(1)}) - \ln(\frac{c_2(3)}{c_1(3)})}{4}\}$$

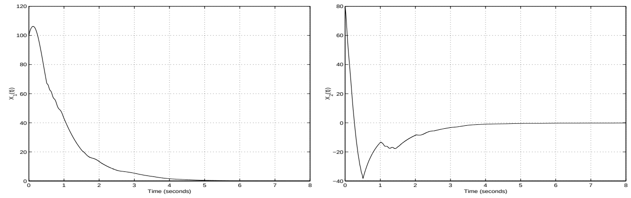

Figure 1: The time responses of the system state $X(t) = [X_1(t)\ X_2(t)]^T$.

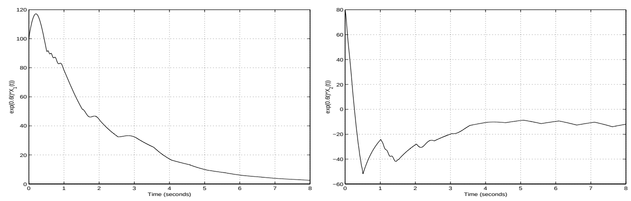

Figure 2: The time responses $e^{3t/5}X(t)$ of the system.

$$\begin{aligned} &= \frac{0.8 - \ln(2)}{4} \\ &= 0.0267 \end{aligned}$$

and

$$\begin{aligned} \gamma &= \min\{e^{0.4(\Delta T_1 - 2\Delta T_0) + 0.4(\Delta T_2 - 2\Delta T_0)}, e^{0.4(\Delta T_2 - 2\Delta T_0) + 0.4(\Delta T_3 - 2\Delta T_0)}, e^{0.4(\Delta T_1 - 2\Delta T_0) + 0.4(\Delta T_3 - 2\Delta T_0)}\} \\ &= 1.4294 \end{aligned}$$

Note that Assumption 3 is satisfied. By Theorem 2, the system can be globally exponentially stabilized by the following switching law.

$$\begin{aligned} t_0^c &= \bar{t}_0 = \{t \le 0.0267 \| \|X(t)\| \le 1.1956\|X(0)\|\} \\ t_j^c &= \bar{t}_j = \sup_t\{t_j^{se} \le t \le t_j^{se} + 0.0267 \| \|X(t)\| \le 1.1956\frac{c_2(m(t_{j-1}^c))}{c_1(m(t_{j-1}^c))}e^{-(t-t_{j-1}^c)}\|X(t_{j-1}^c)\|\} \end{aligned}$$

where the jth controller is the controller corresponding to the subsystem determined by the state knowledge within $[t_j^{se}, t_j^c]$, and

$$t_j^{se} = \sup_t\{t \ge t_{j-1}^c | X(t) = X_{s_{m(t_{j-1}^c)}}(t), \|X(t)\| \le \frac{c_2(m(t_{j-1}^c))}{c_1(m(t_{j-1}^c))}e^{-(t-t_{j-1}^c)}\|X(t_{j-1}^c)\|\}$$

For example, we consider the following simulation. The switched nonlinear system is composed of Subsystem 2 and Subsystem 3. Suppose that the initial subsystem is Subsystem 3. The simulation results are illustrated in Figures 3 and 4. Clearly, the closed-loop system is globally exponentially stable. It is necessary to note that Subsystem 3 cannot be stabilized under the control of $u(t) = 0$ and $u^*(X(t), 2)$. In other words, the following two modes are unstable.

$$\text{Unstable Mode 1:} \begin{cases} \dot{X}_1(t) = -X_1(t)/2 + X_1(t)X_2^2(t) \\ \dot{X}_2(t) = X_2^3(t) \end{cases}$$

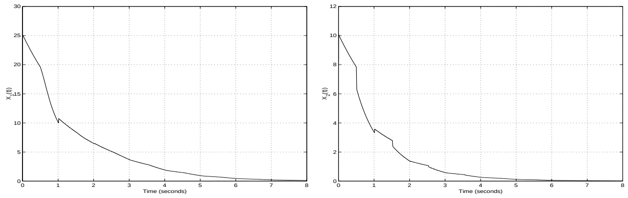

Figure 3: The time responses of the system state $X(t) = [X_1(t)\ X_2(t)]^T$.

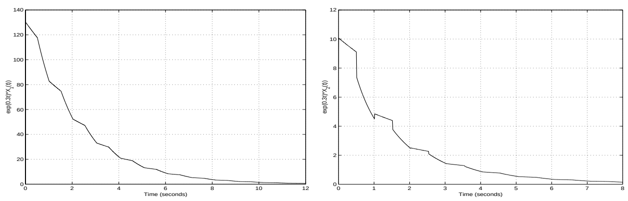

Figure 4: The time responses $e^{3t/10}X(t)$ of the system.

Unstable Mode 2:

$$\begin{cases} \dot{X}_1(t) = -X_1(t)/2 + X_1(t)X_2^2(t) + \\ \qquad\qquad X_1(t)X_2(t)(1-\cos(X_2(t))) \\ \dot{X}_2(t) = X_2^3(t) + (X_2^2(t) + 1/2)(1-\cos(X_2(t))) \end{cases}$$

6 Conclusion

In this paper, we have considered the globally exponential stabilization of switched nonlinear systems with arbitrary infinite "switchings". We handled both the case where the switchings of the system coincide exactly with the switchings of the controllers, i.e. the case of synchronization, and the case where the switchings of the system do not coincide exactly with the switchings of the controllers, i.e. the case of asynchronization. We presented an approach to design a switched controller for a switched nonlinear system in the second case. Some sufficient conditions were derived to globally exponentially stabilize the switched nonlinear system in both cases.

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