Robust global stabilization of an underactuated marine vehicle on a linear course by smooth time-invariant feedback

Giovanni Indiveri[‡], Michele Aicardi[†], Giuseppe Casalino[†]

[‡]GMD-AiS, German National Research Center on Information Technologies,
Institute for Autonomous Intelligent Systems, Schloss Birlinghoven,
53754 Sankt Augustin, Germany
giovanni.indiveri@gmd.de (corresponding author)

[†]DIST-Department of Communications Computers and Systems Science,
Università di Genova, via all'Opera Pia 13, 16145 Genova, Italy
{michele,pino}@dist.unige.it

Abstract

The stabilization of an underactuated marine vehicle on a linear course is considered. In spite of being controllable and that Brocketts Theorem does not prevent the existence of a smooth pure feedback control solution, standard tools of nonlinear control as feedback linearization cannot be applied. A smooth, time invariant, globally converging control solution is designed on the basis of a simple and novel idea to derive motion control laws for underactuated systems. Robustness with respect to model parameter uncertainty and state measurement errors is discussed both analytically and through simulations.

1 Introduction

The issue of stabilizing a marine system on a given linear course while traveling at fixed cruise surge speed is of fundamental importance in practical applications. The main difficulty in designing a closed loop steering law to achieve this objective is related to the underactuated nature of the marine vehicles. If the linear course to be stabilized is assumed to lie on the x axis of a fixed frame (see figure (1)), stabilizing the vehicle on it means stabilizing the heading ϕ, the sway speed v and the lateral distance y to zero having only two control inputs, namely the yaw torque τ_N and the surge speed u. As observed by K.Y.Pettersen and H.Nijmeijer [1] "tracking control of surface vessels have mainly been based on linear models, steering the same number of degrees of freedom as the number of control inputs available, and giving local results". Indeed traditional autopilots stabilize the heading only. On the other hand there are examples [2] of more elaborate control strategies that take environmental disturbances explicitly into account, but they fail to control in closed loop all the degrees of freedom. The main reason for which there are still only very few results regarding the solution to this problem is that the strongly nonlinear marine vehicle models are *not* feedback linearizable, thus one of the most effective tools of nonlinear control theory is totally ruled out and alternative design methods must be developed. In this respect most interesting is the approach suggested in [1] where a change of coordinates of the standard cartesian model is performed so that an integrator backstepping technique may be applied to design a time-invariant, continuous, semi-global state-tracking controller. This integrator backstepping approach was possible in spite of the non existence of any feedback linearizing transformation because the reference yaw velocity was assumed to be always non null. As a consequence this solution refers to the only case of non null curvature paths, i.e. it may *not* be applied to track a linear course which is by far the most common marine navigation requirement. A marine craft tracking solution for a wider class of curvature paths has been presented in [3].

In this paper attention is focused on the linear path tracking only and other important application-oriented issues as robustness to modeling and measurement errors are discussed. The control design methodology is based on a novel idea first suggested in [4] to solve the point stabilization problem of the unicycle kinematic model. It basically consists of two steps: (i) first a velocity vector field is defined such that an ideal point free from any nonholonomic constraint, i.e. able to move freely in all directions at each instant, would exponentially achieve the control objective by applying such velocity. (ii) then a steering law is designed such that the given underactuated vehicle is exponentially stabilized parallel to the above defined velocity vector field. It turns out that this simple control design technique is indeed effective in solving also the motion control problem at hand yielding a simple, time-invariant, globally stable solution which has a most clear physical interpretation.

Notice that the point stabilization, i.e. "parking", problem of a marine vehicle may not be solved via smooth, pure feedback control and either discontinuous [5] or time-varying [6] [7] solutions must be considered.

This paper extends the results presented in [8][9].

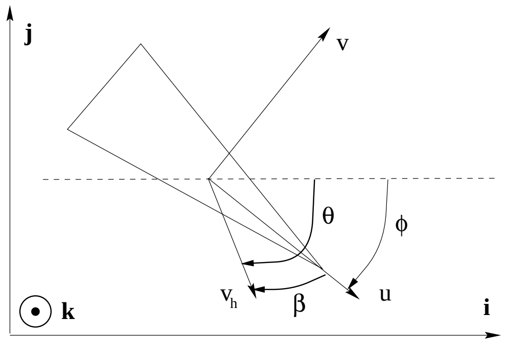

Figure 1: The Model

2 Model and problem definition

Consider the standard [6] underactuated marine vehicle model

$$\begin{aligned} m_{22}\dot{v} &= -m_{11}ur - k_v v - k_{v|v|}v|v| && (1)\\ \dot{y} &= v\cos\phi + u\sin\phi && (2)\\ \dot{\phi} &= r && (3)\\ m_{33}\dot{r} &= (m_{11}-m_{22})uv - k_r r - k_{r|r|}r|r| + \tau_N && (4)\\ u(t) &= u \text{ (constant) } > 0 \ \forall\, t && (5) \end{aligned}$$

being v, u, r the sway, surge and yaw velocities, m_{11}, m_{22}, m_{33} the surge, sway and yaw inertia including added mass, $k_v, k_{v|v|}, k_r, k_{r|r|}$ the linear and quadratic hydrodynamic drag coefficients for sway and yaw, ϕ the orientation of the vehicle with respect to the x axis of a fixed reference frame and y its $y-$coordinate with respect to the same frame (see figure (1)). τ_N is the applied yaw torque. The systems is assumed to move only forward at a non null, bounded surge (5).

2.1 Problem definition

Given the system described by equations (1-5) find a smooth time invariant closed loop control law for τ_N such that $(v, y, \phi, r)^T$ is globally exponentially stabilized to $(0, 0, 0, 0)^T$.

In a backstepping-like fashion, first a control law for the yaw velocity $r = \bar{r}(v, y, \phi)$ will be designed such that under its influence the vector $(v, y, \phi)^T$ is stabilized to zero and then a control law for τ_N will be derived such that the state r robustly converges to the previously defined "virtual" input $\bar{r}$.

Remark 1: The sub-system given by equations (1), (2), (3), (5) and having r as control input is controllable, *not* subject to Brocketts [10] theorem yet not input-state feedback linearizable [8]. ■

The fact that Brocketts Theorem does not prevent the existence of a smooth, pure feedback control law follows from the observation that the mapping $\gamma(\mathbf{x}, r) \longmapsto \dot{\mathbf{x}} : \mathbf{x} = (v, y, \phi)^T$ given by equations (1), (2) and (3) is onto $(0, 0, 0)^T$. The non existence of a feedback linearizing transformation is a known fact in the literature [5][7] and follows from the observation that if equations (1), (2) and (3) are written in standard nonlinear control form, i.e. $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \sum_{i=1}^{m} \mathbf{g}_i(\mathbf{x})u_i : m = 1, u_1 = r, \mathbf{x} = (v, y, \phi)^T$, then the necessary condition for the existence of a feedback linearization transformation, namely that the set $\{\mathbf{g}, [\mathbf{f}, \mathbf{g}]\}$ is involutive [11], is not fulfilled [8]. On the contrary as $\{\mathbf{g}, [\mathbf{f}, \mathbf{g}], [\mathbf{f}, [\mathbf{f}, \mathbf{g}]]\}$ span $\mathcal{R}^3$ the system is controllable [8].

The interesting and perhaps uncommon case is found of an underactuated system which is not prevented from having a pure feedback smooth stabilizing control solution by Brocketts Theorem, yet standard methods of nonlinear control as feedback linearization cannot be applied. Notice moreover that the control objective to achieve is of most interesting practical interest.

3 Stabilization of the reduced model

As stated above, first a control law for the yaw velocity r is searched such that $(v, y, \phi)^T$ is stabilized to zero. Consider the smooth velocity vector field:

$$\mathbf{v}_h = u\,\mathbf{i} - ky\,\mathbf{j} \ : \ k > 0 \qquad (6)$$

being u the vehicle surge speed (5), $\mathbf{i}$ and $\mathbf{j}$ the unit vectors of the x and y fixed reference axis as shown in figure (1) and k a positive constant gain. Denoting with θ the angle between $\mathbf{i}$ and $\mathbf{v}_h$, the following hold:

$$\begin{aligned} \mathbf{i}\cdot\mathbf{v}_h &= \|\mathbf{v}_h\|\cos\theta = \sqrt{u^2 + k^2y^2}\cos\theta = u && (7)\\ \mathbf{i}\wedge\mathbf{v}_h &= \mathbf{k}\,\|\mathbf{v}_h\|\sin\theta = \\ &= \mathbf{k}\,\sqrt{u^2 + k^2y^2}\sin\theta = -\mathbf{k}\,ky && (8) \end{aligned}$$

being $\mathbf{k} = \mathbf{i} \wedge \mathbf{j}$ the unit vector of the z axis of the fixed reference.

Remark 2: The vector field $\mathbf{v}_h$ defined by equation (6) is smooth and globally defined. By construction any ideal point moving with velocity $\mathbf{v}_h$ would be exponentially stabilized on the x axis of the fixed frame. Moreover form equations (5) and (7) it follows that $\theta \in (-\pi/2, \pi/2)$ $\forall\, t$ and $\forall$ finite y and $\theta = 0 \Leftrightarrow y = 0$. ■

From equations (2), (7) and (8) it follows that:

$$\dot{\theta} = -\frac{ku(u \sin\phi + v \cos\phi)}{u^2 + k^2 y^2} \tag{9}$$

thus the vehicle will be globally exponentially steered parallel to $\mathbf{v}_h$ with time constant $1/\gamma_\beta$ by the smooth velocity r:

$$r = \gamma_\beta\, \beta - \frac{ku(u \sin\phi + v \cos\phi)}{u^2 + k^2 y^2} \quad : \quad \gamma_\beta > 0 \tag{10}$$

being β the angle between the vehicles surge axis and $\mathbf{v}_h$ (refer to figure 1), i.e.

$$\beta = \theta - \phi.$$

Indeed equation (10) guarantees that β evolves in closed loop according to $\dot{\beta}$=-$\gamma_\beta\, \beta$.

3.1 Local exponential stability

By the very definition of r as given by equation (10) β converges globally and exponentially to zero, i.e. $\phi \to \theta$ globally and exponentially. It may be shown that given the structure of equation (10), the signals $v(t)$, $y(t)$ and $\phi(t)$ do *not* admit a finite escape time. Stability may be thus analysed in the limit $\beta \to 0$ that will always occur. Moreover from Remark 2 it follows that $y \approx 0 \Leftrightarrow \theta \approx 0$. As a consequence in the limit $\beta \to 0$, the local ($y \approx 0$) stability of the subsystems given by equations (1), (2), (3), (5) is guaranteed if $(v, \theta)^T \to (0, 0)^T$ under the influence of the control r (10) when $y \approx 0$, $\phi \to \theta \approx 0$. In such limits from equations (1), (2), (3), (5) and (10) it follows that the closed loop evolution of $(v, \theta)^T$ is governed by the linear time-varying (LTV) system:

$$\begin{pmatrix} \dot{v} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} (ak - b) - c|v| & aku \\ -k/u & -k \end{bmatrix} \begin{pmatrix} v \\ \theta \end{pmatrix} \tag{11}$$

$$a \equiv \frac{m_{11}}{m_{22}}; \quad b \equiv \frac{k_v}{m_{22}}; \quad c \equiv \frac{k_{v|v|}}{m_{22}}. \tag{12}$$

Denoting with $\mathbf{A}$ the matrix in equation (11), the exponential convergence and stability of $(v, \theta)^T$ to $(0, 0)^T$ is guaranteed if the eigenvalues of $1/2(\mathbf{A} + \mathbf{A}^T)$ are strictly negative at all times [11]. By direct calculation it follows that this condition is indeed satisfied if the following hold:

$$b \;>\; k(a - 1) \tag{13}$$

$$b \;>\; \frac{k}{4}\left(au + \frac{1}{u}\right)^2. \tag{14}$$

Notice that condition (13) is trivially satisfied in practical cases as $a < 1$ and $b, k > 0$. Condition (14) can be always satisfied by a proper choice of k and $u > 0$ for any value of the structural parameters a and b.

3.2 Global stability

Having proved that the proposed law guarantees local ($y \approx 0$) exponential stability of the origin, to prove global stability it is now sufficient to show that the proposed control law drives y towards zero from any initial configuration. This analysis may be limited to the case $\beta \to 0$ due to the already mentioned fact that the signals $v(t)$, $y(t)$ and $\phi(t)$ do *not* admit a finite escape time. Intuitively it appears that once that $\beta \to 0$ if the surge speed and sway drag are sufficient to overcome an eventual sway drift "pushing" the vehicle far from the y axis, the system should indeed approach the y axis starting from any initial configuration thus yielding global stability. This physical insight may be more formally expressed: consider the candidate Lyapunov function $V_y = 1/2y^2$ and its time derivative in the limit $\beta \to 0$, namely:

$$\lim_{\beta \to 0} \dot{V}_y = \lim_{\beta \to 0} y\dot{y} = yv\cos\theta - \frac{kuy^2}{\sqrt{u^2 + k^2 y^2}} \tag{15}$$

where equations (2) and (8) have been used. As $\cos\theta > 0 \quad \forall\, t$ (Remark 2) from equation (15) it follows that a sufficient condition to prove that y will globally asymptotically converge towards zero is that $\lim_{\beta \to 0} vy \le 0$. From equations (1), (10) and (12) consider the closed loop equation for $\dot{v}$ in the limit $\beta \to 0$, namely:

$$\lim_{\beta \to 0} \dot{v} = (au\mu \cos\theta - b - c|v|)v - au^2 \mu \frac{ky}{\sqrt{u^2 + k^2 y^2}} \tag{16}$$

where equation (8) has been used and the strictly positive function

$$\mu = \frac{ku}{u^2 + k^2 y^2} \quad : \quad \max_y \mu = \mu|_{y=0} = k/u \tag{17}$$

has been introduced. Notice that if $b > ak \;\Rightarrow\; au\mu\cos\theta - b - c|v| < 0 \;\; \forall\;\; \theta, v, y$ equations (15) and (16) imply that both y and v are bounded in the limit $\beta \to 0$.

Remark 3 If $b > ak$ then from equations (16) and (17) it follows that all the asymptotic extreme values $v_{ext} \;:\; \lim_{\beta \to 0} \dot{v}|_{v = v_{ext}} = 0$ of v have the opposite sign of y for some suitable μ, θ and y. ■

Assuming $b > ak$ consider the three hypothesis:

H1: $\exists \lim_{\beta \to 0} \operatorname{sgn}(v) = +1$ or -1

H2: $\exists \lim_{\beta \to 0} \operatorname{sgn}(v) = 0$

H3: $\nexists \lim_{\beta \to 0} \operatorname{sgn}(v)$

being the sgn function defined in the standard fashion, i.e. equal to the sign of its argument for non-null arguments and equal to zero otherwise.

If hypothesis H1 or H2 should hold true then it would follow that $\lim_{\beta\to 0} yv \le 0$ and, from equation (15), that $y \to 0$. This is due to the fact that if the sign of the continuous and bounded signal $v(t)$ should converge to a constant value (hypothesis H1) it would be the same one of its extreme points v_{ext} (Remark 3), i.e. opposite to the sign of y.
On the other hand if hypothesis H3 should be true, being v continuous and bounded, there would be a sequence of local extrema v_{ext} having opposite sign between each other, i.e. by the Remark 3 a sequence of configurations corresponding to values of y having opposite signs. Being the time evolution of the system continuous this is sufficient to prove that y will be globally driven arbitrarily close to zero (actually "across" zero if H3 should hold) where the local stability argument can be used to prove convergence. This concludes the proof. In particular the following may be stated:

Remark 4 If

$$b > \max\left\{ ak, \frac{k}{4}\left(au + \frac{1}{u}\right)^2 \right\}$$

the system given by equations (1), (2), (3), (5) is made globally asymptotically (locally exponentially) stable by the smooth control r given by equation (10). ■

The above is indeed the most significant result of this work: it could not have been found with feedback linearization or related standard nonlinear control methods. A different proof of stability and convergence of this solution may be found in [9]. Having obtained a smooth law for r that globally stabilizes the reduced model it is not difficult to design a robust controller for τ_N such that the yaw asymptotically copies such "reference" (10) thus stabilizing the complete model as requested.

4 Robust stabilization of the complete model

Calling now $\bar{r}$ the signal (10), i.e.

$$\bar{r} = \gamma_\beta \beta + \dot{\theta} \qquad (18)$$

a control law for τ_N may be found such that $(r - \bar{r})$ is exponentially driven to zero thus guaranteeing global stability of the whole systems as requested. Consider $V = 1/2(r-\bar{r})^2$ such that:

$$\dot{V} = (r-\bar{r})(\dot{r} - \dot{\bar{r}}) = (r-\bar{r})(f + \tau_N - m_{33}\dot{\bar{r}})\frac{1}{m_{33}} \qquad (19)$$

being

$$f = (m_{11} - m_{22})uv - k_r r - k_{r|r|} r|r| \qquad (20)$$

$$\dot{\bar{r}} = \gamma_\beta(\dot{\theta} - r) + \ddot{\theta}. \qquad (21)$$

Equation (19) suggests for τ_N the following structure:

$$\tau_N = -\hat{f} + \hat{m}_{33}\hat{\dot{\bar{r}}} + g \qquad (22)$$

where the ˆ symbol on a variable denotes its estimate due to the imperfect knowledge of the model parameters. In particular

$$\hat{f} = (\hat{m}_{11} - \hat{m}_{22})uv - \hat{k}_r r - \hat{k}_{r|r|} r|r| \qquad (23)$$

$$\hat{\dot{\bar{r}}} = \gamma_\beta(\dot{\theta} - r) + \hat{\ddot{\theta}}. \qquad (24)$$

With reference to equation (24), notice that while $\dot{\theta}$ as given by (9) does not depend explicitly on any model parameter, its derivative does. In particular:

$$\hat{\ddot{\theta}} = 2y\frac{k}{u}\dot{\theta}^2 + \qquad (25)$$
$$-\frac{ku\left[r\left((1-\hat{a})u\cos\phi - v\sin\phi\right) - (\hat{b} + \hat{c}|v|)v\cos\phi\right]}{u^2 + k^2y^2}.$$

The g term in equation (22) can be chosen in accordance to the accuracy with which the model parameters are known.

4.1 "Perfect" model knowledge

In case of perfect model knowledge, i.e. all the estimated quantities are identical to the true values, g may be simply chosen to be proportional to $r - \bar{r}$. In such case

$$\tau_N = -f + m_{33}\dot{\bar{r}} - \gamma_\tau(r - \bar{r}) \;\; : \;\; \gamma_\tau > 0 \qquad (26)$$

together with equation (19) implies that r converges exponentially to the desired reference $\bar{r}$ value. Replacing equations (18) and (21) in (26) τ_N is found to have a PD (proportional - derivative) structure

$$\tau_N = -f + (m_{33}\gamma_\beta + \gamma_\tau)(\dot{\theta} - r) + \gamma_\beta\gamma_\tau(\theta - \phi) + m_{33}\ddot{\theta}. \qquad (27)$$

Of course the same solution may be applied if the model parameter knowledge is only approximated. In this case increasing the value of the γ_τ gain may be useful to compensate for this model uncertainty, but only if it is small.

4.2 Imperfect model knowledge

A more common technique (sliding mode) to deal with the model parameter uncertainty is to chose g in (22) as:

$$g = -F \,\mathrm{sgn}(r - \bar{r}) \qquad (28)$$

being F a known upper-bound to the modeling error, i.e.

$$F > |f - \hat{f} + \hat{m}_{33}\hat{\dot{\bar{r}}} - m_{33}\dot{\bar{r}}|. \qquad (29)$$

Replacing equation (28) in (22) and then in (19) it follows that $\dot{V} < -|r - \bar{r}|$ and thus that r asymptotically tends towards $\bar{r}$. To limit the chattering phenomena related to the implementation of the resulting

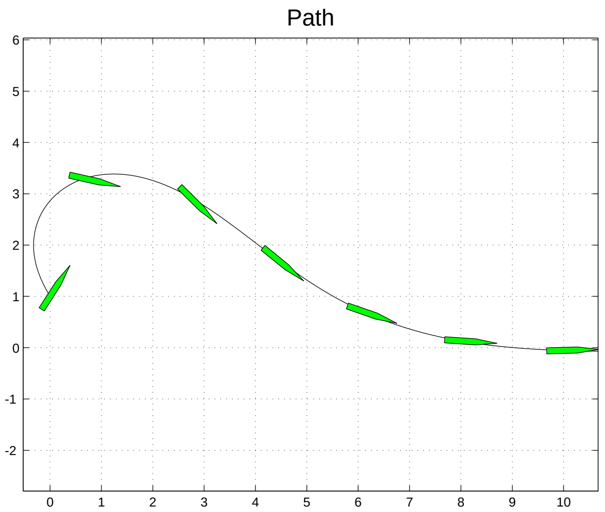

Figure 2: The effect of the large initial sway is clearly visible at the beginning of the path.

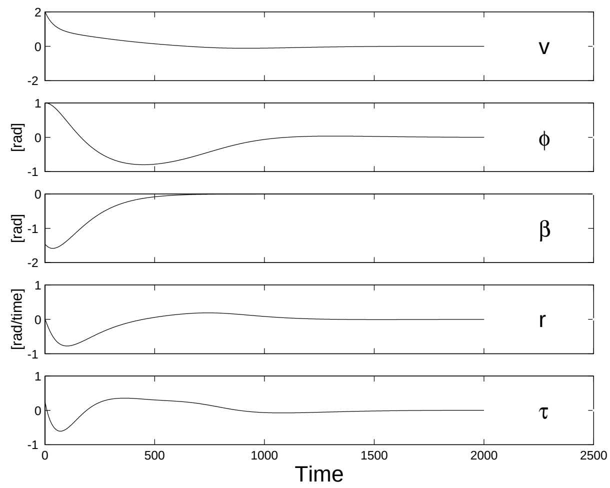

Figure 3: Data relative to the path plotted in figure (2)

discontinuous control law the $\text{sgn}(s)$ function in equation (28) may be replaced by its smooth approximation $\tanh(\mu s)$. Notice that $\lim_{\mu \to +\infty} \tanh(\mu s) = \text{sgn}(s)$. This can be viewed as a smooth version of the classical boundary-layer method [11] to reduce chattering. The trade-off between tracking accuracy and high frequency control action will be modulated by the μ parameter that must be accordingly selected. The final control law for the uncertain model results in:

$$\tau_N = -\hat{f} + \hat{m}_{33}\hat{\dot{\bar{r}}} - F \tanh(\mu(r - \bar{r})) \qquad (30)$$

where F satisfies equation (29).

5 Conclusions

The proposed controller has been tested with some simulations: first the controller given by equation (26) has been tested assuming perfect knowledge of the model parameters and no measurement noise. This has been done mainly to better illustrate the behaviour of the proposed approach. All the model parameters, (inertia, drag coefficients and constant surge u) in equations (1-5) and all the gains where fixed to 1 except for $m_{22} = 2$ and $k = 1/2$ which was necessary to satisfy the conditions of Remark 4. All the estimated quantities correspond exactly to the respective real values. The starting position and orientation was $(x_0, y_0) = (0, 1)$ and $\phi_0 = 1[rad]$ and the initial sway velocity was twice the surge, i.e. $v_0 = 2$. All the differential equations involved have been integrated by Eulers method with a time step $\delta t = 0.01$. The resulting path and the time histories of the relevant quantities are reported in figures (2) and (3). At last a more realistic simulated experiment has been performed using the model parameters of a Mariner Class Vessel (about 172m in length, 15

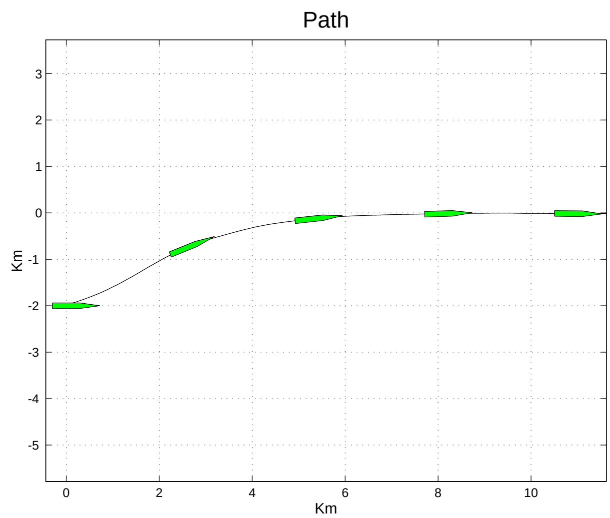

Figure 4: Initial phase of the maneuver, i.e. about 25 min of simulated navigation.

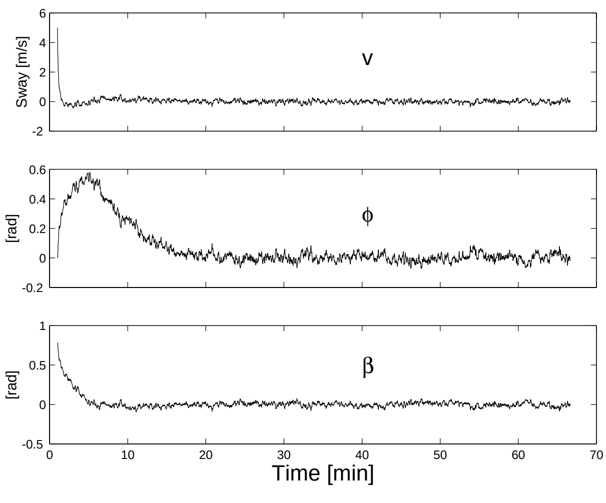

Figure 5: Data relative to the path plotted in figure (4)

knots designed speed, 18541m^3 designed displacement) taken from [12](appendix E). In particular the controller given by equation (30) has been adopted where all the relevant quantities are defined by equations (23), (24), (25) and (29). A realistic uncertainty on the model parameter knowledge has been considered, namely all the estimated quantities used to compute the control torque τ_N (30) differed from the values used to simulate the dynamics of the plant by a randomly chosen relative error in the range $21\% - 30\%$, i.e. errors perhaps even larger than usual experimental ones [13]. Moreover normally distributed, zero mean noise has been added on all the measurement channels. In particular denoting with σ_z the standard deviation of the noise added on the generic variable z, the following hold: $\sigma_y = 5m$, $\sigma_v = 1m/s$, $\sigma_r = 0.3deg/s$, $\sigma_\phi = 2deg$. The surge speed was fixed to $u = 7m/s$, i.e. about 13.6 knots. The initial sway was of the same order of magnitude of the surge, i.e. $v_0 = 5m/s$. The starting position and orientation was $(x_0, y_0) = (0, -2)Km$ and $\phi_0 = 0rad$. The constant gain parameters k and F were chosen in accordance to Remark 4 and equation (29) while μ was fixed to 10^2. As expected, high values of μ, by example $\mu = 10^3$, gave rise to chattering. About one hour of navigation has been simulated integrating the model by Eulers rule with a $1s$ time step. The results are shown in figures (4) and (5).

A novel idea to design time-invariant, closed loop motion controllers for nonlinear underactuated systems has been applied to smoothly, globally and robustly stabilize an underactuated marine vehicle on a linear course taking explicitly into account all the state variables. Conventional marine vehicle guidance and control solutions are based on linearized models in which the sway axis is neglected [12]. On the other hand standard methods of nonlinear control as feedback linearization may not be applied and indeed to the best of the authors knowledge no other solution to this problem is known in the literature. Realistic simulations are provided to illustrate the controllers behaviour.

References

[1] K. Y. Pettersen and H. Nijmeijer, "Tracking control of an underactuated surface vessel," in *37th Conference on Decision and Control, CDC'98*, (Tampa, Florida, USA), pp. 4561–4566, December 1998.

[2] T. I. Fossen, J. M. Godhavn, S. P. Berge, and K. P. Lindegaard, "Nonlinear control of underactuated ships with forward speed compensation," in *IFAC NOLCOS'98*, (Enschede, The Netherlands), pp. 732–738, July 1-3 1998.

[3] P. Encarnação, A. Pascoal, and M. Arcak, "Path following for autonomous marine craft," in 5th *IFAC Conference on Marine Craft Maneuvering and Control, MCMC2000*, (Aalborg, Denmark), August 23-25 2000.

[4] M. Aicardi, G. Cannata, G. Casalino, and G. Indiveri, "On the stabilization of the unicycle model projecting a holonomic solution," in *8th Int. Symposium on Robotics with Applications, ISORA 2000*, (Maui, Hawaii, USA), June 11-16 2000.

[5] I. Fantoni, R. Lozano, F. Mazenc, and K. Y. Pettersen, "Stabilization of a nonlinear underactuated hovercraft," in *38th IEEE Conference on Decision and Control CDC'99*, (Phoenix, Arizona, USA), December 1999.

[6] K. Y. Pettersen and O. Egeland, "Exponential stabilization of an underactuated surface vessel," in *35th Conference on Decision and Control, CDC'96*, (Kobe, Japan), pp. 967–972, December 1996.

[7] K. Y. Pettersen and H. Nijmeijer, "Global practical stabilization and tracking for an underactuated ship - a combined averaging and backstepping approach," in *IFAC Conf. on Systems Structure and Control*, (Nantes, France), pp. 59–64, July 1998.

[8] G. Indiveri, "Linear course tracking for underactuated marine vehicles: a time-invariant nonlinear controller," Tech. Rep. GMD Report 83 (ISSN 1435-2702), GMD - German National Research Center on Information Technologies, Scloß Birlinghoven, D-53754 Sankt Augustin, Deutschland, December 1999.

[9] G. Indiveri, M. Aicardi, and G. Casalino, "Nonlinear time-invariant feedback control of an underactuated marine vehicle along a straight course," in 5th *IFAC Conference on Marine Craft Maneuvering and Control, MCMC2000*, (Aalborg, Denmark), August 23-25 2000.

[10] R. Brockett, *Differential Geometric Control Theory*, ch. Asymptotic Stability and Feedback Stabilization, by Brockett, R. W., pp. 181–191. Birkhauser, Boston, USA, 1983.

[11] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control*. Prentice-Hall, Inc. New Jersey, USA, 1991.

[12] T. I. Fossen, *Guidance and Control of Ocean Vehicles*. John Wiley & Sons, England, 1994.

[13] G. Indiveri, *Modelling and Identification of Underwater Robotic Systems*. Ph.D. Thesis, DIST Università di Genova, Italy and CNR-IAN, Genova, Italy, 1998.