Invariant Sliding Sectors for Discrete Sliding Mode Control

Xinghuo Yu and Shuanghe Yu

Faculty of Informatics and Communication
Central Queensland University, Rockhampton, QLD 4702, Australia
Email: {x.yu and s.yu}@cqu.edu.au

Abstract: A new concept, "invariant sliding sector", for the design of discrete sliding mode control (DSMC) is proposed. The construction of the first order invariant sliding sectors for the second order DSMC systems is first presented, followed by an extension of the first order sector concept to higher order DSMC systems by developing a recursive structure built on a set of nested first order invariant sliding sectors.

1. Introduction

An ideal sliding mode exists only when the system state satisfies the dynamic equation governing the sliding mode for all time, which requires an infinite fast switching. It is obvious that the definition for the continuous time sliding mode cannot be applied to the discrete time case since the concept of velocity vectors of the system state trajectories is not valid (Yu, 1994). Moreover, the limited switching frequency causes zigzagging motion. In some cases chaotic behaviors may occur (Yu, 1997, 1998).

Discrete sliding mode control has been studied extensively. Early works include the first condition (Milosavljevic, 1985) $\nabla s(k)s(k) < 0$ where $s(k)$ is the sliding function and $\nabla s(k) = s(k+1) - s(k)$, sufficient conditions $|s(k+1)| < |s(k)|$ (Sarpturk et al, 1987), $s^2(k+1) < s^2(k)$ (Furuta 1990) and $|s(k)s(k+1)| < s^2(k)$ (Sira-Ramirez, 1991). It was shown (Yu and Potts 1992; Spurgeon 1992) that these conditions and their variants are only sufficient for the existence of quasi-sliding modes (or pseudo-sliding modes). Recently further work has been done in the design of DSMC (Wang et al, 1994; Misawa 1997, Tan and Misawa 1998).

An open question is about the invariance property, which was lost for most DSMC systems. In the continuous time sliding mode control, the sliding mode is an invariant manifold which gives rise to the robustness. This invariant manifold is guaranteed by the ideally infinite switching frequency. In DSMC, because of the zigzagging behaviors due to the time delay and finite switching frequency, exact "sliding" on predefined switching manifolds is impossible. Therefore the invariance property in the discrete sliding mode is meaningless.

In this short paper, we contemplate the DSMC problem from a somehow different viewpoint. We will propose the concept of *invariant sliding sector* (ISS) for DSMC systems and investigate its feasibility in the design of DSMC systems.

2 The Concept of first Order ISS

The discrete system under the DSMC is unlikely to stay in the switching manifolds, hence the concept of the invariant sliding mode does not exist at all. However, one remedy is, one can define a sliding sector, similar to those proposed by Furuta and Pan (1999) for continuous time systems, and make this sliding sector invariant.

Consider the linear second order system

$$\begin{aligned} x_1(k+1) &= x_2(k) \\ x_2(k+1) &= -a_1 x_1(k) - a_2 x_2(k) + u \end{aligned} \tag{1}$$

where a_1, a_2 are constants (or may contain uncertainties) and u is a scalar control.

Definition 1: A sliding sector in the phase plane $(x_1(k), x_2(k))$ is defined as $S[\alpha,\beta]$, $\alpha < \beta$, if for any function $x_2(k) = \phi(x_1(k))$ belonging to the sector,

$$\alpha x_1^2(k) \le x_1(k)\phi(k,x) \le \beta x_1^2,\ \forall k \ge 0,\ \forall x_1 \in R \tag{2}$$

Definition 2: $S[\alpha,\beta]$ is said to be an ISS, if for any $x(k) \in S[\alpha,\beta]$, $x(k+i) \in S[\alpha,\beta]$ for $i > 0$.

Theorem 1: If $|\alpha| < 1$, $|\beta| < 1$ and $\alpha < \beta$, and $S[\alpha,\beta]$ is invariant for system (1), then $x(k) \to 0$ asymptotically.

Proof sketch: Consider a $x(k) = (x_1(k), x_2(k)) \in S[\alpha,\beta]$. Since system (1) under control is assumed to be in ISS, then for $x(k+1) = (x_2(k), x_2(k+1))$, we must have $x_2(k+1) = \gamma_1\ x_2(k)$ where $|\gamma_1| < 1$. Therfore after m iterations, $x_2(k+m) = \prod_{i=0}^{m} \gamma_i\ x_1(k) \to 0$ for $m \to \infty$.

Theorem 1 shows that if the system can be forced into $S[\alpha,\beta]$ regardless of uncertainties, then the system will converge to zero asymptotically – a some kind of invariant property. The DSMC task becomes to design a control to force the system state into $S[\alpha,\beta]$ and make it an ISS. A preliminary study has recently been successfully completed and robustness in system variations has been shown (Yu and Yu 2000).

3 Extension to Higher Order DSMC Systems

Consider the *n*th order controllable system

$$\begin{aligned} x_1(k+1) &= x_2(k) \\ x_2(k+1) &= x_3(k) \\ &\vdots \\ x_n(k+1) &= f(x(k)) + u \end{aligned} \tag{3}$$

To extend the first order ISS for (2), we propose

$$\begin{aligned} s_1(k) &= s_0(k+1) - \phi_0(s_0(k)) \\ s_2(k) &= s_1(k+1) - \phi_1(s_1(k)) \\ &\vdots \\ s_{n-1}(k) &= s_{n-2}(k+1) - \phi_{n-2}(s_{n-2}(k)) \end{aligned} \tag{4}$$

where $s_0(k) = x_1(k)$. Assume that $\phi_0, \phi_1, \cdots, \phi_{n-2}$ satisfy $\phi_i \in S[\alpha_i, \beta_i]$ with $\alpha_i < \beta_i$ and $|\alpha_i| < 1, |\beta_i| < 1$ $(i = 0, \cdots, n-2)$. By applying Theorem 1, one can conclude that if we can force $s_{n-1}(k) = 0$, since $\phi_{n-2}(s_{n-2}) \in S[\alpha_{n-2}, \beta_{n-2}]$, then s_{n-2} will tend to zero. Subsequently $s_{n-3,}, s_{n-4}, \cdots, s_0$ will tend to zero. Therefore, the entire system will be asymptotically stable. This structure can be depicted by a block diagram of a set of nested first order ISS. It can be derived that the transfer function of (4) is

$$s_{n-1} = (z + \phi_{n-2}(\cdot))(z + \phi_{n-3}(\cdot)) \cdots (z + \phi_0(\cdot)) x_1(k) \tag{5}$$

where z is the z-operator and $\phi(\cdot)y(k) = \phi(y(k))$. It can be seen that (5) is confined within the two hyperplanes (Chu, 1996)

$$\begin{aligned} s'_{n-1} &= (z + \alpha_{n-2})(z + \alpha_{n-3}) \cdots (z + \alpha_0) x_1(k) \\ s''_{n-1} &= (z + \beta_{n-2})(z + \beta_{n-3}) \cdots (z + \beta_0) x_1(k) \end{aligned} \tag{6}$$

Since these two hyperplanes are asymptotically stable (from the assumption), then we can conclude that any states confined within the generalized sector will be asymptotically stable).

4. Conclusion

The new concept of invariant sliding sectors is attractive but there are a number of questions remain open and need to be fully addressed. The most important one is the design issue. For 2nd order DSMC systems, it has been shown in (Yu and Yu, 2000) that the existence of the ISS can be guaranteed by imposing some mild conditions on control parameters. Furthermore, the resulting controllers exhibit certain robustness in system variations. This mechanism may be extended to higher order systems by making use of the dynamical properties of the asymptote hyperplanes (Yu, 1994). This issue will be further studied in the future.

References

Chu, V. K.-S., 1996. Sliding Mode Control with Nonlinear Sliding Surfaces: A B-spline Fuzzy Logic Approach, PhD Thesis, University of California at Berkeley.

Furuta, K., 1990. “Sliding mode control of a discrete system.” Systems and Control Letters, Vol. 14, pp. 145-152.

Furuta, K., and Pan, Y., 1999. “Sliding sector for variable structure system.” Variable Structure Systems, Sliding Mode and Nonlinear Control, Lecture Notes in Control and Information Sciences, Young K. D. and Ozguner U. (Eds), Vol. 247, pp. 1-26.

Milosavljevic, C., 1985. “General conditions for the existence of a quasi-sliding mode on the switching hyperplane in discrete variable structure systems.” Automation and Remote Control, Vol. 46, pp. 307-314.

Misawa, E. A., 1997. “Discrete time sliding mode control: The linear case.” ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 119, No. 4, pp. 819-921.

Sarpturk, S. Z., Istefanopulos, Y., Kaynak, O., 1987. “The stability of discrete time sliding mode control systems.” IEEE Transactions on Automatic Control, Vol. 32, pp. 930-932.

Sira-Ramirez, H., 1991. “Nonlinear discrete variable structure systems in quasi-sliding mode.” International Journal of Control, Vol. 54, pp. 1171-1187.

Spurgeon, S. K., 1992. “Hyperplane design techniques for discrete time variable structure control systems.” International Journal of Control, Vol. 54, pp. 445-456.

Tang, C. Y. and Misawa, E. A., 1999. “On discrete variable structure control with switching sector.” Variable Structure Systems, Sliding Mode and Nonlinear Control, Lecture Notes in Control and Information Sciences, Young K. D. and Ozguner U. (Eds) Vol. 247, pp. 27-42.

Wang, W. J. et al, 1994. “Variable structure control design for uncertain discrete-time systems.” IEEE Transactions on Automatic Control, Vol. 39, No. 1, pp. 99-102.

Yu, X. and Potts, R.B., 1992. “Analysis of discrete variable structure systems with pseudo sliding modes.” International Journal of Systems Science, Vol. 23, pp. 503-516.

Yu, X. and Yu, S., 2000. “Discrete sliding mode control design with invariant sliding sectors,” ASME Journal of Dynamic Systems, Measurement, and Control, Special Issue on Variable Structure Control, accepted for publication.

Yu, X., 1997. "Bifurcation and chaotic behaviours in the variable structure system with unlimited control magnitudes." International Journal of Bifurcation and Chaos, Vol. 7, No. 8, pp. 1897-1993.

Yu, X., 1998. "Discretization effect on a sliding mode control system with bang-bang type switching," International Journal of Bifurcation and Chaos, Vol. 8, No. 6, pp. 1245-1257.

Yu. X., 1994. “Digital variable structure control with pseudo sliding modes.” Variable Structure and Lyapunov Control (A. S. I. Zinober ed.), pp. 133-155, Springer-Verlag.