OBSERVER-BASED FAULT DIAGNOSIS FOR STRUCTURED SYSTEMS

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Abstract

We consider here the Fault Detection and Isolation (FDI) problem for linear systems with disturbances. We try to design a set of residuals through a bank of observers, in such a way that the transfer from the disturbances to the residuals is zero and the transfer from the faults to the residuals is diagonal. We deal with this problem when the system under consideration is structured, that is, the entries of the matrices which define the system are either fixed zeros or free parameters. To structured systems one can associate in a natural way a directed graph. We are then able to give a necessary and sufficient condition under which the FDI problem has a solution for almost any value of the free parameters. This condition is simply expressed in terms of particular paths in the associated graph.

1 Introduction

This paper is concerned with the Fault Detection and Isolation (FDI) problem which has received considerable attention [1, 2, 8, 13] in the past ten years. In the model-based approach for FDI two steps are distinguishable: the generation of the residuals which are sensitive to the faults, and the isolation of the faults. A bank of observer-based method for residual generation is here considered [8], and we will design a dedicated residual set [1] using this bank of observers. Many works have been done on observer-based approaches, using either robust design [1, 2, 8, 9] or a structural approach [1, 3, 9, 16].

The FDI problem is solved here for linear structured systems which represent a large class of parameter dependent linear systems [11]. Such linear systems are represented by some matrices whose entries are either fixed zeros or free parameters. For these systems, generic properties are studied, that is properties which are true for almost all values of the parameters. The analysis of such systems is of interest because it gives nice graph conditions to generically solve classical control problems [4, 14] and then it allows to use some efficient algorithms to check the solvability of such problems [10].

Using a structural approach introduced by Lin [11], we will here derive some graph solvability conditions of the FDI problem, when each fault is detected and isolated by its residual. This work follows [6, 5] where conditions for generically solving the FDI problem with disturbances were given. In [5] the transfer matrix between faults and residuals is required to be triangular where the residual is designed using a unique observer. In this case a necessary and sufficient solvability condition is given in terms of number of disjoint paths in the system associated graph. In [6] a sufficient condition has been given such that the transfer matrix between faults and residuals is required to be diagonal where the residual is designed using a unique observer. In this paper we tackle the FDI problem with disturbances such that the transfer matrix between faults and residuals is required to be diagonal, where the residuals are designed using a bank of observers.
We prove that the necessary and sufficient condition for the diagonal case with a bank of observers is the same as the condition for the triangular one with a unique observer.

The outline of this paper is as follows. The problem is formulated in section 2. The linear structured systems are presented in section 3. Solvability conditions for the unique observer-based diagonal and triangular FDI problem with disturbances are recalled in section 4. Our main contribution , i.e. solvability conditions for the bank of observer-based diagonal FDI problem with disturbances, is given in section 5. An illustrative example is presented in section 6. Some concluding remarks end the paper.

2 Problem formulation

Let us consider the following linear time-invariant system :

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) + E_1 d(t) + F_1 f(t) \\ y(t) = Cx(t) + Du(t) + E_2 d(t) + F_2 f(t) \end{array} \right. \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ the known control input vector, $d(t) \in \Re^q$ the unknown input (or disturbance), $f(t) \in \Re^r$ the fault vector and $y(t) \in \Re^p$ the output vector.
$A, B, C, D, E_1, E_2, F_1$ and F_2 are matrices of appropriate dimensions. In this paper , we will design a dedicated residual set [1] using a bank of r observers for the system defined by (1), according to the dedicated observer scheme given in figure 1.

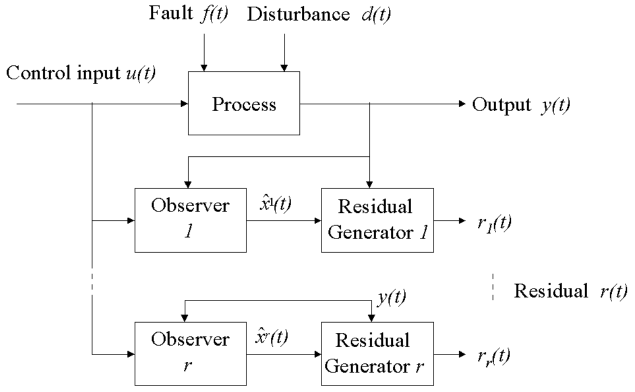

Figure 1: Bank of observers for the generation of residuals.

For this bank of observer-based diagonal FDI problem, it is well known that the control input effects can be taken into account in the observers structure. Then we will assume without loss of generality that matrices B and D are equal to zero, as follows :

$$\Sigma \left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + E_1 d(t) + F_1 f(t) \\ y(t) = Cx(t) + E_2 d(t) + F_2 f(t) \end{array} \right. \tag{2}$$

For designing the bank of r observers, first we consider the system Σ^i obtained from the system Σ defined by (2), as follows :

$$\Sigma^i \left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + E_1^i d^i(t) + F_{1i} f_i(t) \\ y(t) = Cx(t) + E_2^i d^i(t) + F_{2i} f_i(t) \end{array} \right. \tag{3}$$

where $f_i(t)$ is the ith component of $f(t)$ and :

$$d^{iT}(t) = \left[\begin{array}{cccccc} d^T(t) & f_1(t) & \dots & f_{i-1}(t) & f_{i+1}(t) & \dots & f_r(t) \end{array} \right]^T$$

Also F_{1i} (resp. F_{2i}) is the ith column of F_1 (resp. F_2) and E_1^i (resp. E_2^i) is the composite matrix constructed from the matrices E_1 and F_1 (resp. E_2 and F_2) when the ith column of F_1 (resp. F_2) is deleted.

The ith observer of this bank of r observers, designed for the system Σ^i defined by (3) is given as follows :

$$\dot{\hat{x}}^i(t) = A\hat{x}^i(t) + K^i(y(t) - C\hat{x}^i(t)) \tag{4}$$

where $\hat{x}^i(t) \in \Re^n$ is the state of the full-order ith observer, K^i is the matrix to be designed such that $\hat{x}^i(t)$ asymptotically converges to $x(t)$, when no fault and no disturbance are considered.
Using (3) and (4) the estimation error $e^i(t) = x^i(t) - \hat{x}^i(t)$ is given as :

$$\dot{e}^i(t) = (A - K^i C)e^i(t) + (E_1^i - K^i E_2^i)d^i(t) + (F_{1i} - K^i F_{2i})f_i(t)$$

The residual is obtained as follows :

$$\begin{aligned} r_i(t) &= Q^i(y(t) - C\hat{x}^i(t)) \\ &= Q^i C e^i(t) + Q^i E_2^i d^i(t) + Q^i F_{2i} f_i(t) \end{aligned}$$

where Q^i is a $1 \times p$ matrix.
Therefore the transfer matrices from the disturbance to the residual and from the fault to the residual are given by :

$$r_i(s) = \left[\begin{array}{cc} T_{rd}^i(s) & T_{rf}^i(s) \end{array} \right] \left[\begin{array}{c} d^i(s) \\ f_i(s) \end{array} \right] \tag{5}$$

where :

$$T_{rd}^i(s) = Q^i C(sI - A + K^i C)^{-1}(E_1^i - K^i E_2^i) + Q^i E_2^i$$

$$T_{rf}^i(s) = Q^i C(sI - A + K^i C)^{-1}(F_{1i} - K^i F_{2i}) + Q^i F_{2i}$$

The considered problem of bank of observer-based FDI with disturbances consists in finding matrices K^i and Q^i for $i = 1, 2, \dots, r$, such that :

- $$A - K^i C \text{is stable.} \tag{6}$$
- the transfer from the disturbance to the residual is zero :
  $$T_{rd}^i(s) = 0 \tag{7}$$
- the transfer from the fault to the residual is a non zero proper rational transfer function, i.e. :
  $$T_{rf}^i(s) = t_{ii}(s) \neq 0 \tag{8}$$

The conditions (7) and (8) for residual $r_i(s)$ described by (5) are equivalent to the complete representation :

$$r(s) = \left[\begin{array}{cccccccc} 0 & 0 & \cdots & 0 & t_{11}(s) & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & t_{22}(s) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & t_{rr}(s) \end{array} \right] \left[\begin{array}{c} d(s) \\ f(s) \end{array} \right] \tag{9}$$

where $r_i(s)$ is the ith component of $r(s)$, and $t_{ii}(s) \neq 0$ for $i = 1, 2, \dots, r$.

3 Linear structured systems

In this part we recall some definitions and results on linear structured systems. More details can be found in [4, 7].
Consider the linear system described by (2) :

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + E_1 d(t) + F_1 f(t) \\ y(t) = Cx(t) + E_2 d(t) + F_2 f(t) \end{array} \right. \qquad (10)$$

This system is called a linear structured system and denoted by Σ_Λ if the entries of the composite matrix $J = \begin{bmatrix} A & E_1 & F_1 \\ C & E_2 & F_2 \end{bmatrix}$ are either fixed zeros or independent parameters. In this definition Λ is the set of independent parameters of the composite matrix J.
A directed graph $G(\Sigma_\Lambda) = (Z, W)$ can be associated to a structured system Σ_Λ defined by (10) where :
v • the vertex set is $Z = D \cup F \cup X \cup Y$ with D,F,X and Y the disturbance, fault, state and output sets given by $\{d_1, d_2, \ldots, d_q\}$, $\{f_1, f_2, \ldots, f_r\}$, $\{x_1, x_2, \ldots, x_n\}$ and $\{y_1, y_2, \ldots, y_p\}$ respectively.
• and the arc set is $W = \{(d_i, x_j) | e_{1ji} \neq 0\} \cup \{(f_i, x_j) | f_{1ji} \neq 0\} \cup \{(x_i, x_j) | a_{ji} \neq 0\} \cup \{(x_i, y_j) | c_{ji} \neq 0\} \cup \{(d_i, y_j) | e_{2ji} \neq 0\} \cup \{(f_i, y_j) | f_{2ji} \neq 0\}$ where a_{ji} (resp. c_{ji},e_{1ji},e_{2ji},f_{1ji},f_{2ji}) denotes the element (j, i) of the matrix A (resp. C,E_1,E_2,F_1,F_2).
Moreover, recall that a directed path in $G(\Sigma_\Lambda)$ from a vertex $i_{\mu 0}$ to a vertex $i_{\mu q}$ is a sequence of arcs $(i_{\mu 0}, i_{\mu 1}), (i_{\mu 1}, i_{\mu 2}), \ldots, (i_{\mu q-2}, i_{\mu q-1}), (i_{\mu q-1}, i_{\mu q})$ such that $i_{\mu t} \in Z$ for $t = 0, 1, \ldots, q$ and $(i_{\mu t-1}, i_{\mu t}) \in W$ for $t = 1, 2, \ldots, q$. The length of a path is the number of its arcs, each arc being counted the number of times it appears in the sequence. For the last sequence, the path has length q. Occasionally, we denote the path P by the sequence of vertices it consists of, i.e. by :

$$P = (i_{\mu 0}, i_{\mu 1}, \ldots, i_{\mu q-1}, i_{\mu q})$$

Moreover, if $i_{\mu 0} \in D$ (resp. F) and, $i_{\mu q} \in Y$, P is called a disturbance-output (resp. fault-output) path.

Notice that for structured systems, one can study generic properties i.e. properties which are true for almost all values of the parameters [12, 15]. Also, many results have been obtained for these systems on structural controllability, decoupling, disturbance rejection and FDI [4, 6, 5, 7, 11].

We can study some properties of structured systems using their associated graphs.
Consider the system Σ_Λ defined by (10). The system transfer matrix is :

$$T_\Lambda(s) = \left[\begin{array}{cc} T_{yd}(s) & T_{yf}(s) \end{array} \right] \qquad (11)$$

where $T_{yd}(s) = C(sI - A)^{-1} E_1 + E_2$ and $T_{yf}(s) = C(sI - A)^{-1} F_1 + F_2$.
We can calculate the generic rank of $T_\Lambda(s)$ and $T_{yd}(s)$ by using the following theorem [7].

Theorem 1 *Let Σ_Λ be the linear structured system defined by (10) with its associated graph $G(\Sigma_\Lambda)$. The rank of $T_\Lambda(s)$ is generically equal to the maximum number of input-output vertex disjoint paths in $G(\Sigma_\Lambda)$ where the input contains disturbances and faults.*

Example 1 *Let us now present an example to illustrate the previous definitions. Consider the following structured system Σ_Λ :*

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \lambda_1 & \lambda_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, E_1 = \begin{bmatrix} \lambda_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} 0 & 0 \\ \lambda_4 & 0 \\ 0 & 0 \\ \lambda_5 & \lambda_6 \end{bmatrix}, C = \begin{bmatrix} \lambda_7 & \lambda_8 & 0 & 0 \\ 0 & 0 & \lambda_9 & 0 \\ 0 & 0 & 0 & \lambda_{10} \end{bmatrix}$$

The entries of matrices are the free parameters $(\lambda_1, \lambda_2, \ldots, \lambda_{10})$. Its associated graph is given in figure 2.

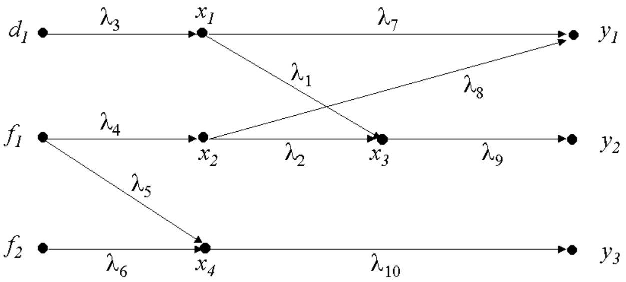

Figure 2: Associated graph of Σ_Λ.

In this example the generic rank of the transfer matrix $T_\Lambda(s)$ is 3 (paths $(d_1, x_1, y_1), (f_1, x_2, x_3, y_2)$ and (f_2, x_4, y_3)).

Let us now come back to our bank of observer-based diagonal FDI problem.
Consider the structured system Σ_Λ described by (10), clearly the system Σ^i defined by (3), is also a structured system with the same set of independent parameters Λ and we have the following structured system Σ^i_Λ:

$$\Sigma^i_\Lambda \left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + E^i_1 d^i(t) + F_{1i} f_i(t) \\ y(t) = Cx(t) + E^i_2 d^i(t) + F_{2i} f_i(t) \end{array} \right. \qquad (12)$$

Thus, the considered problem of bank of observer-based FDI with disturbances for the structured system Σ_Λ defined by (10) consists in finding matrices K^i and Q^i for $i = 1, 2, \ldots, r$, such that the relations (6) and (9) hold true. We will assume in the sequel that the system Σ_Λ is structurally observable. This can be easily checked on the associated graph using the dual of the result given by Lin [11].

In this paper we look for structural conditions under which the above mentioned problem has a solution for almost any value of the parameters (which are the entries of J). The solvability conditions will not depend on the parameters but the solutions (K^i, Q^i) for $i = 1, 2, \ldots, r$ will of course depend on these parameters.

4 Unique observer-based FDI problem

In this part the previous results developed in [6, 5] are recalled.

Consider the system Σ_Λ defined by (10). The observer-based diagonal FDI problem amounts to find a unique observer for designing a residual vector such that the transfer matrix from the disturbance to the residual is zero and the transfer matrix from the fault to the residual is diagonal with non zero diagonal entries.

Recall first the result concerning the diagonal FDI problem [6].

Theorem 2 *Consider the system Σ_Λ defined by (10) and the associated graph $G(\Sigma_\Lambda)$. The unique observer-based diagonal FDI problem is generically solvable if $L = L_q + \sum_{i=1}^{r} l_i$ where:*

- *L is the minimal sum of $q+r$ vertex disjoint (disturbance / fault)-output path lengths in $G(\Sigma_\Lambda)$.*
- *L_q is the minimal sum of q vertex disjoint disturbance-output path lengths in $G(\Sigma_\Lambda)$.*
- *l_i is the minimal length of a fault-output path with initial vertex corresponding to the ith fault in $G(\Sigma_\Lambda)$.*

The given condition is only sufficient and it is rather restrictive as it will be pointed out in section 6. Moreover stability is not insured.

Consider the system Σ_Λ defined by (2). The observer-based triangular FDI problem with stability amounts to find a unique observer for designing a residual vector such that the closed loop system is stable, the transfer matrix from the disturbance to the residual is zero and the transfer matrix from the fault to the residual is triangular with non zero diagonal entries.

Recall now a result concerning the triangular FDI problem [6].

Theorem 3 *Consider the structurally observable system Σ_Λ defined by (2) and the associated graph $G(\Sigma_\Lambda)$. The unique observer-based triangular FDI problem is generically solvable with stability if and only if :*

$$k = k_q + r \tag{13}$$

where :

- *k is the maximum number of (disturbance / fault)-output vertex disjoint paths in $G(\Sigma_\Lambda)$.*
- *k_q is the maximum number of disturbance-output vertex disjoint paths in $G(\Sigma_\Lambda)$.*

The given condition is necessary and sufficient but we can not detect and isolate simultaneous faults.

In the next section we give the necessary and sufficient condition for solving the diagonal FDI problem with stability using a bank of observers.

5 Bank of observer-based FDI problem

We will now state our main result concerning the diagonal FDI problem by using a bank of observers.

Theorem 4 *Consider the structurally observable system Σ_Λ defined by (2) and the associated graph $G(\Sigma_\Lambda)$. The considered bank of observer-based diagonal FDI problem (6,9) is generically solvable with stability if and only if :*

$$k = k_q + r \tag{14}$$

where :

- *k is the maximum number of (disturbance / fault)- output vertex disjoint paths in $G(\Sigma_\Lambda)$*
- *k_q is the maximum number of disturbance-output vertex disjoint paths in $G(\Sigma_\Lambda)$*

It is interesting to notice that the necessary and sufficient condition for this main result is exactly the same as in theorem 3, although in this case we are able to detect and isolate simultaneous faults.

Proof 1 *By using theorem 1, the condition (14) is equivalent to:*

$$rank(T_\Lambda(s)) - rank(T_{yd}(s)) = r \tag{15}$$

where $T_\Lambda(s)$ and $T_{yd}(s)$ are defined by (11). The condition (15) is equivalent to :

$$rank(T_\Lambda(s)) - rank(T^i(s)) = 1 \tag{16}$$

for $i = 1, 2, \ldots, r$ where $T^i(s)$ is :

$$T^i(s) = \begin{bmatrix} T_{yd}(s) & t_1(s) & \cdots & t_{i-1}(s) & t_{i+1}(s) & \cdots & t_r(s) \end{bmatrix}$$

where $t_j(s)$ is the jth column of $T_{yf}(s)$ defined by (11). Clearly, by using again theorem 1, the condition 16 is equivalent to :

$$k = k^i + 1 \tag{17}$$

where :
- *k is the maximum number of (disturbance / fault)-output vertex disjoint paths in $G(\Sigma_\Lambda)$*
- *k^i is the maximum number of (disturbance / fault)-output vertex disjoint paths in $G(\Sigma_\Lambda)$ when $f_i(t) = 0$*

Now, consider the system described by (12). In (12) is a disturbance vector of dimension and a unique fault. By using theorem 3 for the system Σ^i_Λ with its condition represented by (17), there exist matrices K^i and Q^i such that $A - K^iC$ is stable and :

$$r_i(s) = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & t_{ii}\ (s) \end{bmatrix} \begin{bmatrix} d^i(s) \\ f_i(s) \end{bmatrix}$$

where $t_{ii} \neq 0$.

Remember that the matrices K^i and Q^i correspond to the i^th observer and i^th residual generator , respectively according to figure 1. Using this technique for $i = 1, 2, \ldots, r$ one can design the bank of observers such that the relations (6) and (9) hold true.

It is worth stating that, when the condition (14) in theorem 4 is satisfied, a bank of bicausal post compensators can be found which reject the disturbance and isolate the faults. Then the derivation of the observers is standard.

6 Illustrative Example

The following example illustrates our main result in comparison to the previous ones.

Example 2 *Consider again example 1. It is easy to check that this system is structurally observable. Using theorem 2 we obtain :*
- *$L = 7$; paths (d_1, x_1, y_1), (f_1, x_2, x_3, y_2) and (f_2, x_4, y_3)*
- *$L_q = 2$; path (d_1, x_1, y_1)*
- *$l_1 = 2$; path (f_1, x_4, y_3)*
- *$l_2 = 2$; path (f_2, x_4, y_3)*

As $L \neq L_q + l_1 + l_2$ we can not conclude on the solvability of the considered diagonal FDI problem.But in this case it can be shown that no solution exists for this problem.

Using now theorem 3 one has :
- *$k = 3$; paths (d_1, x_1, y_1), (f_1, x_2, x_3, y_2) and (f_2, x_4, y_3)*
- *$k_q = 1$; path (d_1, x_1, y_1)*

The considered problem is therefore generically solvable for the triangular form with stability using a unique observer.

Using now theorem 4 :

As the necessary and sufficient conditions of theorem 4 are exactly the same as in theorem 3, the considered problem is therefore generically solvable for the diagonal form with stability using two observers. The matrices K^1, Q^1, K^2 and Q^2 are for example :

$$K^1 = \begin{bmatrix} a/\lambda_7 & a^2\lambda_8/\lambda_9\bar{\lambda} & 0 \\ 0 & -a^2\lambda_7/\lambda_9\bar{\lambda} & 0 \\ \lambda_1/\lambda_7 & 2a/\lambda_9 & 0 \\ 0 & 0 & a/\lambda_{10} \end{bmatrix}$$

$$Q^1 = \begin{bmatrix} 0 & -1/\lambda_4\lambda_9\bar{\lambda} & 0 \end{bmatrix}$$

$$K^2 = \begin{bmatrix} b/\lambda_7 & b^2\lambda_8/\lambda_9\bar{\lambda} & -b\lambda_4\lambda_8/\lambda_5\lambda_7\lambda_{10} \\ 0 & -b^2\lambda_7/\lambda_9\bar{\lambda} & -b\lambda_4/\lambda_5\lambda_{10} \\ \lambda_1/\lambda_7 & 2b/\lambda_9 & -\lambda_4\bar{\lambda}/\lambda_5\lambda_7\lambda_{10} \\ 0 & 0 & b/\lambda_{10} \end{bmatrix}$$

$$Q^2 = \begin{bmatrix} 0 & 1/\lambda_4\lambda_6\bar{\lambda} & 0 \end{bmatrix}$$

and :

$$r(s) = \begin{bmatrix} 0 & (1/\lambda_7)(s+a)^{-2} & 0 \\ 0 & 0 & (\lambda_9/\lambda_5\lambda_7)(s+b)^{-2} \end{bmatrix} \begin{bmatrix} d(s) \\ f(s) \end{bmatrix}$$

where a and b are arbitrary positive real numbers and $\overline{\lambda}$ is equal to :

$$\bar{\lambda} = \lambda_1\lambda_8 - \lambda_2\lambda_7$$

7 Concluding remarks

In this paper we have considered the diagonal FDI problem with disturbances for linear structured systems using a bank of observers. The necessary and sufficient condition for solving this problem is expressed in terms of input-output paths in the associated graph and is often satisfied in practical situations. The solutions are valid for almost any value of the parameters of the considered structured system.

Appendix A

Consider example 2. In the following we show that the FDI problem has no diagonal solution using a unique observer. Indeed, if there exists such a diagonal solution, then we will have :

$$T_\Lambda(s) = B(s)M(s) \tag{18}$$

where $T_\Lambda(s) \in \Re_p^{3\times3}$ is the transfer matrix of system Σ_Λ described in example 1, $B(s)$ is a bicausal matrix equal to :

$$B(s) = (I + C(sI - A)^{-1}K)Q^{-1}$$

The matrices K and Q are used to design a residual vector using a unique observer and $M(s) \in \Re_p^{3\times3}$ has the following form :

$$M(s) = \left[\begin{array}{c|cc} m_{11}(s) & m_{12}(s) & m_{13}(s) \\ \hline 0 & m_{22}(s) & 0 \\ 0 & 0 & m_{33}(s) \end{array}\right] \tag{19}$$

where $m_{ii} \neq 0$ for $i = 1, 2, 3$. We will show that $B(s)$ cannot be a bicausal matrix. Based upon the result obtained in example 1, the generic rank of $T_\Lambda(s)$ is 3 , i.e. $T_\Lambda(s)$ is invertible. Moreover the relation (18) means that there exists a proper rational postcompensator $B^{-1}(s)$ such that :

$$B^{-1}(s) = M(s) T_\Lambda^{-1}(s) \tag{20}$$

Now consider (20) , as $B^{-1}(s)$ is a proper rational postcompensator and, the second and the third rows of the transfer matrix $M(s)$ described by (19) only have a non zero entry, then one can say that the infinite zeros of m_{22} and m_{33} are greater than or equal to the infinite poles of the second and third row of $T_\Lambda^{-1}(s)$ respectively [4, 7]. First let us calculate $T_\Lambda^{-1}(s)$ for which the infinite poles of its second and third rows are 2 and 2. Thus the relation (18) becomes as follows:

$$\begin{bmatrix} \lambda_3\lambda_7 s^{-1} & \lambda_4\lambda_8 s^{-1} & 0 \\ \lambda_1\lambda_3\lambda_6 s^{-2} & \lambda_2\lambda_4\lambda_9 s^{-2} & 0 \\ 0 & \lambda_5\lambda_{10} s^{-1} & \lambda_6\lambda_{10} s^{-1} \end{bmatrix} =$$

$$\begin{bmatrix} b_{11}(s) & b_{12}(s) & b_{13}(s) \\ b_{21}(s) & b_{22}(s) & b_{23}(s) \\ 0 & b_{32}(s) & b_{33}(s) \end{bmatrix} \begin{bmatrix} m_{11}(s) & m_{12}(s) & m_{13}(s) \\ 0 & k_{22}(s) s^{-\alpha} & 0 \\ 0 & 0 & k_{33}(s) s^{-\beta} \end{bmatrix}$$

where $k_{22}(s)$ and $k_{33}(s)$ are arbitrary bicausal transfer functions and, both α and β are integer numbers greater than or equal to 2. Then we can calculate $b_{32}(s)$ and $b_{33}(s)$ as follows :

$$\begin{aligned} b_{32}(s) &= (\lambda_5\lambda_{10}/k_{22}(s)) s^{\alpha-1} \\ b_{33}(s) &= (\lambda_6\lambda_{10}/k_{33}(s)) s^{\beta-1} \end{aligned}$$

where $\alpha \geq 2$ and $\beta \geq 2$. Therefore, $B(s)$ represented by (18) is not bicausal, i.e. the considered diagonal FDI problem described in example 2 has no solution using a unique observer.

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