Control of Vehicle Speed : a Nonlinear Approach

R. Outbib ‡ and A. Rachid †

‡ CRAN - Université Henri Poincaré - Nancy I
186, rue de Lorraine, 54 400 Cosnes et Romain – France

† Université de Picardie Jules Verne
7, rue du Moulin Neuf, 80 000 Amiens – France

Abstract

In this paper we present a new nonlinear approach to control the speed of a vehicle. We first propose a nonlinear transformation of a vehicle dynamics model, which takes into account the main non linearities of the process, to facilitate the control of the vehicle dynamics. Afterwards, we establish that the nonlinear model can be made asymptotically stable around a desired longitudinal speed.

1 Introduction

There is a real need to improve vehicles in order to make them less pollutant; less consuming; safe; intelligent; comfortable. The dynamics of vehicles is governed by strongly non-linear equations. A challenge that faces vehicle dynamists today is to understand and control such complex non-linear dynamics. The development of specific methodology to vehicle must make it possible to save some expensive and cumbersome sensors, to improve the security and the comfort and to offer the driver a system for assistance.

The control problem of vehicles has attracted the attention of several researchers. A great number of results has been reported (see for instance [1]-[8] and the references therein). However, the complete nonlinear dynamics of the vehicle have not been dealt as a basis for control.

This paper presents a new result on the control problem of vehicle dynamics. This result concerns a classical model (see system (1)) used in the literature (see for instance [2]) to describe the behaviour of the vehicle dynamics. More precisely, we propose a new strategy to control the speed of the vehicle. This approach is based first on a nonlinear transformation which facilitates vehicle dynamics control problem. A nonlinear control design is then discussed to deal with cruise control like problems.

The paper is organized as follows. In the second section we present the vehicle dynamics model under consideration then we propose a useful nonlinear transformation. In the third section we discuss the problem of feedback stabilization of the speed of vehicle. The conclusion is given in the last section.

2 A model transformation

The goal of this section to is to present a strategy, based on a nonlinear transformation, which permits to facilitate vehicle dynamics control problem. We begin by introducing a nonlinear model, used in the literature (see for instance [2]), to describe the behaviour of vehicle dynamics.

Let

$$\left\{\begin{array}{l} \dot{u} = vr - fg + \frac{fk_1-k_2}{M}u^2 + \frac{c_f}{M}(\frac{v}{u} + \frac{ar}{u})\delta + \frac{1}{M}T \\ \dot{v} = -ur - (\frac{c_f+c_r}{M})\frac{v}{u} + (\frac{bc_r-ac_f}{M})\frac{r}{u} + \frac{c_f}{M}\delta + \frac{1}{M}T\delta \\ \dot{r} = -f\frac{Mh}{I_z}ur + (\frac{bc_r-ac_f}{I_z})\frac{v}{u} - (\frac{a^2c_f+b^2c_r}{I_z})\frac{r}{u} \\ \qquad + \frac{ac_f}{I_z}\delta + \frac{a}{I_z}T\delta \\ \dot{x} = u\cos\varphi - v\sin\varphi \\ \dot{y} = u\sin\varphi + v\cos\varphi \\ \dot{\varphi} = r \end{array}\right. \qquad (1)$$

where

- $(x, y) \in \mathbb{R}^2$ denote the vehicle coordinates;
- u is the longitudinal speed ($\in [11, 36]$ m/s);
- v is the lateral speed ($|v| \in [0, 1]$ m/s);
- r is the rotational speed ($|r| \in [0; 0.1]$ rd/s);
- $\varphi \in \mathbb{R}$ is the raw angle in a universe frame.

The control variables are the front wheel steering angle δ and the traction and/or braking force T; the constants M, h and I_z are respectively the full mass of the vehicle, the height from center of gravity (CG) to road, and the initial moment around z-axis ; g is the acceleration of gravity force; f is the rotating friction coefficient; a and b are the distances from front and rear tyres to CG respectively; c_f and c_r are the cornering stiffness coefficients of front and rear tyres respectively; d is the steering angle; k_1 and k_2 are respectively the lift and drag parameters from aerodynamics.

Let

$$\left\{\begin{array}{l} \delta_2 = v(r_1+v)\frac{M\,a}{I_z} - fg + \frac{fk_1-k_2}{M}u^2 + \delta_1 \\ T_2 = -u\,(r_1+v)\frac{M\,a}{I_z} - (\frac{c_f+c_r}{M})\frac{v}{u} \\ \qquad + a(\frac{bc_r-ac_f}{I_z})(\frac{r_1+v}{u}) + T_1\,. \end{array}\right. \qquad (2)$$

with $r_1 = \frac{I_z}{M\,a}\,r - v$, and where

$$\left\{\begin{array}{l} \delta_1 = \frac{c_f}{M}(\frac{v}{u} + \frac{a^2M}{I_z}(\frac{r_1+v}{u}))\delta + \frac{1}{M}T \\ T_1 = \frac{c_f}{M}\delta + \frac{1}{M}T\delta. \end{array}\right. \qquad (3)$$

Direct calculation yields

$$\begin{cases} \dot{u} = \delta_2 \\ \dot{v} = T_2 \\ \dot{r}_1 = \frac{M(a-fh)}{I_z}(v + r_1)u + \frac{(a+b)c_r}{Ma}\frac{v}{u} - \frac{b(b+a)c_r}{I_z}(\frac{r_1+v}{u}) \\ \dot{x} = u\cos\varphi - v\sin\varphi \\ \dot{y} = u\sin\varphi - v\cos\varphi \\ \dot{\varphi} = \frac{Ma}{I_z}(r_1 + v). \end{cases} \tag{4}$$

Our approach consists, first, to answer to the control problem of system (4) where δ_2 and T_2 are the control law. Second, to resolve the system of equations (3) where the unknows are δ and T.

From (3) we get

$$\begin{cases} \frac{-c_f}{M}(\frac{v}{u} + \frac{a^2M}{I_z}(\frac{r_1+v}{u}))\delta^2 + \delta(\frac{c_f}{M} + \delta_1) - T_1 = 0 \\ T = M\delta_1 - c_f(\frac{v}{u} + \frac{a^2M}{I_z}(\frac{r_1+v}{u}))\delta\,. \end{cases} \tag{5}$$

The lefthand side of the first equation of (5) can be seen as a polynomial function of degree two in δ, then, a necessary condition for the existence of a real solution is $\Delta \geq 0$ where

$$\Delta = \left(\frac{c_r}{M} + \delta_1\right)^2 - 4T_1\frac{c_r}{Mu}((1 + \frac{a^2M}{I_z})v + \frac{a^2M}{I_z}r_1)\,.$$

Therefore, if $\Delta \geq 0$ the system of equations (5); where the unkowns are δ and T, has real solutions and the control law to be applied to (1) is given by

$$\delta = \begin{cases} u\frac{MI_z}{2c_f}\frac{(\frac{c_f}{M}+\delta_1-\sqrt{\Delta})}{v(I_z+a^2M)+a^2Mr_1} & \text{if} \quad v \neq -\frac{r_1a^2M}{I_z+a^2\,M} \\ \frac{M\,T_1}{c_f+M\,\delta_1} & \text{elsewhere} \end{cases}$$

and T is given by the second equation of (5).

A simple reasoning shows that if δ_1 and T_1 are continuous and small enough then the function δ, defined above, is continuous.

Now, using realistic numerical values (see for instance [2] page 125) and taking into account the physical vehicle operation domain defind previously, straighforward computation shows that for $\delta_2 = T_2 = 0$ we have $\Delta \geq 0$. Then, by continuity, for δ_2 and T_2 small enough the condition $\Delta \geq 0$ still satisfied.

Finally, the problem control of (1) amounts to solve the control problem of (4) with δ_2 and T_2 small enough.

3 Cruise control

In this section we treat the problem of feedback stabilization of vehicle speed around a given equilibrium point defined by $(u, v, r_1) = (u_0, 0, 0)$, $u_0 > 0$.

Let us consider the system constituted only by the variables concerning the vehicle speed:

$$\begin{cases} \dot{u} = \delta_2 \\ \dot{v} = T_2 \\ \dot{r}_1 = \frac{M(a-fh)}{I_z}(v + r_1)u\frac{(a+b)c_r}{Ma}\frac{v}{u}\frac{b(b+a)c_r}{I_z}(\frac{r_1+v}{u}) \end{cases} \tag{6}$$

Now we state and prove the result concerning the cruise control.

Proposition 1 *The closed-loop system defined from (6) with the control law defined by*

$$\begin{cases} \delta_2 = -k_\delta\frac{u-u_0}{1+(u-u_0)^2} \\ T_2 = -k_\delta(\frac{M(a-fh)}{I_z}u - \frac{b(b+a)c_r}{I_z u})r_1 - k_T\frac{v}{1+v^2} \end{cases} \tag{7}$$

where k_δ , $k_T \in \mathbb{R}_+$, is asymptotically stable around the equilibrium point $(u_0, 0, 0)$.

Sketch of proof. Let

$V(u, v, r_1) = \frac{1}{2}(u - u_0)^2 + \frac{1}{2}v^2 + \frac{k_\delta}{2}\,r_1^2\,.$

$V \geq 0$ and $V = 0 \Leftrightarrow (u, v, r_1) = (u_0, 0, 0)$. By direct considerations on the physical domain we can prove that the time derivative of V verifies

$\dot{V}(u, v, r_1) < 0 \quad \text{for all} \quad (u, v, r_1) \neq (u_0, 0, 0)\,.$

Hence the closed-loop system (6)(7) is asymptotic stability around the equilibrium point $(u_0, 0, 0)$. □

Remark k_δ and k_T can be chosen sufficiently small with respect to any given bounds on δ_2 and T_2.

4 Conclusion

In this paper, the control design problem has been considered on the basis of a nonlinear vehicle dynamics model. A transformation has been proposed and has been shown to be useful for control. An effective solution for the dynamics cruise control problem have also been presented.

Works are in progress to show how this new approach can used for others vehicle dynamics problems.

References

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