# Nonlinear Adaptive Observer Design for Uncertain Dynamical Systems[1]

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**Abstract:**
We address the problem of adaptive state estimation for multivariable nonlinear systems in the presence of parameter uncertainty and bounded disturbances. The nonlinear adaptive observer uses a time varying gain matrix and usual Lyapunov arguments are employed in order to guarantee stability for the state and parameter errors. The presence of perturbations are considered and no SPR assumption is required in order to prove stability.

## 1- Introduction

Adaptive observers design for SISO (single-input single-output) linear time-invariant plant, has been largely investigated [1,2]. However, for nonlinear systems the results are scant. The existent schemes are applicable only to a small class of nonlinear systems subject to strong restrictions on the system structure, nonlinearities, perturbations and so on.

The first nonlinear adaptive observer was proposed in [3], where SISO nonlinear systems are considered and it is assumed that the original system can be transformed into a certain observable canonical form. In [4,5], adaptive observers for a class of MISO nonlinear systems are proposed, and it is assumed that the original system is linear with respect to the unknown parameters and can be transformed to special canonical forms (where the nonlinearities are functions of the output). In [6], an adaptive observer which presents an arbitrary fast exponential rate of convergence for both parameters and state estimates was proposed. However, the same class of MISO systems above mentioned is considered. More recently adaptive observers for particular class of MIMO Lipschitz nonlinear systems were developed by Rajamani et.al. [7,8]. However, the authors considered systems that are linear in the parameters and in which at least some of the measured outputs are such that the transfer matrix between these outputs and the unknown parameters are dissipative or SPR. It should be pointed out that all above mentioned observers can becomes instable in the presence of bounded disturbances.

In this work, we present a Lyapunov-based adaptive observer design for a more general class of nonlinear systems than those in [7,8], where bounded disturbances are not considered. More precisely, we considered multivariable nonlinear systems with a known linear part and known nonlinear perturbation scaled by an unknown linear parameter. Furthermore, it is considered that state and output bounded disturbances are present. The stability properties of the overall scheme are proved by employing usual Lyapunov arguments, which requires a time varying gain matrix. The proofs of the main results of this paper can be found in [9].

## 2- Uncertain system

Consider the class of nonlinear dynamical systems described by

$$\dot{\bar{x}}(t)=\bar{A}\bar{x}(t)+\theta_1\phi_1(\bar{x},u)+v_1(t) \qquad (2.1)$$

$$\bar{y}(t)=\bar{C}\bar{x}(t)+\theta_2\phi_2(\bar{x},u)+v_2(t) \qquad (2.2)$$

where $\bar{x}\in\Re^{\bar{n}}$, $u\in\Re^{m}$, $\bar{y}\in\Re^{\bar{q}}$, $\bar{A}\in\Re^{\bar{n}x\bar{n}}$ and $\bar{C}\in\Re^{\bar{q}x\bar{n}}$ are known matrices, $\theta_1\in\Re^{\bar{n}xl_1}$ and $\theta_2\in\Re^{\bar{q}xl_2}$ are unknown parameters, $\phi_1$, $\phi_2$ are known vector fields and $v_1$, $v_2$ are unknown bounded disturbances.

By redefining the state vector, equations (2.1)-(2.2) can be expressed as

$$\dot{x}(t)=Ax(t)+B\theta\phi(x,u)+v(t) \qquad (2.3)$$

$$y(t)=Cx(t) \qquad (2.4)$$

where $x=\begin{bmatrix}\bar{x}\\ \bar{y}\end{bmatrix}\in\Re^{n}$, $y\in\Re^{q}$, $A=\begin{bmatrix}\bar{A} & 0\\ \overline{CA} & 0\end{bmatrix}\in\Re^{nxn}$, $B\in\Re^{nxn}$ is a diagonal matrix, $C=\begin{bmatrix}0 & I\end{bmatrix}$, $\theta=B^{-1}\begin{bmatrix}\theta_1 & 0\\ \bar{C}\theta_1 & \theta_2\end{bmatrix}\in\Re^{nxl}$, $\phi=\begin{bmatrix}\phi_1\\ \dot{\phi}_2\end{bmatrix}:\Re^{n+m}\mapsto\Re^{l}$, and $v(t)=\begin{bmatrix}v_1\\ \bar{C}v_1+\dot{v}_2\end{bmatrix}\in\Re^{n}$. We impose the following assumptions on (2.3)-(2.4):

**(A1)** $\phi$ is Lipschitz in $x$ with Lipschitz constant $\gamma>0$, i. e.,

$$\left\|\phi(\hat{x},u,\dot{u})-\phi(x,u,\dot{u})\right\|\le\gamma\left\|\hat{x}-x\right\|,\ \forall\hat{x},x\in\Re^{n}$$

**(A2)** $v$ is bounded, i. e., for some positive constant $v_0$,

$$\left\|v(t)\right\|\le v_0\ \ \forall t\ge 0$$

**(A3)** $\theta$ is bounded in the following sense

$$\left\|\theta\right\|_F\le\gamma_1$$

**(A4)** At least some linear combination of the measured outputs is such that the system (2.3)-(2.4) is observable in a neighborhood $U_{x_0}$ of $x_0$, i.e., for some $i\in\{1,2,..,n\}$ $rank\left\{d\left(L_f^j\eta_i^T Cx\right),\ 0\le j\le n-1\right\}=n\ \ \forall x\in U_{\chi_0}$ [4].

## 3- Main result

Our main result reads as follows:

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***Theorem:***

Consider the nonlinear dynamical systems described by (2.3)-(2.4), the assumptions (A1)-(A4) and the following adaptive observer

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{\theta}\hat{\phi} + L\left(\hat{y},\hat{\phi}\right)\eta\tilde{y} \tag{3.1}$$

$$\dot{\hat{\theta}}_i = -\Gamma_i\hat{\phi}\eta_i^T\tilde{y} - \sigma_i\left|\eta_i^T\tilde{y}\right|\Gamma_i\hat{\theta}_i \text{ , for } i=1,2, ...,n \tag{3.2}$$

where, $\tilde{y} = \hat{y} - y$, $\hat{\phi} = \phi(\hat{x},u)$, $\hat{\theta}^T = \left[\hat{\theta}_1 \mid \hat{\theta}_2 \mid \cdots \mid \hat{\theta}_n\right] \in \Re^{lxn}$, $\Gamma_i = \Gamma_i^T > 0 \in \Re^{lxl}$, $\eta^T = [\eta_1 \mid \eta_2 \mid \cdots \mid \eta_n] \in \Re^{qxn}$ and $L(\tilde{y},\phi) \in \Re^{nxn}$ are properly designed matrices, $B = diag(b_i) \in \Re^{nxn}$, $b_i \neq 0 \in \Re$, $\sigma_i > 0 \in \Re$.

If the matrices $L, \eta, B$ are chosen such that

$$A + L\left(\tilde{y},\hat{\phi}\right)\eta C + \left(B - C^T\eta^T\right)^T\left(B - C^T\eta^T\right) + \left[\frac{\gamma^2}{4} + \gamma_1^2\|B\|_F^2 + \varphi\left(\tilde{y},\hat{\phi}\right)\right]I = Q < 0 \tag{3.3}$$

Then, $\tilde{x}, \tilde{\theta} \in L_\infty$.

***Comments:***

**1-** The selection of the matrix $L\left(\tilde{y},\hat{\phi}\right)$ to satisfy (3.3) is carried out simply by using linear design methods such as the LQ scheme or pole placement. Thus ours method is more easily applicable that [7,8] where the selection of gain matrix is not straightforward.

**2-** Note that Assumption (A1) is not restrictive, since most nonlinearities can be Lipschitz if the states can be assumed to be bounded. Furthermore, many nonlinearities, like the sinusoidal terms encountered in robotics, are globally Lipschitz [8].

## 4- Application

Consider the system represented by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \theta f(x,u)$$
$$y = x_1$$

where $\gamma = 2$, $\gamma_1 = 10$. It is clear that $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = I$, $C = [1 \quad 0]$. By using $\eta = [1 \quad 1]^T$ we obtain $Q = \begin{bmatrix} L_{11} + L_{12} + \varphi\left(\tilde{y},\hat{\phi}\right) + 201 & 1 \\ L_{21} + L_{22} & \varphi\left(\tilde{y},\hat{\phi}\right) + 203 \end{bmatrix}$. Consequently, if the entries of gain matrix $L$ are selected such that

$$L_{11} + L_{12} = -2\varphi\left(\tilde{y},\hat{\phi}\right) - 505$$
$$L_{21} + L_{22} = -\left(\varphi\left(\tilde{y},\hat{\phi}\right) + 304\right)\left(\varphi\left(\tilde{y},\hat{\phi}\right) + 203\right) - 10$$

we have that the eigenvalues of $Q$ are –1 and –10. Therefore by using equations (3.1) and (3.2) we obtain the following adaptive observer

$$\dot{\hat{x}}_1(t) = \hat{x}_2(t) - \left(2\varphi\left(\tilde{y},\hat{\phi}\right) + 505\right)\tilde{y}$$
$$\dot{\hat{x}}_2(t) = \hat{\theta}\hat{\phi} - \left(\varphi\left(\tilde{y},\hat{\phi}\right) + 304\right)\left(\varphi\left(\tilde{y},\hat{\phi}\right) + 203\right)\tilde{y} - 10\tilde{y}$$
$$\dot{\hat{\theta}} = -\Gamma\hat{\phi}\tilde{y} - \sigma|\tilde{y}|\Gamma\hat{\theta}$$

## 5- Conclusions

In this work we have proposed a simple method for adaptive state estimation of multivariable nonlinear systems. The proposed observer relies on properly designed matrices in order to guarantees stability. Application example is considered in order to illustrate the theory.

## References

[1] K.S. Narendra & A.M. Annaswamy, *Stable Adaptive Systems*, New Jersey, Prentice Hall Inc., 1989.

[2] P.A. Ioannou, & J. Sun, *Robust Adaptive Control*, Englewood Cliffs, NJ: Prentice Hall, 1995.

[3] G. Bastin & M.R. Gevers, "Stable Adaptive Observers for Nonlinear Time-Varying Systems," *IEEE Transactions on Automatic Control*, Vol. 33, No. 7, p. 650-658, July 1988.

[4] R. Marino, "Adaptive Observers for Single Output Nonlinear Systems," *IEEE Transactions on Automatic Control*, Vol. 35, No. 9, p. 1054-1058, September 1990.

[5] R. Marino & P. Tomei, "Global Adaptive Observers for Nonlinear Systems via Filtered Transformations," *IEEE Transactions on Automatic Control*, Vol. 37, No. 8, p. 1239-1245, August 1992.

[6] R. Marino & P. Tomei, "Adaptive Observers with Arbitrary Exponential Rate of Convergence for Nonlinear Systems," *IEEE Transactions on Automatic Control*, Vol. 40, No. 7, p. 1300-1304, July 1995.

[7] R. Rajamani & K. Hedrick "Adaptive Observer for Active Automotive Suspensions: Theory and Experiment," *Transactions on Control, Systems Technology*, Vol. 3, No. 1, p. 86-93, March 1995.

[8] Y. Man Cho, & R. Rajamani, "A Systematic Approach to Synthesis for Nonlinear Systems," *IEEE transactions on Automatic Control*, Vol. 42, p. 534-537, April 1997.

[9] J. A. R. Vargas & E. M. Hemerly, "Nonlinear Adaptive Observer Design for Uncertain Dynamical Systems, to be submitted to .*IEEE transactions on Automatic Control.*