# Global Configuration Stabilization for the VTOL Aircraft with Strong Input Coupling

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### Abstract

Trajectory tracking and configuration stabilization for the VTOL aircraft (vertical take off and landing) in the literature has been only considered for the case of slight or zero input coupling. In this paper, our main contribution is to address global configuration stabilization for the VTOL aircraft with a strong input coupling using a smooth static state feedback.

## 1 Introduction

In the past recent years, trajectory tracking and configuration stabilization of the VTOL aircraft has been extensively studied by many researchers [1], [2], [5]. Here, we consider configuration stabilization of the VTOL aircraft from any arbitrary initial configuration and speed to any position with zero roll angle and zero speed. The simplified dynamics of the VTOL aircraft is given in [2] as the following

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & -u_1\sin(\theta)+\epsilon u_2\cos(\theta) \\ \dot{y}_1 & = & y_2 \\ \dot{y}_2 & = & u_1\cos(\theta)+\epsilon u_2\sin(\theta)-g \\ \dot{\theta} & = & \omega \\ \dot{\omega} & = & u_2 \end{array} \tag{1}$$

where $\epsilon \neq 0, \theta$ is the roll angle and the plane moves in a vertical $(x_1, y_1)$-plane. In [1], approximate linearization techniques were used that ignore the coupling between the first two second-order subsystems in (1) and the $(\theta,\omega)$-subsystem and then treat the system as a slightly non-minimum phase system. Under a similar assumption, for $\epsilon = 0$ and sufficiently small $|\epsilon|$, stabilization of the origin for the VTOL aircraft is considered in [5]. Here, we are interested in the case $\epsilon \neq 0$ with arbitrarily large $|\epsilon|$ or the *strong input coupling case*. The main key point in control design for (1) is the decoupling of the first two second order subsystems $(x_1, x_2)$ and $(y_1.y_2)$ and the third subsystem $(\theta, \omega)$ with respect to the control input $u_2$. This is the main contribution of this paper. After, applying this decoupling global change of coordinates, the control design for the system in new coordinates is straightforward and can be done using standard backstepping procedure. Here is an outline of the paper. First, we explain our decoupling method. Then, we give the control design. Finally, we provide simulation results and concluding remarks.

## 2 Decoupling Method

To decouple three second-order subsystems of the VTOL aircraft in (1), we use a change of coordinates obtained from theorem 1 in [4] (also, see [3]) as the following

$$\begin{array}{rcl} z_1 & = & x_1-\epsilon\sin(\theta) \\ z_2 & = & x_2-\epsilon\cos(\theta)\omega \\ w_1 & = & y_1+\epsilon(\cos(\theta)-1) \\ w_2 & = & y_2-\epsilon\sin(\theta)\omega \\ \xi_1 & = & \theta \\ \xi_2 & = & \omega \end{array} \tag{2}$$

in new coordinates, we have

$$\begin{array}{rcl} \dot{z}_1 & = & z_2 \\ \dot{z}_2 & = & -\sin(\xi_1)\bar{u}_1 =: v_1 \\ \dot{w}_1 & = & w_2 \\ \dot{w}_2 & = & \cos(\xi_1)\bar{u}_1-g =: v_2 \\ \dot{\xi}_1 & = & \xi_2 \\ \dot{\xi}_2 & = & u_2 \end{array} \tag{3}$$

where $\bar{u}_1 = u_1 - \epsilon\xi_2^2$ is a new control. Now, we are ready to present our control design method.

## 3 Control Design

In this section, we present our control design method for configuration stabilization of the VTOL aircraft. Here is our main result:

**Proposition 3.1.** *There exists a smooth static state feedback in explicit form that globally asymptotically and locally exponentially stabilizes any desired configuration of the VTOL aircraft in (1) with zero speed and roll angle.*

**Proof.** Without loss of generality assume the desired configuration is $q = 0$ where $q = (x_1, y_1, \theta)$. Define

$$\begin{array}{rcl} r_1 & = & c_{11}z_1+c_{12}z_2 \\ r_2 & = & c_0\sigma(c_{21}w_1+c_{22}w_2) \end{array}$$

where $c_{i1}, c_{i2} < 0$ for $i = 1, 2$, $0 < c_0 < g$, and $\sigma(\cdot) = \tanh(\cdot)$. Given, $v_1 = r_1$ and $v_2 = r_2$, $z = 0$ and $w = 0$ are globally asymptotically stable for the $z$ and $w$ subsystems, respectively (Later on, we explain why $r_2$ should be bounded). This means that taking

$$\begin{aligned} \bar{u}_1 &= k_1(z, w) := \sqrt{r_1^2 + (r_2 + g)^2} \\ \tan \xi_1 &= k_2(z, w) = \frac{-r_1}{r_2 + g} \end{aligned}$$

$(z, w) = (0, 0)$ is globally asymptotically stable for the $(z, w)$-subsystem of (3). To avoid singularity of $\tan(\xi_1)$ in the last equation, we use a bounded control $r_2$ with a bound $c_0 < g$. At this point, a straightforward use of the backstepping procedure proves that a globally stabilizing static feedback law exists for (3). But we prefer to take a different approach for simplicity of calculations. After applying the change of coordinates and control

$$\begin{aligned} \mu_1 &= \tan \xi_1 - k_2(z, w) \\ \mu_2 &= (1 + \tan^2 \xi_1)\xi_2 - \dot{k}_2 \\ \bar{u}_2 &= (1 + \tan^2 \xi_1)(u_2 + 2 \tan \xi_1 \xi_2^2) - \ddot{k}_2 \end{aligned}$$

we get

$$\begin{aligned} \dot{\mu}_1 &= \mu_2 \\ \dot{\mu}_2 &= \bar{u}_2 \end{aligned}$$

Thus, applying $\bar{u}_2 = -d_1\mu_1 - d_2\mu_2$ with $d_1, d_2 > 0$ globally exponentially stabilizes $(\mu_1, \mu_2) = (0, 0)$ for the $\mu$-subsystem. The dynamics of the closed loop system is in the form

$$\begin{aligned} \dot{\eta} &= f(\eta, \mu) \\ \dot{\mu} &= A\mu \end{aligned} \tag{4}$$

where $A$ is Hurwitz and given $\mu = 0$ for $\dot{\eta} = f(\eta, 0)$, $\eta = 0$ is globally asymtotically and locally exponentially stable. It can be shown that for any solution of the $\mu$-subsystem the solution of the $\eta$-subsystem is uniformly bounded and the asymptotic stability of $(\eta, \mu) = 0$ for the cascade system in (4) follows from a theorem in [6] due to Sontag. Therefore, global asymptotic stabilization and local exponential stabilization of the origin is achieved for the VTOL aircraft. Explicit expression for $\ddot{k}_2$, can be obtained which is omitted due to space limitation. □

Figure 1 shows the trajectories of the VTOL aircraft from initial condition $(2, 3, 4, 1, \pi/3, 1)$ with $\epsilon = 1$. Also, Figure 2 shows the trajectory of the VTOL aircraft in the vertical $(x_1, y_1)$-plane for the same initial condition. It can be observed that the stabilizing maneuver in Figure 2 is rather aggressive and such a performance cannot be achieved using a linear controller.

## 4 Conclusion

In this paper, we considered global configuration stabilization of the VTOL aircraft with an arbitrary $\epsilon \neq 0$.

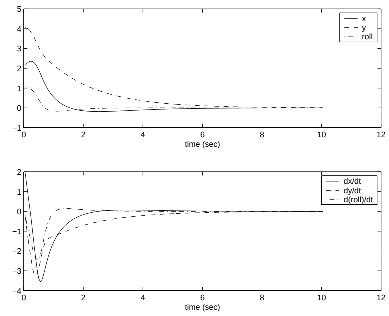


**Figure 1:** The state trajectory of the VTOL aircraft

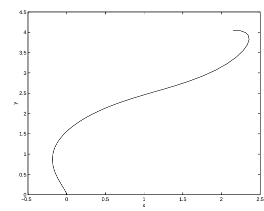

**Figure 2:** The state trajectory of the VTOL aircraft in $xy$-plane

We showed a key point in control of the VTOL aircraft is in decoupling of its three second-order subsystems using a global change of coordinates introduced in [4]. Then, we obtained a globally stabilizing smooth static state feedback law for the configuration of the VTOL aircraft in closed-form. Simulation results were presented for a difficult initial condition with initial roll angle of $\pi/3$.

### References

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