On Robust Nonlinear H_∞ Control Problem with Matched Parameter Uncertainty

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Abstract

This paper addresses robust H_∞ control problem for nonlinear systems with matched parameter uncertainty. The necessary and sufficient conditions for existence of global strong robust H_∞ dynamic compensators are obtained based on nonlinear matrix inequality (NLMI) approach. It reduces a robust H_∞ control problems to a standard H_∞ problem for an auxiliary nonlinear system.

1 Introduction

H_∞ control problems in linear systems and nonlinear systems have been reduced to problems of solving Riccati-type equations (AREs) and Hamilton-Jacobi equations (HJEs) (or inequalities HJIs) in state-space framework, respectively [1, 3]. It has recently been emphasized [2] that H_∞ control problem in linear systems can be cast into the form of first-order matrix polynomial inequalities called linear matrix inequalities (LMIs) instead of AREs, which belong to the group of convex problem and thus can efficiently find feasible and global solutions to them via inter-point methods [2]. Subsequently, motivated by LMI approach, the solution of nonlinear H_∞ control problem are characterized by means of nonlinear matrix inequalities (NLMIs) instead of the HJEs or HJIs in [4]. H_∞ control problems in nonlinear systems with uncertainty were discussed in [5, 6] and the conditions were reduced to HJEs (or HJIs). Motivated by the discussion above, in this paper we focus on robust H_∞ control problem in nonlinear systems with uncertainty by NLMI approach. The objective of this paper is to obtain the necessary and sufficient conditions for the existence of so-called strong robust H_∞ controllers in terms of NLMI.

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2 Robust H_∞-control problem

Consider the following nonlinear systems with uncertainty.

$$
\begin{aligned}
\dot{x} &= [A(x) + \Delta A(x,t)]x + B_1(x)w \\
&\quad +[B_2(x) + \Delta B_2(x,t)]u, \qquad (1)\\
z &= C_1(x)x + D_{12}(x)u, \\
y &= C_2(x)x + D_{21}(x)w,
\end{aligned}
$$

where the dimensions of x, w, u, z and y are n, p_1, p_2, q_1 and q_2 respectively. For all $x \in \mathbf{X}$, $A(x), B_i(x), C_i(x)$ and $D_{ij}(x)$ $(i,j = 1,2)$ are C^0 matrix-valued functions. $\Delta A(x,t)$ and $\Delta B(x,t)$ satisfy the following assumption.

Assumption 1 *Suppose $E(x) \in R^{n\times \tilde{p}_1}$, $H_i(x)$ $(i = 1,2)$ are known C^0 matrix-valued functions, and*

$$\left(\Delta A(x,t) \quad \Delta B(x,t) \right) = E(x)G(x,t) \left(H_1(x) \quad H_2(x) \right),$$

where $G(x,t) \in \Omega$ is unknown matrix-valued functions with

$$
\begin{aligned}
\Omega \;:=\; &\{G(x,t) | G'(x,t)G(x,t) \le I \;\; x \in \mathbf{X},\; t \in R^+, \\
&\text{the elements of } G(x,t) \text{ Lebesgue measurable}\}.
\end{aligned}
$$

For systems (1), we are interested in constructing the form of the dynamic compensator as follows

$$
\begin{aligned}
\dot{\xi} &= \hat{A}(\xi)\xi + \hat{B}(\xi)y, \\
u &= \hat{C}(\xi)\xi + \hat{D}(\xi)y,
\end{aligned} \qquad (2)
$$

where $\xi \in \mathbf{X_0}$ and $\mathbf{X_0}$ is a convex open subset in R^k $(k \le n)$ containing the origin. In this case, the closed-loop systems (1) and (2) can be rewritten as the following forms.

$$
\begin{aligned}
\dot{x}_c &= A_c x_c + B_c w \\
z &= C_c x_c + D_c w
\end{aligned} \qquad (3)
$$

where $x_c = \left(x' \quad \xi' \right)'$, $A_c = A_c(x_c), B_c = B_c(x_c), C_c = C_c(x_c), D_c = D_c(x_c)$ can be easily determined.

Inspired by [4], we concentrate on so-called strong robust H_∞-performance in this paper. We give the definition of strong robust H_∞-performance as follows.

Definition 1 *The closed-loop systems (3) are said to have strong robust H_∞-performance if there exists a C^0*

positive definite matrix-valued function $P_c = P_c(x_c) = P_c'(x_c) > 0$ *which satisfies the following inequality*

$$\begin{pmatrix} A_c'P_c + P_cA_c & P_cB_c & C_c' \\ B_c'P_c & -I & D_c' \\ C_c & D_c & -I \end{pmatrix} < 0 \tag{4}$$

and $\frac{\partial V_c(x_c)}{\partial x_c} = 2x_c'P_c$ *for all* $x_c \in \mathbf{X_c} = \mathbf{X} \times \mathbf{X_0} \subset R^n \times R^k$*, all uncertainty* $G \in \Omega$ *and some* C^1 *positive definite function* $V_c : \mathbf{X_c} \to \mathbf{R}^+$.

We give the necessary and sufficient conditions of existence of the dynamic compensator (2) such that the resulting closed-loop systems (3) have global strong robust H_∞ performance.

Theorem 2 *There exists a dynamic compensator (2) such that the resulting closed-loop systems have global strong robust* H_∞ *performance if and only if there exists a* C^0 *(or* C^∞*) positive function* $\epsilon = \epsilon(x_c)$ *for all* $x_c \in \mathbf{X_c}$ *such that the resulting closed-loop systems of (2) and the following systems (5) have global strong* H_∞ *performance.*

$$\begin{aligned} \dot{x} &= A(x)x + \begin{pmatrix} B_1(x) & \frac{1}{\sqrt{\epsilon}}E(x) \end{pmatrix} \tilde{w} + B_2(x)u, \\ \tilde{z} &= \begin{pmatrix} C_1(x) \\ \sqrt{\epsilon}H_1(x) \end{pmatrix} x + \begin{pmatrix} D_{12}(x) \\ \sqrt{\epsilon}H_2(x) \end{pmatrix} u, \\ y &= C_2(x)x + \begin{pmatrix} D_{21}(x) & 0 \end{pmatrix} \tilde{w}, \end{aligned} \tag{5}$$

where disturbance input $\tilde{w} \in R^{p_1+\tilde{p}_1}$.

Sketch of the proof: Necessity: Divide the LHS of (4) into two parts: $M_0(x_c)$ is independent of $G(x,t)$, and $M_\Delta(x_c)$ depends on $G(x,t)$. Suppose $\eta \neq 0$ is a column vector with appropriate dimension, we have $\eta' M_0(x_c)\eta < -\eta' M_\Delta(x_c,t)\eta$. Using the similar technique to [7] and after some manipulations, we can choose two semi-definite positive of continuous matrices $M_1(x_c)$ and $M_2(x_c)$ independent of $G(x,t)$ such that for each $x_c \in \mathbf{X_c}$, $[\eta' M_0(x_c)\eta]^2 > 4\eta' M_1(x_c)\eta\eta' M_2(x_c)\eta$. Then from [7], we have that for every $v \in \mathbf{X_c}$, there exists a positive constant ϵ_v such that $\epsilon_v^2 M_1(v) + \epsilon_v M_0(v) + M_2(v) < 0$. It follows from the continuity of every element of $M_i(x_c)$ ($i = 0,1,2$) for all $x_c \in \mathbf{X_c}$ that there exists a closed ball $B(v)$ centered at v such that for all $x_c \in B(v)$, $\epsilon_v^2 M_1(x_c) + \epsilon_v M_0(x_c) + M_2(x_c) < 0$, then there is a partition of unity $\{B_i, \phi_i\}$, where $B_i = B_i(v_i)$, the union of their interiors covers $\mathbf{X_c}$, ϕ_i is a smooth map on $\mathbf{R^{n+k}}$ supported on B_i with $\phi_i \geq 0$ and $\sum_{i=1}^\infty \phi_i(v) = 1$. Let $\epsilon = \epsilon(x_c) = \sum_{i=1}^\infty \epsilon_{v_i}\phi_i(x_c)$. Then $\epsilon(x_c)$ is well-defined and smooth on $\mathbf{X_c}$ since any given $x_c \in \mathbf{X_c}$, there is a definite number of index i such that $x_c \in B_i$, that is, there exist some positive integers $n_N > n_{N-1} > \cdots > n_1$ such that $\sum_{i=1}^N \phi_{n_i}(x_c) = 1$. Using the following inequality: $(\sum_{i=1}^N \mu_i a_i)^2 \leq \sum_{i=1}^N \mu_i^2 a_i$, where $\sum_{i=1}^N a_i = 1$ with $a_i \geq 0$ and $\mu_i \geq 0$, $i = 1,2,\cdots,N$. Therefore for each $x_c \in \mathbf{X_c}$, we have

$$\begin{aligned} & \epsilon^2(x_c)M_1(x_c) + \epsilon(x_c)M_0(x_c) + M_2(x_c) \\ = \; & [\sum_{i=1}^N \epsilon_{v_{n_i}}\phi_{n_i}(x_c)]^2 M_1(x_c) \\ & + \sum_{i=1}^N \epsilon_{v_{n_i}}\phi_{n_i}(x_c)M_0(x_c) + M_2(x_c) \\ \leq \; & \sum_{i=1}^N \phi_{n_i}(x_c)[\epsilon_{v_{n_i}}^2 M_1(x_c) + \epsilon_{v_{n_i}}M_0(x_c) + M_2(x_c)] \\ < \; & 0. \end{aligned} \tag{6}$$

From Schur complement, we can show that (6) implies that the close-loop systems (2) and (5) have strong global H_∞ performance [4]. This completes the proof of necessity.

Sufficiency: similar to [8], so the proof is omitted.

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