Noncausal robust set-point regulation of nonminimum-phase scalar systems [1]

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Abstract

In this paper we propose a method, based on dynamic inversion, for the set-point regulation of uncertain nonminimum-phase scalar systems. In particular, the worst-case settling time is minimized taking into account an amplitude constraint on the control variable and limits on the overshoot and undershoot of the output function. The application of the devised methodology yields to the connected design of both the controller and the reference command input. The latter is obtained by solving a special stable inversion problem on the nominal dynamic system that leads to a noncausal signal, causing the so-called "preaction control". Eventually, an optimization problem arises and its solution is gained by means of genetic algorithms. A simulation example shows the effectiveness of the overall methodology, despite the inherent difficulty of the addressed problem.

1 Introduction

Many works, mainly devoted to robust performance analysis, have appeared in the literature on the subject of robust regulation of linear systems affected by parametric uncertainties [1, 2, 3]. However, for the case of nonlinear uncertainties, effective techniques to synthesize robust controllers achieving predefined performances are apparently not available. Aiming to provide new tools for robust synthesis, in this paper we focus our efforts on the synthesis of a feedforward/feedback strategy for the set-point regulation of nonminimum-phase scalar systems nonlinearly depending on uncertain parameters. Specifically, the addressed problem is the suboptimal determination of a command (reference) signal and of a feedback controller that minimize the worst-case settling time subject to given overshoot and undershoot constraints and to an amplitude limit for the plant control actuator (see e.g. [4] for the problem of stabilizing a linear system using bounded controls). The general idea behind the provided solution for this problem is the combined synthesis of the controller and of the command signal using a dynamic inversion methodology. Previous work on this theme restricted to minimum-phase systems only has been reported in [5, 6], where the superiority of the dynamic inversion approach over conventional control schemes has also been shown. In this paper we extend the approach of [5] by admitting general nonminimum-phase plants so that the required system inversion performed on the nominal plant leads to a special stable inversion problem, for which a solution is provided using a Laplace transform technique. In the control literature, the necessity to pose and solve stable inversion problems, i.e. determining bounded inputs that generate given output functions, was pointed out especially for the nonlinear systems by Devasia and Paden [7, 8], Hunt and Meyer [9] and their coworkers [10, 11, 12, 13]. This stable inversion process determines the so-called "noncausal" inputs that imply actuator exertion before the actual output begins moving toward new desired values (preaction control).

The final step of the proposed design procedure requires the solution of a nonlinear optimization problem. This problem admits a solution (Theorem 2) by virtue of specially devised outputs [14] and an approximate solution can be gained using genetic algorithms [15].

Paper's organization. Section 2 formally expose the addressed problem. The synthesis of the controller and of the desired outputs, depending on the positive parameters α and τ respectively, is reported in Section 3. The subsequent section is devoted to the fundamental stable inversion problem for deriving the appropriate bounded reference signal (Theorem 1). The overall design procedure is exposed in Section 5, where the main result is given by a solvability condition to the final optimization problem (Theorem 2). An illustrative example is given in Section 6 and Section 7 concludes the paper.

2 The set-point constrained regulation problem

In the context of linear, time-invariant, continuous-time systems, consider an uncertain scalar strictly proper nonminimum-phase plant whose transfer func-

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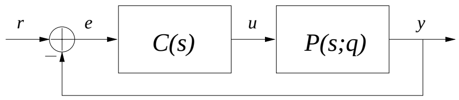

Figure 1: The unity-feedback system.

tion is:

$$P(s;\mathbf{q}) = \frac{b(s;\mathbf{q})c(s;\mathbf{q})}{s^h a(s;\mathbf{q})} \quad h \in \{0,1\} \tag{1}$$

where

$$\begin{aligned} a(s;\mathbf{q}) &= \textstyle\sum_{i=0}^{n-h} a_i(\mathbf{q})s^i, \\ b(s;\mathbf{q}) &= \textstyle\sum_{i=0}^{p} b_i(\mathbf{q})s^i, \\ c(s;\mathbf{q}) &= \textstyle\sum_{i=0}^{m} c_i(\mathbf{q})s^i, \end{aligned} \tag{2}$$

and $\mathbf{q} = [q_1, \ldots, q_g]^T$ belongs to a given multidimensional interval (box) $Q = [q_1^-, q_1^+] \times \cdots \times [q_g^-, q_g^+]$. The order of the plant is n, its relative order is $\rho = n-m-p$. Polynomials $a(s;\mathbf{q})$, $b(s;\mathbf{q})$ and $c(-s;\mathbf{q})$ are (Hurwitz) stable for all $\mathbf{q} \in Q$ and their coefficients are continuous nonlinear functions over Q. In order to avoid degeneracies, we assume

$$\begin{gathered} a_{n-h}(\mathbf{q}) > 0 \quad \forall \mathbf{q} \in Q, \\ b_p(\mathbf{q}) > 0 \quad \forall \mathbf{q} \in Q, \\ \{c_m(\mathbf{q}) > 0 \quad \forall \mathbf{q} \in Q\} \vee \{c_m(\mathbf{q}) < 0 \quad \forall \mathbf{q} \in Q\} \end{gathered} \tag{3}$$

In the context of a unity-feedback system (see Figure 1), we search for a feedforward/feedback control strategy in order to obtain a "robust" transition from a previous set-point value y_0 to a new one y_1. Without loss of generality in the following we will assume $y_0 = 0$. Obviously, the first requirement to be satisfied is the robust stability of the closed-loop over the uncertain domain Q. Moreover, this transition has to satisfy an overshoot and an undershoot limitation, an amplitude constraint on the control variable $u(t)$ and has to minimize the (worst-case) settling time. In other words, the above problem can be stated as follows: determine a reference function $r(t)$ and a controller $C(s)$ such that

1. the closed-loop system is stable for all $\mathbf{q} \in Q$;
2. $\lim_{t\to\infty} y(t) = y_1$ for all $\mathbf{q} \in Q$ (steady-state condition);
3. the overshoot in response to $r(t)$ is bounded by a given $\bar{O}$ for all $\mathbf{q} \in Q$;
4. the undershoot in response to $r(t)$ is bounded by a given $\bar{U}$ for all $\mathbf{q} \in Q$;
5. the absolute value of the manipulative input $u(t)$ is bounded by a given u_{sat} for all $\mathbf{q} \in Q$;
6. it is minimized the worst-case settling time.

Searching for the true global solution of the above problem is extremely difficult, so that in the following we search for a practicable sub-optimal but effective solution based on the concept of dynamic inversion.

3 Controller's structure and desired outputs

Introduce the "nominal" parameter vector $\mathbf{q}^0 := \text{mid}(Q)$, i.e. the midpoint center of the box Q (see Section 6). Then, the following biproper parameterized controller, based on nominal stable pole-zero cancellations, is chosen:

$$C(s;\alpha) := \text{sign}(c(0;\mathbf{q}^0))\alpha\frac{a(s;\mathbf{q}^0)}{sb(s;\mathbf{q}^0)d(s)} \quad \text{if} \quad h=0$$

$$C(s;\alpha) := \text{sign}(c(0;\mathbf{q}^0))\alpha\frac{a(s;\mathbf{q}^0)}{b(s;\mathbf{q}^0)d(s)} \quad \text{if} \quad h=1$$

where $\alpha \in \mathbb{R}^+$ and $d(s)$ is a user-chosen monic polynomial of degree $n-p-1$. In any case ($h=0$ or $h=1$) the closed-loop characteristic polynomial is:

$$\begin{aligned} &sb(s;\mathbf{q}^0)d(s)a(s;\mathbf{q})+ \\ &\text{sign}(c(0;\mathbf{q}^0))\alpha a(s;\mathbf{q}^0)c(s;\mathbf{q})b(s;\mathbf{q}), \end{aligned} \tag{4}$$

which has degree $2n$ if $h=0$ and $2n-1$ if $h=1$.

Proposition 1 *Assume that the monic polynomial $d(s)$ is Hurwitz stable. Then, there exists a sufficiently small $\alpha \in \mathbb{R}^+$ such that the closed-loop system is stable for all $\mathbf{q} \in Q$.*

Proof. Omitted for brevity.

The output function $y(t)$ is chosen as a polynomial, in such a way that we have a smooth transition between 0 and y_1, to be completed without undershooting and overshooting in the interval $[0,\tau]$. Specifically, we consider over the domain $[0,\tau]$ a polynomial $y(t)$ of order $2v+1$:

$$y(t) = c_{2v+1}t^{2v+1} + c_{2v}t^{2v} + \cdots + c_1 t + c_0,$$

where the $2v+2$ coefficients are determined by solving the following parameterized system with $2v+2$ linear equations:

$$\begin{cases} y(0) = 0; & y(\tau) = y_1 \\ y^{(1)}(0) = 0; & y^{(1)}(\tau) = 0 \\ \vdots & \\ y^{(v)}(0) = 0; & y^{(v)}(\tau) = 0 \end{cases}$$

The above algebraic system always admits a unique solution which is given by the following closed-form expression [14]:

$$y(t;\tau) = y_1\beta(v,\tau)\int_0^t \sigma^v(\tau-\sigma)^v d\sigma, \tag{5}$$

where the positive coefficient $\beta(v,\tau)$ is given by

$$\beta(v,\tau) := \left(\int_0^\tau \sigma^v(\tau-\sigma)^v d\sigma\right)^{-1} = \frac{(2v+1)!}{(v!)^2\tau^{2v+1}}. \tag{6}$$

Derivation of (5) is easy by considering that

$$y^{(1)}(t;\tau) = y_1\beta(v,\tau)t^v(\tau-t)^v.$$

The order of the polynomial is selected to obtain a function $y(t;\tau)$ belonging to $C^{(v)}$ over $(-\infty,+\infty)$. Outside the interval $[0,\tau]$ the function $y(t;\tau)$ is equal to 0 for $t<0$ and equal to y_1 for $t>\tau$.

4 Solving the stable inversion problem

The devised function $y(t;\tau)$ of formulae (5) and (6) is the ideal plant output to be obtained by injecting an appropriate reference signal in the unity-feedback system of Figure 1. A natural choice is to compute the reference, or command, signal by means of a dynamic inversion technique. Specifically, we consider the nominal plant transfer function so that the overall dynamics from r to y is simplified by exact stable pole-zero cancellations between the controller and the plant. The nominal open-loop transfer function is

$$L(s;\alpha) := \mathrm{sign}(c(0;\mathbf{q}^0))\alpha\frac{c(s;\mathbf{q}^0))}{sd(s)} \tag{7}$$

and the corresponding transfer function from r to y is given by

$$T_{yr}(s;\alpha) := \frac{\mathrm{sign}(c(0;\mathbf{q}^0))\alpha c(s;\mathbf{q}^0)}{sd(s)+\mathrm{sign}(c(0;\mathbf{q}^0))\alpha c(s;\mathbf{q}^0)} \tag{8}$$

The order of $T_{yr}(s;\alpha)$ is $n-p$ and its relative order is ρ. Denote with $Y(s;\tau)$ the Laplace transform of $y(t;\tau)$ and, performing the inverse Laplace transformation, compute

$$r_u(t;\alpha,\tau) := \mathcal{L}^{-1}[T_{yr}^{-1}(s;\alpha)Y(s;\tau)]. \tag{9}$$

First, we note that, due to the relative degree of $T_{yr}(s;\alpha)$, $r_u(t;\alpha,\tau)$ is continuous over $(-\infty,+\infty)$ only if $v\geq\rho$, i.e. the desired output $y(t;\tau)$ has to belong to $C^{(\rho)}$ at least. Secondly, we observe that $r_u(t;\alpha,\tau)$ is the reference signal computed with the standard dynamic inversion technique, i.e. the method that consider the overall system at the equilibrium for $t=0$.
Unfortunately, due to the unstable zero dynamics, $r_u(t;\alpha,\tau)$ is unbounded over $[0,\infty)$, so that it is useless for reference signal purposes. As a consequence, it is necessary to introduce the following problem.
Stable Inversion Problem. Assume that the nominal unity-feedback system is at the equilibrium for $t=-\infty$. Determine a bounded reference signal defined over $(-\infty,+\infty)$ such that the corresponding output is exactly given by $y(t;\tau)$.
In the following we give an explicit constructive solution to the above problem. For notational simplicity we assume that all the zeros of $c(s;\mathbf{q}^0)$ are distinct. Hence, polynomial $c(s;\mathbf{q}^0)$ can be factorized according to:

$$\begin{array}{l} c(s;\mathbf{q}^0) = c_z(s-z_1)\cdot\dots\cdot(s-z_l)\cdot \\ (s^2-2\delta_1\omega_1 s+\omega_1^2)\cdot\dots\cdot(s^2-2\delta_h\omega_h s+\omega_h^2) \end{array} \tag{10}$$

where $z_i\in\mathbb{R}^+$, $i=1,\dots,l$ and $\omega_j\in\mathbb{R}^+$, $\delta_j\in(0,1)$, $j=1,\dots,h$. Define the reference function $r(t;\alpha,\tau)$ as follows:

$$\begin{cases} r(t;\alpha,\tau) := r_u(t;\alpha,\tau)+r_c(t;\alpha,\tau) & \text{for} \quad t\leq\tau \\ r(t;\alpha,\tau) := y_1 & \text{for} \quad t>\tau \end{cases} \tag{11}$$

where

$$\begin{array}{l} r_c(t;\alpha,\tau) := k_1 e^{z_1(t-t_1)}+\dots+k_l e^{z_l(t-t_l)}+ \\ k_{l+1}e^{\delta_1\omega_1(t-t_{l+1})}\sin(\omega_1\sqrt{1-\delta_1^2}(t-t_{l+1})+\phi_1)+\cdots \\ +k_{l+h}e^{\delta_h\omega_h(t-t_{l+h})}\sin(\omega_h\sqrt{1-\delta_h^2}(t-t_{l+h})+\phi_h). \end{array} \tag{12}$$

Theorem 1 *There exist $t_i\in\mathbb{R}$, $k_i\in\{-1,0,+1\}$, $i=1,\dots,l+h$ and $\phi_j\in[0,\pi)$, $j=1,\dots,h$ such that the function $r(t;\alpha,\tau)$ defined in (11) and (12) is the solution to the stable inversion problem.*

Proof. A constructive proof of Proposition 2 is briefly sketched. The coefficients t_i, k_i and ϕ_i appearing in (12) can be determined by solving the following equation system (note that $m=l+2h$):

$$\begin{cases} r_c(\tau;\alpha,\tau) = y_1 - r_u(\tau^+;\alpha,\tau) \\ Dr_c(\tau;\alpha,\tau) = -Dr_u(\tau^+;\alpha,\tau) \\ \vdots \\ D^{m-1}r_c(\tau;\alpha,\tau) = -D^{m-1}r_u(\tau^+;\alpha,\tau) \end{cases} \tag{13}$$

where $(i=0,1,\dots,p-1)$

$$D^i r_u(\tau^+;\alpha,\tau) := \lim_{t\to\tau^+} D^i r_u(t;\alpha,\tau). \tag{14}$$

For brevity, we expose the case of $m=l=2$ (see the example in Section 6).
From equations (13) we deduce

$$\begin{cases} k_1 e^{z_1(\tau-t_1)}+k_2 e^{z_2(\tau-t_2)} = y_1-r_u(\tau^+;\alpha,\tau) \\ k_1 z_1 e^{z_1(\tau-t_1)}+k_2 z_2 e^{z_2(\tau-t_2)} = -Dr_u(\tau^+;\alpha,\tau) \end{cases} \tag{15}$$

Hence, defining

$$\eta_1 := z_2[y_1-r_u(\tau;\alpha,\tau)]+Dr_u(\tau^+;\alpha,\tau) \tag{16}$$

$$\eta_2 := -z_1[y_1-r_u(\tau;\alpha,\tau)]-Dr_u(\tau^+;\alpha,\tau) \tag{17}$$

it follows that

$$k_1 e^{z_1(\tau-t_1)} = \eta_1(z_2-z_1)^{-1}, \tag{18}$$

$$k_2 e^{z_2(\tau-t_2)} = \eta_2(z_2-z_1)^{-1}. \tag{19}$$

The coefficients k_1 and k_2 are computed according to

$$k_1 = \mathrm{sign}[\eta_1(z_2-z_1)^{-1}], \tag{20}$$

$$k_2 = \mathrm{sign}[\eta_2(z_2-z_1)^{-1}]. \tag{21}$$

Therefore, from (18)-(19) and using (20)-(20) we obtain

$$t_1 = \tau - z_1^{-1}\ln[\eta_1 k_1(z_2-z_1)^{-1}], \tag{22}$$

$$t_2 = \tau - z_2^{-1}\ln[\eta_2 k_2(z_2-z_1)^{-1}]. \tag{23}$$

It is evident from the above passages that the found solution (20)-(23) corresponds to a unique function $r_c(t;\alpha,\tau)$ satisfying system (15). This fact also holds in full generality ($l,h \in \mathbb{N}$).

In the following, consider the nominal unity-feedback system at the equilibrium for $t=-\infty$. Applying the reference signal given by (12), defined on $(-\infty,+\infty)$, we evidently have that the output signal is zero over $(-\infty,+\infty)$, for any choice of the involved coefficients t_i, k_i and ϕ_j. Hence, by virtue of linear superposition, applying $r_u(t;\alpha,\tau)+r_c(t;\alpha,\tau)$ as a reference signal, defined over $(-\infty,+\infty)$, the output is given by $y(t;\tau)$. Now it is shown that, for the special choice of coefficients t_i, k_i and ϕ_j satisfying system (13), also the reference signal defined in (11) gives the signal $y(t;\tau)$ as corresponding output. Indeed, the nominal unity-feedback system is at the equilibrium for $t>\tau$.

Denote the transfer function $T_{yr}(s;\alpha)$ as

$$T_{yr}(s;\alpha)=\frac{\mu_m s^m+\cdots+\mu_1 s+1}{\lambda_{n-p}s^{n-p}+\cdots+\lambda_1 s+1}$$

so that the differential equation describing the nominal unity-feedback dynamics is

$$\begin{aligned}&\lambda_{n-p}D^{n-p}y+\cdots+\lambda_1 Dy+y=\\&\mu_m D^m r+\cdots+\mu_1 Dr+r.\end{aligned}$$

This equation is satisfied over $(-\infty,+\infty)$ by functions $r_u(t;\alpha,\tau)+r_c(t;\alpha,\tau)$ and $y(t;\tau)$. Hence, for $t>\tau$ it follows that

$$\begin{aligned}&\mu_m D^m[r_u(t;\alpha,\tau)+r_c(t;\alpha,\tau)]+\cdots+\mu_1 D[r_u(t;\alpha,\tau)+\\&r_c(t;\alpha,\tau)]+r_u(t;\alpha,\tau)+r_c(t;\alpha,\tau)=y_1.\end{aligned}$$

Taking into account that $r_c(t;\alpha,\tau)\in C^\infty$, from (13) we have

$$\begin{cases}r_u(\tau^+;\alpha,\tau)+r_c(\tau^+;\alpha,\tau)=y_1\\ D[r_u(\tau^+;\alpha,\tau)+r_c(\tau^+;\alpha,\tau)]=0\\ \vdots\\ D^{m-1}[r_u(\tau^+;\alpha,\tau)+r_c(\tau^+;\alpha,\tau)]=0\end{cases}$$

Consequently, by virtue of the uniqueness result of ordinary differential equations,

$$r_u(t;\alpha,\tau)+r_c(t;\alpha,\tau)=y_1\quad\forall t>\tau.$$

Evidently, $r_u(t;\alpha,\tau)$ is bounded over $(-\infty,\tau]$ and, considering that all the roots of $c(s;\mathbf{q}^0)$ have positive real parts, also $r_c(t;\alpha,\tau)$ is bounded over $(-\infty,\tau]$. Therefore, for the special performed choice of the coefficients, $r_u(t;\alpha,\tau)+r_c(t;\alpha,\tau)$ is bounded over $(-\infty,+\infty)$ and coincides with the function $r(t;\alpha,\tau)$ defined in (11). □

In order to practically use the synthesized function (11) it is necessary to truncate $r(t;\alpha,\tau)$ resulting in an approximate generation of the desired output $y(t;\tau)$. This can be done with arbitrarily precision given any small parameter $\varepsilon_0>0$. Indeed, compute

$$t_{oc}:=\max\{t_c:|r_c(t;\alpha,\tau)|\le\varepsilon_0\quad\forall t\in(-\infty,t_c]\}$$

and define

$$t_0:=\min\{0,t_{oc}\}.$$

Hence, the approximate reference signal to be used is

$$r_a(t;\alpha,\tau):=\begin{cases}0 & \text{for}\quad t<t_0\\ r(t;\alpha,\tau) & \text{for}\quad t\ge t_0.\end{cases}$$

Note that t_0 depends on both α and τ. Moreover, it can happen that $t_0<0$, resulting in the so-called "pre-action control" [16, 10, 17].

5 Design procedure

In the previous section we have determined a controller $C(s;\alpha)$ and a command signal $r_a(t;\alpha,\tau)$ that depend on the free positive parameters α and τ. The overall design procedure to solve the set-point constrained regulation problem posed in Section 2 can then be outlined as follows.

- Choose the monic polynomial $d(s)$ ensuring robust closed-loop stability for at least one positive value of α.
- Determine the optimal parameters α^* and τ^* that minimize the worst-case settling time subject to all the required constraints.

Step 1 can be simply accomplished by choosing a (Hurwitz) stable $d(s)$. Indeed, Proposition 1 assures that robust stability is achieved for a sufficiently small α. It is worth noting that $d(s)$ has not to be necessarily stable; for example, see the design example in Section 6. In any case, the control engineer may select the polynomial $d(s)$ on the grounds of bandwidth considerations, root locus reasoning, physical judgement, etc.

Denote by $\xi(s;\alpha,\mathbf{q}):=\sum_{i=0}^{n_c}\xi_i(\alpha,\mathbf{q})s^i$ ($n_c:=2n$ if $h=0$ and $n_c:=2n-1$ if $h=1$) the characteristic polynomial appearing in (4). $H_i(\alpha,\mathbf{q})$ designates the ith order Hurwitz determinant related to $\xi(s;\alpha,\mathbf{q})$. Robust stability can be taken into account by using the following result.

Proposition 2 *The closed-loop system is stable for all $\mathbf{q}\in\mathcal{Q}$ if and only if*

$$\xi_0(\alpha,\mathbf{q})>0\quad\forall\mathbf{q}\in\mathcal{Q};\tag{24}$$

$$H_{n_c-1}(\alpha,\mathbf{q})>0\quad\forall\mathbf{q}\in\mathcal{Q};\tag{25}$$

$$\xi_1(\alpha,\mathbf{q}^0)>0,\quad\xi_3(\alpha,\mathbf{q}^0)>0,\ldots,\xi_v(\alpha,\mathbf{q}^0)>0;\tag{26}$$

$$H_{n_c-3}(\alpha,\mathbf{q}^0)>0,\ \ H_{n_c-5}(\alpha,\mathbf{q}^0)>0,\ldots,H_w(\alpha,\mathbf{q}^0)>0\tag{27}$$

with $v:=n_c-1$, $w:=3$ if n_c is even and $v:=n_c-2$, $w:=2$ if n_c is odd.

Proof. Omitted for brevity.

Define $y(t;\alpha,\tau,\mathbf{q})$ and $u(t;\alpha,\tau,\mathbf{q})$ the output and control signals respectively when the command function is $r_a(t;\alpha,\tau)$. The associated settling time can be pertinently defined as

$$\begin{aligned}t_s(\alpha,\tau,\mathbf{q}):=|t_0|+\min\{s\in\mathbb{R}^+:\\|y(t;\alpha,\tau,\mathbf{q})-y_1|\le0.02y_1\quad\forall t\ge s\}\end{aligned}\tag{28}$$

Consequently, the worst-case settling time is

$$t_{wcs}(\alpha,\tau) := \max_{\mathbf{q}\in\mathcal{Q}} t_s(\alpha,\tau,\mathbf{q}). \tag{29}$$

Without conservativeness, the optimal choice of α and τ entails solving the following semi-infinite optimization problem

$$\min_{\alpha,\tau\in\mathbb{R}^+} t_{wcs}(\alpha,\tau) \tag{30}$$

subject to:

$$\xi_0(\alpha,\tau) \geq \varepsilon \quad \forall \mathbf{q}\in\mathcal{Q}; \tag{31}$$

$$H_{n_c-1}(\alpha,\tau) \geq \varepsilon \quad \forall \mathbf{q}\in\mathcal{Q}; \tag{32}$$

$$\xi_1(\alpha,\mathbf{q}^0) \geq \varepsilon, \quad \xi_3(\alpha,\mathbf{q}^0) \geq \varepsilon, \ldots, \xi_v(\alpha,\mathbf{q}^0) \geq \varepsilon; \tag{33}$$

$$H_{n_c-3}(\alpha,\mathbf{q}^0) \geq \varepsilon, \ H_{n_c-5}(\alpha,\mathbf{q}^0) \geq \varepsilon, \ldots, H_w(\alpha,\mathbf{q}^0) \geq \varepsilon; \tag{34}$$

$$y(t;\alpha,\tau,\mathbf{q}) \leq (1+0.01\bar{O})y_1 \quad \forall t\geq 0 \quad \forall \mathbf{q}\in\mathcal{Q}; \tag{35}$$

$$y(t;\alpha,\tau,\mathbf{q}) \geq -0.01\bar{U}y_1 \quad \forall t\geq 0 \quad \forall \mathbf{q}\in\mathcal{Q}; \tag{36}$$

$$|u(t;\alpha,\tau,\mathbf{q})| \leq u_{sat} \quad \forall t\geq 0 \quad \forall \mathbf{q}\in\mathcal{Q}; \tag{37}$$

where ε is a sufficiently small threshold parameter.

Remark. Note that the settling time definition incorporates the preaction time $|t_0|$ even though during the interval $(t_0, 0)$ the output signal is almost identically zero. This appears technically sound because during $(t_0, 0)$ the overall system is out of equilibrium.

Solving problem (30) means to find solution to the complete set-point regulation problem. Indeed all the control requirements are incorporated as inequality constraints of (30) with the exception of the steady-state regulation condition that is guaranteed through robust stability by virtue of the controller structure (the internal model principle is satisfied). The main result of this section is the following.

Theorem 2 *For any given overshoot limit $\bar{O} \in \mathbb{R}^+$ and any undershoot limit $\bar{U} \in \mathbb{R}^+$, optimization problem (30) has a solution if*

$$u_{sat} > (\min_{\mathbf{q}\in\mathcal{Q}} |P(0,\mathbf{q})|)^{-1}|y_1| \quad \text{when} \quad h=0, \tag{38}$$

$$u_{sat} > 0 \quad \text{when} \quad h=1. \tag{39}$$

Proof. Omitted for brevity.

Problem (30) is a nonlinear semi-infinite optimization problem for which an approximate solution can be obtained by relaxing the semi-infinite constraints. For example, the box $\mathcal{Q}$ can be substituted with its vertexes and a genetic algorithm [15] can be adopted for estimating α^* and τ^*. This approach requires, as explained in [5] and [6], an algorithmic post-processing to ensure the feasibility of the solution. A more effective but effortful, approach could be using the genetic/interval algorithm of Guarino Lo Bianco and Piazzi [19, 20].

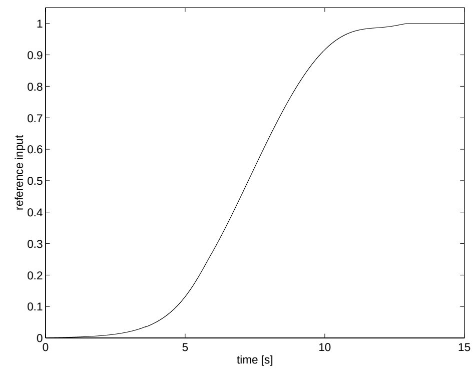

Figure 2: The reference command input.

6 A worked example

As an illustrative example we consider a plant with $h = 0$:

$$P(s;\mathbf{q}) = \frac{(s-q_1)(s-q_2)}{(s^2+2q_3s+1)(s+2)} \tag{40}$$

where $\mathbf{q} = [q_1, q_2, q_3] \in Q = [0.8, 1.2] \times [1.6, 2.4] \times [0.5, 0.7]$. We also have fixed $y_1 = 1$, $\bar{S} = 5\%$, $\bar{U} = 3\%$ and $u_{sat} = 2$. Following the methodology described in the paper about the overall control system design, we set $a(s;\mathbf{q}^0) = (s^2 + 2q_3^0 s + 1)(s+2) = (s^2 + 1.2s + 1)(s+2)$, $b(s;\mathbf{q}^0) = 1$, and $c(s;\mathbf{q}^0) = (s-q_1^0)(s-q_2^0) = (s-1)(s-2)$. Hence, the following controller results (note that $m = 2$):

$$C(s;\alpha) = \alpha\frac{(s^2+1.2s+1)(s+2)}{sd(s)} \tag{41}$$

We selected $d(s) = s^2 + 35$, so that all the poles of $C(s;\alpha)$ are purely imaginary. In order to have $r(t;\alpha,\tau) \in C^{(0)}$, a third-order polynomial ($v = 1$, note that the plant relative order is $\rho = 1$) has been chosen as output function, so that we have:

$$y(t;\tau) = -\frac{2}{\tau^3}t^3 + \frac{3}{\tau^2}t^2 \qquad t\in[0,\tau].$$

The optimal values of α and τ have been determined by means of a genetic algorithm [15]. The resulting values are $\alpha^* = 7.57$ and $\tau^* = 6.99$, which results in an optimal worst-case settling-time t^*_{wcs} equal to 16.44s, with preaction time $t_0^* = -6.2$s ($\varepsilon_0 = 10^{-3}$). The reference input $r_a(t;\alpha^*,\tau^*)$ is plotted in Figure 2, while the worst-case output (which occurs when $\mathbf{q} = [0.8, 1.6, 0.5]$) is shown in Figure 3 and the corresponding control variable is reported in Figure 4. For technical convenience, in all the plots the zero time has been shifted to t_0^*.

7 Conclusions

In this paper we proposed a dynamic inversion based approach for the set-point constrained regulation of nonminimum-phase scalar systems subject to structured uncertainties. The methodology provides the coupled design of the controller and of the reference

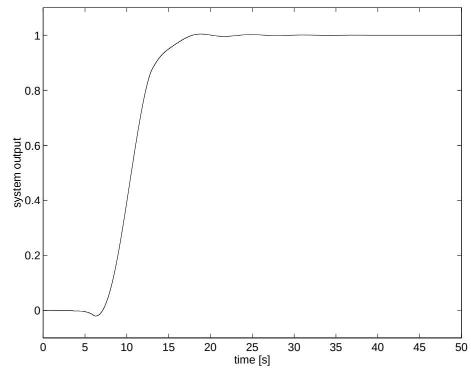

Figure 3: The worst-case output.

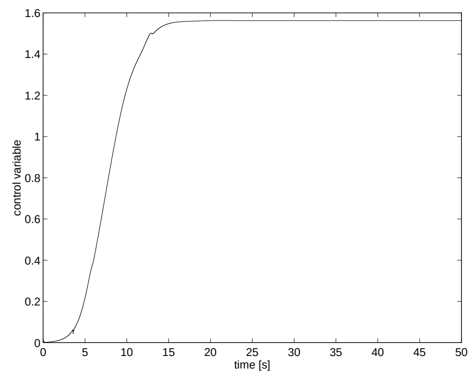

Figure 4: The control variable corresponding to the worst-case output.

command input, for which the concept of "preaction control" is exploited, in order to achieve a stable dynamic inversion. A key role in the overall technique is played by the choice of a polynomial function as the desired output signal (see [21] for the application of such a function in the context of motion planning for a vibratory system). This assures an inherent robustness to the control system, so that high performances can be attained, as demonstrated by the given example.

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