Robust Control of Nonlinear Systems by Estimating Time Variant Uncertainties

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Abstract

In this paper, robust control design is considered for nonlinear systems with time variant uncertainties. Instead of assuming that bounding function on uncertainties is either known or parameterizable in terms of unknown constants, uncertainties or their bounding function will be estimated. It is shown that bounded uncertainties from a known or partially known exo-system can be estimated as part of a globally stabilizing robust control. The proposed method extends the existing results of adaptive robust control, and it makes robust control more applicable by requiring less information on uncertainties.

1 Introduction

Robust control of nonlinear uncertain systems have attracted a lot of attention. Much of the interests stem from the fact that nonlinear and uncertain dynamics are common in many applications and that robust control is the design method to guarantee stability and performance. Robust stabilizability in terms of structure properties of systems and their uncertainties, design procedures, robust stability and performance, properties of robust control, and robust optimality are among of the subjects studied in [5, 2, 1, 3, 11, 12, 6, 7, 8, 4, 17, 15, 13]. Although exact knowledge of the plant is not required, robust control designs have to be done according to the extent of information that is known. Information such as bounding function on and structural properties of the uncertainties in the system is required in most of the existing results. It is adequate to assume in analytical analysis that uncertainties be bounded in a certain sense, but getting information about the size of uncertainties could be very difficult in many applications. Since uncertainties are uncertain by nature, the less information about uncertainty we need to know in design the more applicable the resulting robust control becomes.

In a standard robust control design result such as [2], bounding function on nonlinear uncertainty is assumed to be known. In this case, robust control is designed to compensate for the worst uncertainty within the bounding function. In case that bounding function can be parameterized in terms of unknown constants, adaptive robust control can be designed to adaptively estimate the bounding function [3]. While this result represents a major step forward in reducing required information on uncertainty, it is often restrictive and conservative as the unknowns have to be constants. In this paper, we investigate how to design robust control for systems whose uncertainties or their bounding functions are parameterized in terms of unknown outputs from a known or partially known exo-systems. Using the proposed method, time variant signals, not just unknown constants, will be estimated and a globally stabilizing robust control can be found.

2 Problem Statement

An uncertain system consider in the paper is of form

$$\begin{aligned}\dot{x} &= f(x,t) + \Delta f_u(x,\upsilon,t) + \Delta B_u(x,\upsilon,t)u \\ &\quad + B(x,t)[\Delta f_m(x,\upsilon,t) + \Delta B_m(x,\upsilon,t)u + u], \quad (1)\end{aligned}$$

where $x(t) \in \Re^n$ is the state, $u(t) \in \Re^m$ is the control to be designed, $\Omega \subset \Re^p$ is a bounded set, $\upsilon(t) \in \Omega$ denote the time variant uncertainties, $f(x,t)$ and $B(x,t)$ are known parts of system dynamics, $\Delta f_u(x,\upsilon,t)$, $\Delta B_u(x,\upsilon,t)$, $\Delta f_m(x,\upsilon,t)$ and $\Delta B_m(x,\upsilon,t)$ are uncertainties, and the subscripts $_u$ and $_m$ in (1) denote the so-called unmatched and matched uncertainties [2, 13], respectively.

The robust control problem is to design a control $u(x,t)$ such that the resulting closed loop system is stable (in the sense of either asymptotical stability or stability of uniform ultimate boundedness [2, 13]) for all possible values of bounded uncertain vector $\upsilon(t)$ in the prescribed set Ω. Robust control design requires several technical assumptions. Typically, robust control design is based upon stability or stabilizability of known dynamics. Specifically, the system consisting of known dynamics

$$\dot{x} = f(x,t) + B(x,t)u. \quad (2)$$

is referred to as the nominal system of system (1). The first assumption, given below, is on stability of the nominal system.

Assumption 1: *The origin, $x = 0$, is globally asymptotically stable for the uncontrolled nominal system $\dot{x} = f(x,t)$. Therefore, there exists a C^1 function $V(x,t) : \Re^n \times \Re \rightarrow \Re^+$ such that $\gamma_1(\|x\|) \leq V(x,t) \leq \gamma_2(\|x\|)$ and $\partial V(x,t)/\partial t + \nabla_x^T V(x,t) f(x,t) \leq -\gamma(\|x\|)$ where $\gamma_1, \gamma_2, \gamma : \Re^+ \rightarrow \Re^+$ are class $\mathcal{K}_\infty$ functions.*

It is easy to show that assumption 1 is equivalent to the nominal system being stabilizable under a known, nominal control. Assumption 1 is important as it provides Lyapunov function $V(x,t)$ used to synthesize robust control.

The second assumption, originally defined in [5, 2] and given below, is the standard matching condition which ensures robust stabilizability. It has been shown in [13] and in the references therein that, in several cases, robust stability can be achieved without the matching condition. Nonetheless, the assumption is employed here so we can focus our attention on estimating uncertainties.

Assumption 2: *Uncertain system (1) satisfies the matching conditions (MCs). That is, there exists a positive constant ϵ_b such that, for all $(x,\upsilon,t) \in \Re^n \times S \times \Re^+$, $\Delta f_u(x,\upsilon,t) = 0$, $\Delta B_u(x,\upsilon,t) = 0$, and $\|\Delta B_m(x,\upsilon,t)\| \leq 1 - \epsilon_b$. To make mathematical derivations simpler, it is also assumed that $\Delta B_m(x,\upsilon,t) = 0$.*

The third assumption will be on the size of uncertainty $\Delta f_m(x,t)$ because, as stated in the definition of robust control problem, uncertain variable vector $\upsilon(t)$ is bounded. In principle, the assumption is necessary as a successful robust control has to compensate for the potentially destabilizing uncertainty and to be bounded itself. In other words, control in the presence of unbounded uncertainty is not only mathematically impossible but also physically unrealistic. However, there are several ways by which the assumption on uncertainty size can be made, and the choices will have major impact on whether and how robust control can be successfully applied.

Typically, uncertainties are handled and compensated for by defining or estimating their size bounding functions. The first option in making the third assumption, originally described in [5, 2] and restated below, is to assume that size information on uncertainty is known.

Assumption 3A: *The uncertainty is bounded in Euclidean norm by a known nonlinear function as, for all $(x,\upsilon,t) \in \Re^n \times \Omega \times \Re^+$, $\|\Delta f_m(x,\upsilon,t)\| \leq \rho_m(x,t)$, where $\rho_m(x,t)$ is Caratheodory, uniformly bounded with respect to t, and locally uniformly bounded with respect to x.*

Assumption 3A states that, although uncertain vector $\upsilon(t)$ is unknown, its range of variation is known, and its contribution in $\Delta f_m(\cdot)$ and $\Delta B_m(\cdot)$ can be quantified so that known bounding functions can be found. Although this assumption is reasonable in many cases, uncertainties are unknown by nature and, in other cases, finding bounding functions may become the major obstacle to applying robust control. To overcome this difficulty, it was proposed in [3] that an adaptive version of robust control could be developed. Similar to the standard adaptive control results [9, 10, 14, 16, 8], adaptive robust control is applicable if bounding function $\rho_m(x,t)$ is parameterizable in terms of a set unknown but constant parameters as described by the following assumption.

Assumption 3B: *The uncertainty is bounded in Euclidean norm as follows: for all $(x,\upsilon,t) \in \Re^n \times \Omega \times \Re^+$, $\|\Delta f_m(x,\upsilon,t)\| \leq \rho_m(x,t)$, and*

$$\rho_m(x,t) = W^T(x,t)\phi, \quad (3)$$

where vector $\phi \in \Re^l$ contains all multiplicative and additive, unknown constant parameters, and $W(x,t)$ is a vector consisting of know functions that are Caratheodory, uniformly bounded with respect to t, and locally uniformly bounded with respect to x.

Since uncertain vector $\upsilon(t)$ is bounded, choosing constant vector ϕ as its magnitude vector can always be done. However, as will be shown in the subsequent section, such a treatment implies trade-off. First, demanding a constant upper bound introduces conservatism for any time variant uncertainty. Second, adaptive robust control under assumption 3B may not be one that is both continuous and asymptotically stabilizing. These limitations prompt us to study better designs of robust control. The approach we take in the paper is to properly estimate time-varying uncertainties. To this end, we introduce the following assumptions as the new options of defining bounding functions on uncertainties. Compared with assumptions 3A and 3B, bounding functions are now allowed to be in terms of unknown time varying parameters or to be simply the output of a known model.

Assumption 3C: *The uncertainty is bounded in Euclidean norm as follows: for all* $(x, \upsilon, t) \in \Re^n \times \Omega \times \Re^+$,

$$\|\Delta f_m(x, \upsilon, t)\| \leq W_1^T(x,t)\phi_1(t), \tag{4}$$

where unknown vector $\phi_1(t) \in \Re^{l_1}$ *is assumed to be the bounded output of a quasi-linear system*

$$\dot{\phi}_1 = g_1(x,t)\phi_1 + g_2(x,t), \tag{5}$$

vector $W_1(x,t)$ *and functions* $g_i(x,t)$ *are known, they are uniformly bounded with respect to* t *and locally uniformly bounded with respect to* x*, and there exists a constant, positive definite matrix* P_1 *such that, for all* $x \in \Re^n$ *and for* t*, matrix*

$$P_1 g_1(x,t) + g_1^T(x,t)P_1 \leq 0 \tag{6}$$

is negative semi definite.

Assumption 3D: *The uncertainty is bounded in Euclidean norm as follows: for all* $(x, \upsilon, t) \in \Re^n \times \Omega \times \Re^+$, $\|\Delta f_m(x, \upsilon, t)\| \leq W_2^T(x,t)\phi_2(t)$*, and unknown vector* $\phi_2(t) \in \Re^{l_2}$ *is the bounded output of a nonlinear exo-system*

$$\dot{\phi}_2 = h_1(x, \phi_2, t) + h_2(x,t), \tag{7}$$

where vector $W_2(x,t)$ *and functions* $h_i(\cdot)$ *are known, they are uniformly bounded with respect to* t *and locally uniformly bounded with respect to* x*, and vector function* $h_1(\cdot)$ *has the property that, for all bounded* x *and for all* z, w, t*,*

$$(z-w)^T P_2 \left[h_1(x,z,t) - h_1(x,w,t)\right] \leq 0 \tag{8}$$

is negative semi-definite for a known positive definite matrix P_2*.*

It is obviously that, if $g_i(x,t) = 0$, assumption 3C reduces to assumption 3B and that assumption 3D includes assumption 3C as the special case that $h_1(x, \phi_2, t)$ is linear with respect to ϕ_2. Note that a bound on $\phi_1(t)$ or $\phi_2(t)$ can only be developed by solving analytically the corresponding differential equation (which may be nonlinear and whose initial condition is unknown but bounded), and that the resulting bound would be in general a function of state x. Therefore, although $\phi_1(t)$ and $\phi_2(t)$ are assumed to be bounded (if x is bounded) and the model of exo-system is known, finding function $W(x,t)$ in (3) from either (4) or (7) could be quite difficult. In other words, the extensions from assumption 3B to 3C and 3D are not trivial.

Assumption 3C and 3D imply that bounding function on uncertainty $\Delta f_m(x, \upsilon(t), t)$ is generated by a known exosystem, either linear or nonlinear. Various conditions in the assumptions are to ensure boundedness of the bounding function. Since $\upsilon(t)$ is the source of time variant uncertainty and since time variant uncertainty generated from a known exo-system can be estimated, robust control design may be pursued without the operation of developing a bounding function, which the subject of section 4. The issue of further relaxing assumption 3C (or 3D) to admit an uncertain model for exogenous signal $\upsilon(t)$ will be studied in section 5.

3 Robust Control Designs

We begin with two of existing results on robust control designs. The first one is the standard result in [5, 2].

Lemma 1: *Consider system (1) satisfying assumptions 1, 2 and 3A. Then the closed loop system is either uniformly ultimate bounded (if* $\epsilon > 0$*) or asymptotically stable (if* $\epsilon = 0$*) under control*

$$u(x,t) = -\rho_m(x,t)\frac{\mu(x,t)}{\|\mu(x,t)\| + \epsilon}, \tag{9}$$

where $\epsilon \geq 0$ *is a design constant, and* $\mu(x,t) \triangleq B^T(x,t) \bigtriangledown_x V(x,t)\rho_m(x,t)$*.*

It is known that, if bounding function is known, robust control compensates for uncertainties through size domination and that there are many robust controls equivalent to (9. In case that bounding function is not known, an adaptive robust control can be designed using the certainty-equivalence principle as did in standard adaptive control design. The main result in [3] can be restated as the following lemma.

Lemma 2: *Consider system (1) satisfying assumptions 1, 2 and 3B. Then the closed loop system is either uniformly ultimate bounded (if $k_a > 0$ or $k_\epsilon = 0$) or asymptotically stable (if $k_\epsilon > 0$ and $k_a = 0$) under control*

$$u(x,t) = -\hat{\rho}_m(x,t)\frac{\hat{\mu}(x,t)}{\|\hat{\mu}(x,t)\| + \epsilon}, \qquad (10)$$

where $\hat{\rho}_m(x,t) = W^T(x,t)\hat{\phi} \geq 0$, $\hat{\mu}(x,t) = B^T(x,t) \bigtriangledown_x V(x,t)\hat{\rho}_m(x,t)$, ϵ is a design parameter given by

$$\dot{\epsilon} = -k_\epsilon \epsilon \qquad (11)$$

with $\epsilon(t_0) > 0$, $\hat{\phi}$ is the estimate of ϕ and is generated by adaptation law

$$\dot{\hat{\phi}} = W^T(x,t)\|B^T(x,t) \bigtriangledown_x V(x,t)\| - k_a\hat{\phi}, \quad (12)$$

and $k_a \geq 0$ and $k_\epsilon \geq 0$ are gains.

The above adaptive robust control scheme provides an avenue for us to apply robust control to the cases that uncertainties are bounded by a parameterizable nonlinear function. Since uncertainties are often generated by exo-systems, assumption 3C or 3D is introduced in order to make robust control more applicable while reducing conservatism. Robust control designs under these two assumptions are given by the following two theorems.

Theorem 1: *Consider system (1) satisfying assumptions 1, 2 and 3C. Then the closed loop system is either uniformly ultimate bounded (if $k_a > 0$ or $k_\epsilon = 0$) or asymptotically stable (if $k_\epsilon > 0$ and $k_a = 0$) under control*

$$u(x,t) = -\hat{\rho}_{m1}(x,t)\frac{\hat{\mu}_1(x,t)}{\|\hat{\mu}_1(x,t)\| + \epsilon}, \qquad (13)$$

where $\hat{\rho}_{m1}(x,t) = W_1^T(x,t)\hat{\phi}_1 \geq 0$, $\hat{\mu}_1(x,t) = B^T(x,t) \bigtriangledown_x V(x,t)\hat{\rho}_{m1}(x,t)$, ϵ is a design parameter given by (11), $\hat{\phi}_1$ is the estimate of $\phi_1(t)$ and is given by adaptation law

$$\begin{aligned}\dot{\hat{\phi}}_1 &= g_1(x,t)\hat{\phi}_1 + g_2(x,t) + P_1^{-1}W_1^T(x,t) \\ &\quad \times\|B^T(x,t) \bigtriangledown_x V(x,t)\| - k_a P_1^{-1}\hat{\phi}_1, \quad (14)\end{aligned}$$

and $k_a \geq 0$ and $k_\epsilon \geq 0$ are gains.

Proof: Consider the Lyapunov function $L_1(x,t,\phi_1,\hat{\phi}_1) = V(x,t) + 0.5\tilde{\phi}_1^T P_1\tilde{\phi}_1 + k_l\epsilon$, where $\tilde{\phi}_1 = \phi_1 - \hat{\phi}_1$ is the output estimation error, and $k_l = 0$ if $k_\epsilon = 0$ and $k_l = 1/k_\epsilon$ if otherwise. It follows from (14) and (4) that $\dot{\tilde{\phi}}_1 = g_1(x,t)\tilde{\phi}_1 - P_1^{-1}W_1^T(x,t)\|B^T(x,t) \bigtriangledown_x V(x,t)\| - k_a P_1^{-1}\tilde{\phi}_1 + k_a P_1^{-1}\phi_1$. Therefore, by (13) and (6) that

$$\begin{aligned}\dot{L}_1 &\leq -\gamma(\|x\|) + \epsilon - \|\hat{\mu}_1(x,t)\| + \|\mu_1(x,t)\| \\ &\quad +\tilde{\phi}_1^T P_1\dot{\tilde{\phi}}_1 + k_l\dot{\epsilon} \\ &\leq -\gamma(\|x\|) - \frac{k_a}{2}\|\tilde{\phi}_1\|^2 + \frac{k_a}{2}\|\phi_1\|^2 \\ &\quad +(1 - k_l k_\epsilon)\epsilon, \qquad (15)\end{aligned}$$

from which the stability results can be concluded using the stability theorems in [2, 13]. □

Theorem 2: *Consider system (1) satisfying assumptions 1, 2 and 3D. Then the closed loop system is either uniformly ultimate bounded (if $k_a > 0$ or $k_\epsilon = 0$) or asymptotically stable (if $k_\epsilon > 0$ and $k_a = 0$) under control*

$$u(x,t) = -\hat{\rho}_{m2}(x,t)\frac{\hat{\mu}_2(x,t)}{\|\hat{\mu}_2(x,t)\| + \epsilon}, \qquad (16)$$

where $\hat{\rho}_{m2}(x,t) = W_2^T(x,t)\hat{\phi}_2 \geq 0$, $\hat{\mu}_2(x,t) = B^T(x,t) \bigtriangledown_x V(x,t)\hat{\rho}_{m2}(x,t)$, ϵ is a design parameter given by (11), $\hat{\phi}_2$ is the estimate of ϕ_2 and is given by adaptation law

$$\begin{aligned}\dot{\hat{\phi}}_2 &= h_1(x,\hat{\phi}_2,t) + h_2(x,t) + P_2^{-1}W_2^T(x,t) \\ &\quad \times\|B^T(x,t) \bigtriangledown_x V(x,t)\| - k_a P_2^{-1}\hat{\phi}_2, \quad (17)\end{aligned}$$

and $k_a \geq 0$ and $k_\epsilon \geq 0$ are gains.

Proof: Consider the Lyapunov function $L_2(x,t,\phi_2,\hat{\phi}_2) = V(x,t) + \frac{1}{2}\tilde{\phi}_2^T P_2\tilde{\phi}_2 + k_l\epsilon$, where $\tilde{\phi}_2 = \phi_2 - \hat{\phi}_2$ is the estimation error, and $k_l = 0$ if $k_\epsilon = 0$ and $k_l = 1/k_\epsilon$ if otherwise. It follows from (17) that $\dot{\tilde{\phi}}_2 = [h_1(x,\phi_2,t) - h_1(x,\hat{\phi}_2,t)] - P_2^{-1}W_2^T(x,t)\|B^T(x,t) \bigtriangledown_x V(x,t)\| + k_a P_2^{-1}\hat{\phi}_2$. Under assumption 3D, we know from (15) and (16) that $\dot{L}_2 \leq -\gamma(\|x\|) - \frac{k_a}{2}\|\tilde{\phi}_2\|^2 + \frac{k_a}{2}\|\phi_2\|^2 + (1 - k_l k_\epsilon)\epsilon$, from which the two types of stability can be concluded using the stability theorems in [2, 13]. □

4 Direct Estimation of Uncertainty

If system uncertainty is bounded by a function parameterized in terms of outputs (rather than constants only) from a known exo-system, we can use the results in theorems 1 and 2 to reduce conservatism in developing bounding functions. In the theorems, global boundedness of the estimation error and asymptotic convergence of state x can be concluded only if, in robust controls (13) and (16), design parameter ϵ is set to an exponential decaying function as defined by (11). It has been shown in [13] that, if $\epsilon(t)$ is an L_1 time function, robust control of form (13) becomes discontinuous in the limit unless $W_i(0,t)=0$. Thus, trade-off between asymptotic stability of state x and continuity of robust control is needed in an application of the theorems.

Possible discontinuity of robust control is due to the use of bounding function in its design. As defined in (3) and (4), uncertainty $\Delta f_m(x,\upsilon,t)$ is known to be bounded in norm by a function. In robust control design and stability analysis, every possibility of the uncertainty within the given bounding function must be considered. The worst uncertainty that is admissible by inequality of form (4) is one of those that change arbitrarily fast within the bound and may even become discrete or jump dynamics in the limit. Therefore, to compensate asymptotically for such uncertainty, robust control must be capable of becoming discontinuous itself.

The extension from assumption 3B to 3C (or 3D) brings about another way of handling uncertainties in robust control. Since uncertainties are general time-varying (due to their own dynamics), assumption 3A or 3B must be made if only unknown constants can be estimated in the design. By admitting unknown time variant output of a known exo-system, we can get rid of the process of developing bounding function (which is conservative by its nature) and focus upon directly estimating time variant uncertainties. The following corollary corresponds to theorem 1, its proof is almost identical to that of theorem 1, and its resulting control will be both continuous and asymptotically stabilizing. Theorem 2 can be re-stated in a similar fashion.

Corollary 1: *Consider system (1) satisfying assumptions 1 and 2. If the uncertainty is generated as: for all $(x,\upsilon,t)\in\Re^n\times\Omega\times\Re^+$, $\Delta f_m(x,\upsilon,t)=W_1(x,t)\phi_1(t)$, where unknown vector $\phi_1(t)\in\Re^{l_1}$ is the bounded output of exo-system (5), matrix $W_1(x,t)\in\Re^{m\times l_1}$ and functions $g_i(x,t)$ are known and continuous, they are uniformly bounded with respect to t and locally uniformly bounded with respect to x, and $g_1(x,t)$ satisfies (6) for a constant, positive definite matrix P_1. Then, the closed loop system is either uniformly ultimate bounded (if $k_a>0$) or asymptotically stable (if $k_a=0$) under control*

$$u(x,t)=-W_1(x,t)\hat{\phi}_1, \tag{18}$$

where $\hat{\phi}_1$ is the estimate of ϕ_1 and $\dot{\hat{\phi}}_1 = g_1(x,t)\hat{\phi}_1 + g_2(x,t) + P_1^{-1}W_1^T(x,t)B^T(x,t)\bigtriangledown_x V(x,t)-k_aP_1^{-1}\hat{\phi}_1$ for a constant scalar gain $k_a\geq 0$.

5 Estimation Based on Uncertain Model of Exo-System

As for system (1), dynamics of the exo-system with output $\phi_i(t)$ may not be known exactly. If the model of exo-system is a black box (completely unknown), the corresponding robust control problem is in general ill-defined. This is because, unlike a typical model identification problem or state observation problem, output of the exo-system is itself an uncertainty to be estimated. Thus, in this section, we shall investigate the problem of robust control design under the following assumption and extend the result of theorem 2 to corollary 2 (its proof is omitted for briefness). In essence, the assumption implies that the exo-system is stable (for all bounded x), is partially known, and has a bounded output. Stability results in theorem 1 and corollary 1 can be extended in a similar fashion.

Assumption 3E: *Uncertainty $\Delta f_m(x,\upsilon,t)$ is bounded by inequality (7) where unknown vector $\phi_2(t)\in\Re^{l_2}$ is assumed to be the bounded output of a nonlinear system*

$$\dot{\phi}_2=h_1(x,\phi_2,t)+\Delta h_1(x,\phi_2,t)+h_2(x,t)+\Delta h_2(x,t), \tag{19}$$

vector $W_2(x,t)$ and functions $h_1(x,\phi_2,t)$ and $h_2(x,t)$ are known, but functions $\Delta h_1(x,\phi_2,t)$ and $\Delta h_2(x,t)$ are uncertain. Furthermore, vector function $h_1(\cdot)$ has the property that, for constant $\sigma>1$, for a known positive definite matrix P_2, and for a non-negative function $\beta(\cdot)$ and

$$\begin{aligned}&(z-w)^TP_2\left[h_1(x,z,t)-h_1(x,w,t)\right]\\ \leq\;&-\beta(\|x\|,\|z\|,\|w\|)\|z-w\|^\sigma,\end{aligned}$$

and uncertainties $\Delta h_1(x,\phi_2,t)$ and $\Delta h_2(x,t)$ are bounded as, for all x,t and for a known scalar function $\alpha(\cdot)$, $\|\Delta h_1(x,\phi_2,t)+\Delta h_2(x,t)\|\leq\alpha(\|x\|)$.

Corollary 2: *Consider system (1) satisfying assumptions 1, 2 and 3E. Then, the closed loop system can be stabilized under robust control*

$$u(x,t)=-\hat{\rho}_{m2}(x,t)\frac{\hat{\mu}_2'(x,t)}{\|\hat{\mu}_2'(x,t)\|+\epsilon}, \tag{20}$$

where $\hat{\rho}_{m2}(x,t) = W_2^T(x,t)\hat{\phi}_2 \geq 0$, $\hat{\mu}_2'(x,t) = k_vV^b(x,t)B^T(x,t)\bigtriangledown_x V(x,t)\hat{\rho}_{m2}(x,t)$, ϵ is a design parameter given by (11), $b>-1$ is another *design parameter, $\hat{\phi}_2$ is the estimate of ϕ_2 and is given by adaptation law*

$$\dot{\hat{\phi}}_2 = h_1(x, \hat{\phi}_2, t) + h_2(x,t) + k_v V^b(x,t) P_2^{-1} \times W_2^T(x,t) \|B^T(x,t) \nabla_x V(x,t)\| - k_a P_2^{-1} \hat{\phi}_2,$$

and $k_v > 0$, $k_a \geq 0$ and $k_\epsilon \geq 0$ are gains. More specifically, the closed loop system is globally and asymptotically stable under control (21) by setting $k_a = 0$ and $k_\epsilon > 0$ if gain $k_v > 0$ and constant $b > -1$ can be chosen such that, for all $s \geq 0$ and for all (x,t), $V^b(x,t) W_2^T(x,t) \|B^T(x,t) \nabla_x V(x,t)\|$ is well defined and

$$\inf_{\lambda_1, \lambda_2 \geq 0} k_v \beta^{\frac{1}{\sigma-1}}(s, \lambda_1, \lambda_2) \gamma_1^b(s) \gamma(s) > (\sigma - 1) 2^{\frac{1}{\sigma-1}} \sigma^{-\frac{\sigma}{\sigma-1}} \lambda_{max}^{\frac{\sigma}{\sigma-1}}(P_2) \alpha^{\frac{\sigma}{\sigma-1}}(s);$$

or, the closed loop system is globally, uniformly and ultimately bounded under control (21) by simply choosing gains $k_a > 0$, $k_v > 0$ and constant $b \geq 0$ such that

$$\lim_{s \to +\infty} \frac{k_v k_a \gamma_1^b(s) \gamma(s)}{\lambda_{max}^2(P_2) \alpha^2(s)} > 1.$$

6 Conclusions

Robust control can be made more applicable by designing it in conjunction with estimation of nonlinear time variant uncertainties. It is proposed in the paper that, despite of their nonlinearity and time variance, uncertainties or their bounding functions can be estimated as long as they are generated by exo-systems whose models are either known or partially known. Technical conditions are found in the paper under which estimation and stability can be achieved. Compared with the existing results on adaptive robust control, the proposed result represents a step forward in handling nonlinear time variant uncertainties. An illustrative example will be presented at the conference.

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