Optimal Control of Systems with Delayed Observation Sharing Patterns via Input-Output Methods[1]

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Abstract

In this paper we present an input-output point of view of certain optimal control problems with constraints on the processing of the measurement data. In particular, we consider norm minimization optimal control problems under the so-called one-step delay observation sharing pattern. We present a Youla parametrization approach that leads to their solution by converting them to nonstandard, yet convex, model matching problems. This conversion is always possible whenever the part of the plant that relates controls to measurements possesses the same structure in its feedthrough term with the one imposed by the observation pattern on the feedthrough term of the controller, i.e., (block-)diagonal. When that is not the case, it amounts to the so-called non-classical information pattern problems. For the $\mathcal{H}^\infty$ case, using loop-shifting ideas, a simple sufficient condition is given under which the problem can be still converted to a convex, model matching problem. We also demonstrate that there are several nontrivial classes of problems satisfying this condition. Finally, we extend these ideas to the case of a N-step delay observation sharing pattern.

Keywords: optimal, decentralized, input-output, discrete-time, lifting

1 Introduction

Optimal control under decentralized information structures is a topic that, although it has been studied extensively over the last forty years or so, still remains a challenge to the control community. The early encounters with the problem date back in the fifties and early sixties under the framework of team theory (e.g., [7, 8].) Soon it was realized that, in general, optimal decision making is very difficult to obtain when decision makers have access to private information, but do not exchange their information [17]. Nonetheless, under particular decentralized information schemes such as the partially nested information structures [6] certain optimal control problems admit trackable solutions. Several results exist by now when exchange of information is allowed with a one-step time delay (which is a special case of the partially nested information structure.) To mention only a few we refer to [3, 9] where LQG criteria are of interest, [10, 11, 12, 13] where linear exponential-quadratic Gaussian (LEQG) problems are considered and certain connections to minimax quadratic problems are furnished. The interested reader may further refer to [1] for further bibliographical information

In this paper, in contrast to the state-space viewpoint of the works previously cited, we undertake an input-output approach to optimal control under the information scheme known as the one-step delay observation sharing pattern (e.g., [3]). Under this pattern, measurement information can be exchanged between the decision makers with a delay of one time step. In the paper we provide an approach for solving the ℓ^1, $\mathcal{H}^\infty$ and $\mathcal{H}^2$ (or LQG) optimal disturbance rejection in the case of quasi-classical information exchange [2]. This is the case whenever the part of the plant that relates controls to measurements possesses the same structure in its feedthrough term with the one imposed by the observation pattern on the feedthrough term of the controller, i.e., (block-)diagonal. The key ingredient in this approach is the transformation of

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the decentralization constraints on the controller to *linear* constraints on the Youla parameter used to characterize all controllers. Hence, the resulting problems in the input-output setting are, although nonstandard, convex. These problems are of the same form as the ones appearing in optimal control of periodic systems when lifting techniques are employed [4, 14], and can be solved analogously by employing Duality, Nehari and Projection theorems respectively.

When the part of the plant that relates controls to measurements *does not* possess the same structure in its feedthrough term with the one imposed by the observation pattern on the feedthrough term of the controller, the information exchange is nonclassical (e.g., [3]). The previous approach leads in general to nonconvex problems since the constraints on the Youla parameter are not linear any more. For the $\mathcal{H}^\infty$ case however, using loop-shifting ideas, a simple sufficient condition is given under which the problem can be still converted to a convex, model matching problem. We also demonstrate that there are nontrivial classes of problems satisfying this condition. These ideas can be extended in the case of a N-step delay observation sharing pattern using lifting techniques to obtain similar simple conditions for convexity [15].

2 Problem Definition

The standard block diagram for the disturbance rejection problem is depicted in Figure 1. In this figure, P denotes some fixed linear time invariant (LTI) causal plant, C denotes the compensator, and the signals w, z, y, and u are defined as follows: w, exogenous disturbance; z, signals to be regulated; y, measured plant output; and u, control inputs to the plant. In what follows we will assume that both P and C are LTI systems; we comment on this restriction on C later. Furthermore, we assume that there is a predefined information structure that the controller C has to respect when operating on the measurement signal y. The particular information structure is precisely defined in the sequel.

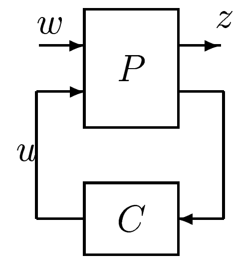

Figure 1: Block Diagram for Disturbance Rejection.

The one-step delay observation sharing pattern
To simplify our analysis we will consider the case where the control input u and plant output y are partitioned into two (possibly vector) components u_1, u_2 and y_1, y_2 respectively, i.e., $u = (u_1^T \ u_2^T)^T$ and $y = (y_1^T \ y_2^T)^T$. Let $Y_k := \{y_1(0), y_2(0), \ldots, y_1(k), y_2(k)\}$ represent the measurement set at time k. The controllers that we are considering (henceforth, admissible controllers) are such that $u_1(k)$ is a function of the data $\{Y_{k-1}, y_1(k)\}$ and $u_2(k)$ is a function of the data $\{Y_{k-1}, y_2(k)\}$. I.e., there is one-step delay in passing the currently received information $y_2(k)$ to the channel that computes $u_1(k)$ and similarly for $y_1(k)$ and $u_2(k)$. We refer to this particular information processing structure imposed on the controller as the *one-step delay observation sharing pattern.*

Alternatively, partitioning the controller C accordingly as $C = \begin{pmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{pmatrix}$, the one-step delay observation sharing pattern requires that both C_{12} and C_{21} be strictly causal operators. Let now

$$\mathcal{S} := \{C \text{ stabilizing and LTI} : \ C_{12},\ C_{21} \text{ strictly causal}\}$$

and let T_{zw} represent the resulting map from w to z for a given compensator $C \in \mathcal{S}$ which we also denote as the linear fractional map $\mathcal{F}(P, C)$. The problem of interest is

$$\mu := \inf_{C \in \mathcal{S}} \|T_{zw}\| = \inf_{C \in \mathcal{S}} \|\mathcal{F}(P, C)\|$$

where $\|\bullet\|$ represents a system norm such as $\mathcal{H}^2$, $\mathcal{H}^\infty$ or ℓ^1.

To solve the above problems the following standard assumption is introduced. Let $P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$ then,

Assumption 2.1 *P is finite dimensional and stabilizable and the closed loop in Figure 1 is well-posed.*

Hence, if P has a state space description $P \sim (A, (B_1 \ B_2), \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}, \begin{pmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{pmatrix})$ then the pairs (A, B_2) and (A, C_2) are stabilizable and detectable respectively.

3 Problem Solution

We consider separately two cases. In the first case it is assumed that the feedthrough term D_{22} is block diagonal which corresponds to a quasi-classical information pattern. The second case is when D_{22} is *not* block diagonal which corresponds to a non-classical information pattern. Using the Youla parametrization, the problem converts to a convex minimization in the first case, while in the second, sufficient conditions for which convexification is possible are provided.

3.1 Case (i): D_{22} block diagonal

The problem defined in the previous section can be related to problems in periodic systems where additional constraints that ensure causality appear in the so-called lifted system [4, 14]. These constraints are of similar nature as with the problem at hand. A basic step in the solution is the convenient characterization of all controllers that are in $\mathcal{S}$. This is done in the sequel.

Since we have assumed that P is finite dimensional with a stabilizable and detectable state space description we can obtain a doubly coprime factorization (dcf) of P_{22} using standard formulas (e.g., [5, 16]) i.e., having P_{22} associated with the state space description $P_{22} \sim (A, B_2, C_2, D_{22})$ a coprime factorization is $P_{22} = N_r D_r^{-1} = D_l^{-1} N_l$ with

$$\begin{pmatrix} X_l & -Y_l \\ -N_l & D_l \end{pmatrix} \begin{pmatrix} D_r & Y_r \\ N_r & X_r \end{pmatrix} = I$$

where $N_r \sim (A_K, B_2, C_K, D_{22})$, $D_r \sim (A_K, B_2, K, I)$, $N_l \sim (A_M, B_M, C_2, D_{22})$, $D_l \sim (A_M, M, C_2, I)$, $X_r \sim (A_K, -M, C_K, I)$, $Y_r \sim (A_K, B_2, -M, K, 0)$, $X_l \sim (A_M, -B_M, K, I)$, $Y_l \sim (A_M, -M, K, 0)$ with K, M selected such that $A_K = A + B_2 K$, $A_M = A + M C_2$ are stable (eigenvalues in the open unit disk) and $B_M = B_2 + M D_{22}$, $C_K = C_2 + D_{22} K$. Note that the above formulas indicate that the coprime factors of P_{22} have as feedforward terms the matrices D_{22} or I or 0 which are all block diagonal. The following is a well-known result (e.g.,[5, 16]):

Fact 3.1 *All ℓ^p-stabilizing LTI controllers C (possibly not in $\mathcal{S}$) of P are given by*

$$C = (Y_r - D_r Q)(X_r - N_r Q)^{-1} = (X_l - Q N_l)^{-1}(Y_l - Q D_l).$$

where Q is an ℓ^p bounded and causal LTI map.

The above fact characterizes the set of all stabilizing controllers in terms of the so-called Youla parameter Q and it amounts to the observer based parametrization in Figure **??** The set $\mathcal{S}$ of interest is clearly a subset of the set implied by Fact 3.1. The constraints on C amount to the constraint that the feedforward term of C should be block diagonal i.e.,

$$C(0) = \begin{pmatrix} C_{11}(0) & 0 \\ 0 & C_{22}(0) \end{pmatrix}.$$

A simple characterization of such a constraint is possible as the following lemma indicates

Lemma 3.1 *All ℓ^p-stabilizing controllers C in $\mathcal{S}$ of P are given by*

$$C = (Y_r - D_r Q)(X_r - N_r Q)^{-1} = (X_l - Q N_l)^{-1}(Y_l - Q D_l).$$

where Q is an ℓ^p bounded and causal LTI map with $Q(0)$ block diagonal.

Proof: It follows from the particular structure of the doubly coprime factors of P_{22} since $C(0) = -Q(0)(I - D_{22} Q(0))^{-1}$ with D_{22} block diagonal and hence $C(0)$ is block diagonal if and only if $Q(0)$ is block diagonal. ∎

Using the above lemma it follows that all feasible closed-loop maps are given as $T_{zw} = H - UQV$ where H, U, V stable depending only on P, and, Q is ℓ^p-stable with $Q(0)$ block diagonal. Hence the problem transforms to the minimization

$$\mu = \inf_{Q,\ Q(0) \text{ block diagonal}} \|H - UQV\|$$

The problem above is an infinite dimensional, minimization of a convex functional over a convex domain (subspace). It represents a non-standard model matching problem due to the constraint on $Q(0)$. However, this constraint is of the same nature as in the optimal control problems of periodic systems considered and solved in [4] for ℓ^1 via duality theory, and, in [14] for $\mathcal{H}^\infty$ and $\mathcal{H}^2$ via Nehari and Projection Theorems respectively. The solution procedures in [4, 14] carry through exactly for the problem at hand and thus we refer the reader to these references for further details. We would also like to mention that considering smooth nonlinear controllers does not improve performance [15]

3.2 Case (ii): D_{22} not block diagonal

When D_{22} is not block diagonal the constraints on C do not transform to convex constraints on the Youla parameter Q as in Lemma 3.1. Hence the resulting problem becomes a nonlinear and non-convex optimization which is in general difficult to

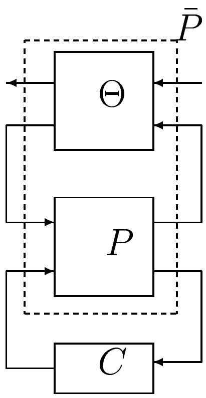

Figure 2: Loop shifting

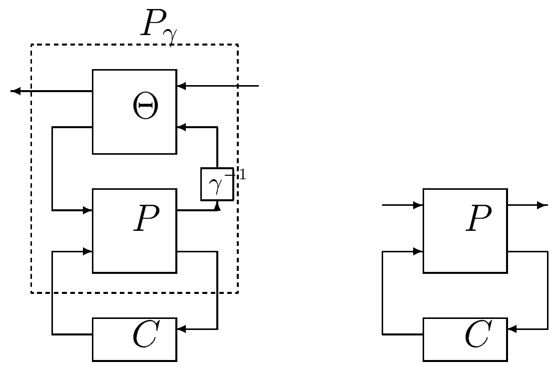

Figure 3: $\|\mathcal{F}(P, C)\| < \gamma$ iff $\|\mathcal{F}(P_\gamma, C)\| < 1$.

handle. The idea in this section is to transform, if possible, the original problem to a problem of Case (i) of the previous section i.e., obtain an equivalent system for which the new D_{22} has the block diagonal structure that ensures convexity of the (new) problem. This is done herein *only for $\mathcal{H}^\infty$ optimal control* by using the so-called loop-shifting property (e.g., [18] chap. 16).

In particular, consider the loop-shifted system $\bar{P}$ of Figure 2 where $\Theta = \begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{pmatrix}$ is an inner matrix i.e., $\Theta^*\Theta = I$ with Θ_{21} invertible. Then the following equivalence holds

$$\|\mathcal{F}(P, C)\| < 1 \iff \left\|\mathcal{F}(\bar{P}, C)\right\| < 1.$$

If $\bar{D}_{22} := \bar{P}_{22}(0)$ i.e., the new D_{22}-term in $\bar{P}$, then it holds that

$$\bar{D}_{22} = D_{22} + D_{21}\Theta_{22}(I - D_{11}\Theta_{22})^{-1}D_{12}.$$

The goal here is to select Θ_{22} so that $\bar{D}_{22}$ is block diagonal and thus apply the methods of the previous subsection to solve a convex yet nonstandard model matching problem. Splitting

$$D_{22} = D_{22,d} + D_{22,u}$$

where $D_{22,d}$ is the block diagonal part and $D_{22,u}$ is the remaining (unwanted) part of D_{22}, we would like to have a Θ_{22} such that

$$D_{22,u} = -D_{21}\Theta_{22}(I - D_{11}\Theta_{22})^{-1}D_{12}.$$

If we assume further that D_{21} and D_{12} have full row and column rank respectively then a matrix Θ_{22} that satisfies the above equation generically exists. However, such a Θ_{22} may not in general lead to an inner Θ: note that for Θ inner it is necessary $\|\Theta_{22}\| \leq 1$. To investigate when an appropriate Θ exists let

$$\rho := \inf\{\|\Theta_{22}\| : D_{22,u} = -D_{21}\Theta_{22}(I - D_{11}\Theta_{22})^{-1}D_{12}\}$$

and let Θ_{22}^o denote a minimizer [1] of the above definition (i.e., $\|\Theta_{22}^o\| = \rho$).

If we *assume* that $\mu\rho < 1$ then letting $\gamma > 0$ be such that $\mu < \gamma < 1/\rho$ where $\mu = \inf_{C \in \mathcal{S}} \left\|\mathcal{F}(\bar{P}, C)\right\|$ we have that for

$$\Theta := \begin{pmatrix} -\gamma\Theta_{22}^o & (I - \gamma^2\Theta_{22}^o\Theta_{22}^{o*})^{1/2} \\ (I - \gamma^2\Theta_{22}^{o*}\Theta_{22}^o)^{1/2} & \gamma\Theta_{22}^{o*} \end{pmatrix}$$

it holds that $\Theta^*\Theta = I$ and Θ_{21} invertible. This leads to

$$\|\mathcal{F}(P, C)\| < \gamma \iff \|\mathcal{F}(P_\gamma, C)\| < 1$$

in Figure 3. Therefore, the above equivalence allows as to solve the original problem by solving for

$$\inf\{\gamma : \|\mathcal{F}(P_\gamma, C)\| < 1\}$$

for each fixed γ which is a Case (i) problem of the previous section since P_γ has a block triangular D_{22} term. Hence, the solution procedure outlined in Case (i) can readily be applied. The following summarizes the developments so far.

Theorem 3.1 *If $\mu\rho < 1$, the $\mathcal{H}^\infty$ optimal disturbance rejection under the one step delay observation sharing pattern is a convex problem.*

[1] in situations where a minimizer does not exist Θ_{22}^o is taken to mean arbitrarily close to ρ in size, i.e., $\left\|\Theta_{22}^o\right\| = \rho + \delta$ with $\delta > 0$ arbitrarily small

To check whether the sufficient condition for convexity holds in the above theorem, one needs to know μ. This would not typically be known. However, one can check using computable upper bounds $\bar{\mu} \geq \mu$, i.e., check if $\bar{\mu}\rho < 1$. For example, if P is stable, a readily obtained upper bound on μ is obviously $\bar{\mu} = \|P_{11}\|$. A more general way could be to solve a *standard* $\mathcal{H}^\infty$ problem restricting C to be strictly causal. That amounts to placing a delay in the measured output (or control input) in both channels in the original system P i.e., augmenting P_{22} to ΛP_{22} where Λ is the delay operator, and design for the $\mathcal{H}^\infty$ optimal cost; it is simple to show that the augmented system is stabilizable given Assumption 2.1 and that the so obtained $\mathcal{H}^\infty$ optimal cost is an upper bound on μ. Obviously, the aforementioned bounds may be very far from μ and better ones can be sought for.

Irrespective of what upper bounds may be available for a-priori checks the following algorithm provides the solution to the problem whenever convexification via the loop shifting procedure described above is possible.

- **step 1**: Find
$$\rho := \inf\{\|\Theta_{22}\| : D_{22,u} = -D_{21}\Theta_{22}(I-D_{11}\Theta_{22})^{-1}D_{12}\}$$
and a minimizer Θ_{22}^o.
- **step 2**: Let $\gamma_0 = \frac{1}{\rho+\epsilon}$ where $\epsilon > 0$ is arbitrarily small
- **step 3**: Solve for $C \in \mathcal{S}$ such that
$$\|\mathcal{F}(P_{\gamma_0}, C)\| < 1$$
using the procedures indicated in Case (i)
- **step 4**: If step 3 feasible then $\mu\rho < 1$ and $\inf_{C\in\mathcal{S}} \|\mathcal{F}(P, C)\|$ is convex: solve equivalent
$$\mu = \inf\{\gamma < \gamma_0 : \|\mathcal{F}(P_\gamma, C)\| < 1\}$$
by the methods indicated in Case (i)
- **step 5**: If step 3 not feasible, then $\mu > 1/\rho$ but convexification fails

An interesting interpretation of Theorem 3.1 can be given in the special case (yet quite common) where $D_{11} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $D_{12} = \begin{pmatrix} 0 \\ I \end{pmatrix}$ and $D_{21} = (0 \quad I)$. Letting
$$D_{22,u} = \begin{pmatrix} 0 & D_{2212} \\ D_{2221} & 0 \end{pmatrix}$$
we get
$$\Theta_{22}^o = \begin{pmatrix} 0 & 0 \\ 0 & -D_{22,u} \end{pmatrix}$$
and $\rho = \|D_{22,u}\| = \max(\|D_{2212}\|, \|D_{2221}\|)$. The sufficient condition for convexity of Theorem 3.1 gives
$$\mu \max(\|D_{2212}\|, \|D_{2221}\|) < 1.$$
This condition implies that if the size of the unwanted terms in D_{22} i.e., the off-block diagonal, are strictly smaller than the inverse of the optimal $\mathcal{H}^\infty$ cost, the problem is convex. If P is stable, $\mu \leq \|P_{11}\|$. Based on that, several problems with non-classical information pattern can be constructed that lead to convex problems. Consider as an example the norm minimization of weighted sensitivity and complementary sensitivity given as
$$\mathcal{F}(P, C) =$$
$$\begin{pmatrix} W_{z_1}(I-P_{22}C)^{-1}W_{w_1} & W_{z_1}P_{22}C(I-P_{22}C)^{-1}W_{w_2} \\ W_{z_2}C(I-P_{22}C)^{-1}W_{w_1} & W_{z_2}C(I-P_{22}C)^{-1}W_{w_2} \end{pmatrix}$$
with W_{z_i}, W_{w_i} stable weights for $i = 1, 2$ satisfying $W_{w_1}(0) = W_{z_1}(0) = 0$ and $W_{z_2}(0) = W_{w_2}(0) = I$ and where P_{22} is stable. In this case $P_{11} = \begin{pmatrix} W_{z_1}W_{w_1} & 0 \\ 0 & 0 \end{pmatrix}$, $P_{12} = \begin{pmatrix} W_{z_1}P_{22} \\ W_{z_2} \end{pmatrix}$, $P_{21} = (W_{w_1} \quad W_{w_2})$. If the weights are selected to satisfy $\|W_{z_1}W_{w_1}\| \max(\|D_{2212}\|, \|D_{2221}\|) < 1$ then $\mu\rho < 1$ and the problem is therefore convex. This shows that there are several nontrivial classes of problems for which the condition of Theorem 3.1 is satisfied.

4 Conclusions

We presented an input-output point of view for norm minimization optimal control problems under the one-step delay observation sharing pattern. In the case where the part of the plant that relates controls to measurements possesses the same structure in its feedthrough term with the one imposed by the observation pattern on the feedthrough term of the controller, i.e., (block-)diagonal, the problem is convex and a procedure that leads to its solution was given. It was also documented in this case of such a quasi-classical information pattern, that smooth nonlinear time-varying controllers do not outperform linear time invariant.

The case where the part of the plant that relates controls to measurements does not posses the same structure in its feedthrough term with the one imposed by the observation pattern on the feedthrough term of the controller, a case of non-classical information pattern, was also considered. For the $\mathcal{H}^\infty$ case, using loop-shifting ideas a simple sufficient condition was given under which the problem can be still converted to a convex, model matching problem. In this condition, the size of the unwanted terms for convexity in the part of the plant that relates controls to measurements is related to the optimal cost. It was also demonstrated that there are several nontrivial classes of problems satisfying this condition and a general high level algorithm was given to solve the problem if convexification with loop-shifting is possible.

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