

Hybrid sliding mode control for a set of interconnected nonlinear components

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Abstract

The paper reports the use of sliding mode control in the design of a hybrid controller for a distributed system comprising a pair of loosely-coupled inverted pendulums. A sliding mode controller is designed to independently drive each pendulum mechanism to provide stabilization and profiled motion control. Then, to accomplish a reschedulable sequence of tasks, a supervisory system is developed using compositional methods and is modelled and analysed using controlled Petri nets. It is shown that using an appropriate coordination strategy it is possible to achieve a stability envelope for the composite system which is greater than that of the individual components.

1 Introduction

The control of sets of independently driven mechanisms, from a simple conveyor belt to a sophisticated robotic manipulator, is traditionally dealt with in a continuous time/continuous state environment. When these mechanisms are used in a real environment, such as a production line, they have to be co-ordinated and synchronised with each other and are required to perform sequences of tasks.

In recent years, much research has been carried out on the concepts of *hybrid systems* and *hybrid control* [3], with the objective of developing an integrated approach to the discrete event and continuous parts of such systems. The need is to combine the study of the continuous domain stability/controllability with the study of the intended discrete domain functionality, such as the supervisory control of tasks sequences.

The paper describes a design study which explores the hybrid control of a distributed system comprising linked inverted pendulums arranged in a production-line style. We start by analysing the static and dynamic continuous domain behaviour of each of these components of a distributed system in isolation and design individual sliding mode controllers for set-point regulation. We then identify conventional switching decisions governing the modes and set-points of the continuous control. Through a process of abstraction we make these decisions an explicit

part of the discrete event or supervisory layer, so that they can be modelled and analysed using discrete event methods such as Automata or Petri nets [2], [5].

A Petri net model of the composite system was designed and was then analysed using Petri net techniques to verify the behaviour of the system. Finally, the analysis was extended to examine the effect of severe disturbances.

2 Introduction to the design problem: an inverted pendulum

Consider an inverted pendulum formed by a rotating triangular frame and two balance weights, as shown in Figure 1. The frame lies in the vertical plane and can rotate freely about an axis through the origin O .

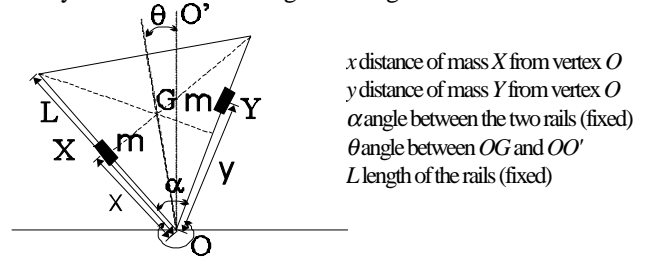


Figure 1. The triangular frame

X and Y are two balance masses that can move along the sides of the frame ('rails' throughout the paper). The movements of X and Y along the rails displace the centre of mass of the triangular frame system.

A simplified mathematical model is the following

$$\begin{cases} \ddot{x} = \frac{F_x}{m} + x\dot{\theta}^2 - g \cos\left(\theta - \frac{\alpha}{2}\right) \\ \ddot{y} = \frac{F_y}{m} + y\dot{\theta}^2 - g \cos\left(\theta + \frac{\alpha}{2}\right) \\ \ddot{\theta} = \frac{1}{\frac{J}{m} + x^2 + y^2} \left[-\dot{x}\dot{\theta} - \dot{y}\dot{\theta} + gx \sin\left(\theta - \frac{\alpha}{2}\right) + gy \sin\left(\theta + \frac{\alpha}{2}\right) + \frac{2ML}{3m} g \cos\frac{\alpha}{2} \sin\theta \right] \end{cases} \quad (1)$$

where J is the inertia of the frame, F_x and F_y the forces applied to X and Y , respectively. Because of the hard constraint represented by the limited length of the rails,

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system (1) is valid only when $0 < x, y < L$. When hitting the boundaries $x = 0, L$, and $y = 0, L$, the quantities \dot{x} and \dot{y} will become 0 instantaneously, while accelerations \ddot{x} and \ddot{y} have to include the constraint reaction forces.

3 The control problem

Consider a composite system formed by placing two inverted pendulums (of the triangular form described above) $T1$ and $T2$, on the same axis of rotation but some distance apart, as shown in Figure 2.

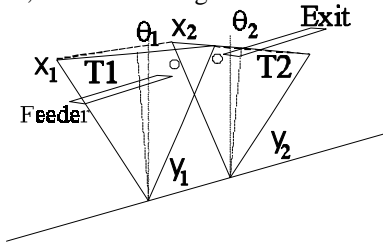


Figure 2

Let the vertices of the triangles be linked by non-elastic constraints or chains which have some slack so that the triangles can rotate relative to each other before they become 'locked' at a constant relative displacement. Let the two loosely coupled triangular frame inverted pendulums plus associated product manipulators be configured into a production-line style system as shown in Figure 2.

3.1 Normal operation mode

The task of the system is to move a product from a "feeder" conveyor to an "exit" conveyor via an arbor on each of the triangular frames. The overall movement that $T1$ and $T2$ have to do to accomplish the target is subdivided into 4 tasks.

[Load_T1] If T1 not loaded, T1 go to loading point: the control objective is to move $T1$ to the set point θ_1 (*loading point*), and align its arbor with the "feeder" conveyor, and then for $T1$ to be held stationary at θ_1 during the loading period.

[Unload_T1] If T1 loaded, T1 go to rendez-vous point: the control objective is to move $T1$ such that it rendez-vous with $T2$ by aligning its arbor with that of $T2$ and then follows it with zero relative velocity for the period of the product transfer from $T1$ to $T2$ (i.e. unload $T1$). Note that, the rendez-vous point is not a fixed position.

[Load_T2] If T2 not loaded, T2 go to rendez-vous point: same as **Unload_T1**, but $T2$ is exchanged with $T1$. As a result $T2$ is loaded.

[Unload_T2] If T2 loaded, T2 go to unloading point: the control objective is to move $T2$ to the set point θ_u (*unloading point*), and align its arbor with the "exit" conveyor and then for $T2$ to be held stationary at θ_u during the unloading period.

We shall assume that the dynamics of the product manipulators can be disregarded. For demonstration purposes, the arbors on frames $T1$ and $T2$ are purpose designed to be asymmetric and are arranged such that the production-line task can only be accomplished if both inverted pendulums cooperate.

3.2 Abnormal circumstances: the rescue function

Now suppose that one of the triangular frames can go past the stability limit angle, perhaps due to an external impulsive disturbance, and start to 'tumble or fall down' (i.e. starts heading towards the stable equilibrium point for the uncontrolled system where it would be dangling from the pivot). When one triangle begins to fall, the chains would tighten and the triangles would become locked and both would then fall. However simple appraisal shows that, if the two frames are identical and the ratio between m and M is suitable, the chains provide a mechanism that allows the non-falling triangular frame to come to the rescue of the falling frame.

For example, given a method of detecting the onset of "falling" the non-falling triangular pendulum could throw itself in the opposite direction in the hope that the chains might prove to be "safety chains" and rescue the falling pendulum.

At this stage, the following policies were introduced:

If $T1$ ($T2$) is outside the stabilizability region, (i.e. $T1$ ($T2$) is falling), then

- $X1$ ($X2$) and $Y1$ ($Y2$) should be moved as fast as possible to $(x, y) = (0, 0)$ (*Falling policy*),
- $T2$ ($T1$), should change its target set-point to a predetermined *rescue set-point* (*Rescue policy*),

when $T1$ ($T2$) is rescued, $T2$ ($T1$) should go back to try to reach the abandoned target set-point.

3.3 Control Strategy

All the various tasks can be expressed in terms of angular position and velocity set-points. Therefore the first objective was to design a set-point regulator capable of achieving

- zero steady state tracking error for all the specified profiles,
- robustness towards significant but bounded disturbances

for one single triangular inverted pendulum. The design process involved determining the stability area of the frames and a suitable structure for the controller.

The second objective was to design, a discrete event or supervisory controller to co-ordinate and synchronise the motion of the pair of linked triangular inverted pendulums and to schedule their motion profiles.

4 Dynamic stabilization and motion control

The frame will have an unstable equilibrium point at $\theta^* \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and a stable one at $\pi - \theta^*$. Due to the geometry of the device, θ^* will belong to a finite sector of the plane, that is determinable by moving each of the two masses to the limit on the rails, i.e. $(x,y)=(0,L)$ or $(x,y)=(L,0)$. Specifically, if the system has initial conditions $(x(0),y(0),\dot{x}(0),\dot{y}(0),\theta,\dot{\theta}(0))=(0,L,0,0,\theta_m,0)$ or, equivalently, $(x(0),y(0),\dot{x}(0),\dot{y}(0),\theta,\dot{\theta}(0))=(L,0,0,0,\theta_M,0)$, then, the limiting values for the equilibrium condition are:

$$\theta_M = -\theta_m = \arctan \left(\tan \frac{\alpha}{2} \left[\frac{1}{1 + \frac{2M}{3m}} \right] \right) \quad (2)$$

It follows that the vertical plane can be divided in three sectors

- $\Omega_1 = \{ \theta \mid \theta \in [-\pi, -\pi + \theta_m] \cup [\pi - \theta_M, \pi] \}$, where all points are stable ones, provided that X and Y are moved to suitable values x, y computable open-loop from a static analysis.
- $\Omega_2 = \{ \theta \mid \theta \in [-\pi + \theta_m, -\theta_m] \cup [\theta_M, \pi - \theta_M] \}$, where the frame surely cannot be stabilized/controlled.
- $\Omega_3 = \{ \theta \mid \theta \in [\theta_m, \theta_M] \}$, the biggest sector in the plane, where the frame can be stabilized/controlled.

Control objective. Given the initial conditions $(\theta_0, \dot{\theta}_0) \in \Omega \subset \Omega_3 \times \mathbb{R}$, the control objective is to make the target position $(\theta_b, 0) \in \Omega \subset \Omega_3 \times \mathbb{R}$, a stable equilibrium point of the controlled system, where Ω is a suitable subset of $\Omega_3 \times \mathbb{R}$ to be determined.

To solve the stated control objective, a three blocks regulator has been designed as shown in Figure 3.

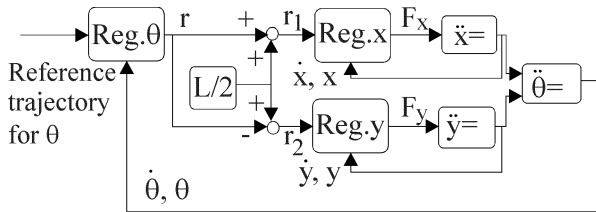


Figure 3. Control Scheme

The position of each mass X, Y on the rails x, y was controlled, $Reg.x, Reg.y$, with a PD, which provides set-point regulation with a sufficiently fast response time. The set-points are generated by the third block $Reg.\theta$. When the boundaries are reached, X and Y are kept at the boundaries by applying a force to counteract the gravity terms, where not already cancelled by the reaction force. With reference to the scheme in Figure 3, the three control modules are

$$Reg.x \quad F_x = \begin{cases} -a(x-r_1) - b\dot{x}, & 0 \leq x \leq L \\ g \cos\left(\theta - \frac{\alpha}{2}\right), & x = L \\ 0, & x = 0 \end{cases}$$

$$Reg.y \quad F_y = \begin{cases} -a(y-r_2) - b\dot{y}, & 0 \leq y \leq L \\ g \cos\left(\theta + \frac{\alpha}{2}\right), & y = L \\ 0, & y = 0 \end{cases}$$

$$Reg.\theta \quad (r_1, r_2) = f(\theta, \dot{\theta}, \theta^*)$$

where r_1 and r_2 are the reference trajectories (control inputs) for the regulated rails, whilst $\theta^* = [\theta_d \ 0]^T$ are the reference trajectories for the variables θ and $\dot{\theta}$.

In choosing an appropriate control strategy for r_1 and r_2 , it is of note that if the frame has to balance at an angle at the left of the line OO' , the mass Y has to be moved up, and mass X has to be moved down. Alternatively, if θ_d is at the right of OO' , mass X should be up, mass Y should be down. Therefore, it is sensible to generally link the two control inputs F_x and F_y by making $r_1 = \frac{L}{2} + r$ and $r_2 = \frac{L}{2} - r$ where r is a function of $e = \theta - \theta_d$, and its first derivative.

4.1 The θ regulator

Summing up, we want precise set-point regulation, possibly with velocity control as well, and robustness towards bounded disturbances. Some experiments with a series of PID for $Reg.\theta$ showed that exact set-point regulation is not possible using PID control due to the highly non linearity of the system. Among other nonlinear control techniques, a discontinuous control has been chosen because of the fast dynamics it can provide for the closed loop system and the simplicity of the overall regulator.

To make the co-operative behaviour of Section 3.2 as easy as possible, if one of the two frames is outside the working (stabilisation) area Ω_3 , then as the rails cannot be used for stabilisation they may as well be used to minimise the angular velocity of the falling frame. With this in mind, $Reg.\theta$ can be designed as a two module regulator (the control is switched between them) as follows:

$$r(\theta, \dot{\theta}, \theta^*) = \begin{cases} 0 & \theta \in \Omega_1 \cup \Omega_2 \\ u(\theta, \dot{\theta}, \theta^*) & \theta \in \Omega_3 \end{cases}$$

Now, $u(\cdot)$ must be specified.

4.2 Sliding mode control

Given a dynamical system $\dot{x} = f(x) + g(x)u$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, a typical *sliding mode control* [6] is set up in two steps

1. Design a switching function $S(x)$, such that for $S(x)=0$ some control objective is satisfied.

- Design a control law $u(x)$ discontinuous on $S(x)=0$ such that the states of the system reach the *sliding manifold* $\{x \in \mathbb{R}^n \mid S(x)=0\}$ in a finite time.

For step 1, a suitable surface for our system is

$$S(\cdot) = \dot{\theta} + c(\theta - \theta_d)$$

where θ_d is the angle set-point. On the manifold the dynamics of the frame is $\dot{\theta} = -c\theta + c\theta_d$, i.e. an asymptotically stable first order system, then, $\dot{\theta} \rightarrow 0$ and $\theta \rightarrow \theta_d$.

4.3 Second order sliding mode control

For step 2, because the control u appears only in the second derivative of S (in fact $\dot{S} = \ddot{\theta} + \dots$, and $\ddot{\theta}$ does not contain u , as can be seen from (1), while $\ddot{S} = \ddot{\ddot{\theta}} + \dots$ contains it through the expressions of \ddot{x} and \ddot{y}), we apply a second order sliding mode algorithm [1], that is

- Let $y_1=S$ and $y_2=\dot{S}$.

- Write the auxiliary system

$$\begin{cases} y_1(t) = y_2(t) \\ y_2(t) = \mathcal{F}[y(t), t] + \mathcal{G}[y(t), t]u(t) \end{cases} \quad (3)$$

with $y(t)^T = [y_1(t) \ y_2(t)]$, $y_2(t)$ unmeasurable, $\mathcal{F}[y(t); t] = \frac{\partial}{\partial x} \left(\frac{\partial S}{\partial x} (f(x)+g(x)) \right) (f(x)+g(x)u)$, and $\mathcal{G}[y(t); t] = \frac{\partial S}{\partial x} g$ uncertain functions.

- Find suitable bounds F , Γ_m and Γ_M such that

$$|\mathcal{F}[y(t); t]| < F, \quad 0 < \Gamma_m \leq \mathcal{G}[y(t); t] \leq \Gamma_M \quad (4)$$

- Apply the control law

$$u(t) = -\beta(t)U_M \operatorname{sign} \left\{ y_1(t) - \frac{1}{2} y_1(t_{M_i}) \right\} \quad (5)$$

$$\alpha(t) = \begin{cases} \alpha^* & \text{if } \left[y_1(t) - \frac{1}{2} y_1(t_{M_i}) \right] \left[y_1(t_{M_i}) - y_1(t) \right] > 0 \\ 1 & \text{if } \left[y_1(t) - \frac{1}{2} y_1(t_{M_i}) \right] \left[y_1(t_{M_i}) - y_1(t) \right] \leq 0 \end{cases}$$

where U_M is the control amplitude to be suitably selected, t_{M_i} is such that $y_2(t_{M_i})=0$, and $y_1(t_{M_i})$ represents the last extremal value of the $y_1(t)$ function, i.e., the last local maximum, local minimum or horizontal flex point of $y_1(t)$.

In [1] it has been proved that, the corresponding sufficient conditions for the finite time convergence to the sliding manifold are

$$\begin{cases} \beta^* \in (0,1] \cap \left(0, \frac{3\Gamma_m}{\Gamma_M} \right) \\ U_M > \max \left(\frac{F}{\beta^* \Gamma_m}, \frac{4F}{3\Gamma_m - \beta^* \Gamma_M} \right) \end{cases} \quad (6)$$

Let us apply this strategy to our case, i.e. let $y_1 = \dot{\theta} + c(\theta - \theta_d)$, and $y_2 = \ddot{\theta}$.

$$\dot{y}_1 = y_2 \quad (7)$$

$$\begin{aligned} \dot{y}_2 = & \frac{1}{J/m + x^2 + y^2} \left[-ax^2\ddot{\theta} - b\dot{x}\ddot{\theta} - \dot{x}^2\ddot{\theta} - ay^2\ddot{\theta} \right. \\ & - b\dot{y}\ddot{\theta} - \dot{y}^2\ddot{\theta} + g\dot{x} \sin \left(\theta - \frac{\alpha}{2} \right) + gx\dot{\theta} \cos \left(\theta - \frac{\alpha}{2} \right) \\ & + g\dot{y} \sin \left(\theta + \frac{\alpha}{2} \right) + gy\dot{\theta} \cos \left(\theta + \frac{\alpha}{2} \right) \\ & + \frac{2MLg}{3m} \dot{\theta} \cos \theta \cos \frac{\alpha}{2} \left. \right] - 3 \left[\dot{x}\ddot{\theta} - \dot{y}\ddot{\theta} + gx \sin \left(\theta - \frac{\alpha}{2} \right) \right. \\ & + g \sin \left(\theta + \frac{\alpha}{2} \right) + \frac{2MLg}{3m} \sin \theta \cos \frac{\alpha}{2} \left. \right] \times \\ & \times \frac{\dot{x}\dot{x} + \dot{y}\dot{y}}{\left(J/m + x^2 + y^2 \right)^2} - a\dot{\theta}(x-y)u \end{aligned} \quad (8)$$

Because of the physical constraints, and of the assumption that we are working in the sector Ω_3 , bounds for the expressions in (8) can be easily found. These quantities are

functions of $\dot{\theta}(t^*)$, where t^* is the time when a new set-point/reference profile is imposed to the system. As the

reader can notice, the control gain is $-\dot{\theta}(x-y)$ i.e. it is a quantity that can be zero and can change its sign, thus it is contrary to the assumptions for the application of the second order sliding mode algorithm describe above. However, this does not prevent the stabilisation of the frame: in fact, the control tends to bring x and y to different values even when the gain is zero. Moreover, the X and Y

movement influences directly $\dot{\theta}$, then the condition $\dot{\theta}(x-y)=0$ will hold for a time of measure zero. As for the sign, because of the geometry of the frame, $\forall \theta \in \Omega_3$, if $x-y > 0$, $\dot{\theta}$ shall become negative, and, if $x-y < 0$, $\dot{\theta}$ shall become positive. Then, apart from transients periods, the applied

control is such that $\dot{\theta}(x-y) < 0$, i.e. $u = \beta(t) U_M \operatorname{sign}(S - 0.5S_{t_{M_i}})$ is applied.

However (8) is the expression of \ddot{S} when the PDs are applied on X and Y . When the ends of the rails are reached, for example $x=L$ and $y=0$,

$$\ddot{\theta} = gL \sin \left(\theta + \frac{\alpha}{2} \right) + \frac{2MLg}{3m} \sin \theta \cos \frac{\alpha}{2} \quad (9)$$

It is easy to see that, when in Ω_3 , $\ddot{\theta} = -|\ddot{\theta}| \operatorname{sign}(y-x) = -|\ddot{\theta}| \operatorname{sign} S$, where $|\ddot{\theta}|$ has a lower bound κ_θ determinable from (9). Then, for each $\theta(t^*)$, we can determine the limit value of $c\dot{\theta}$ for which the term κ_θ is dominating the expression of \dot{S} , i.e. the system satisfies a first order sliding mode condition $S\dot{S} = S(-|\ddot{\theta}| \operatorname{sign} S + c\dot{\theta}) < -\kappa|S|$, κ some positive constant.

4.4 First order sliding mode control

The reasoning at the end of the previous section, suggests that even a first order sliding mode control law could be applied successfully to the system in question. In fact, let us choose the easiest discontinuous control, that is $r=K\text{sign}(S)$. Because of the constraints and of the fast response of the PD applied to X and Y , x exhibits the behaviour of a saturated function, say σ_ε .

$$\sigma_\varepsilon = \begin{cases} L, & S \geq \varepsilon \\ \kappa\gamma(S), & |S| < \varepsilon \\ 0, & S \leq -\varepsilon \end{cases} \quad (10)$$

where $\gamma(S)$ is an increasing function of S , κ is a value proportional to K , and ε is proportional to τ/K , τ the PD time constant. After the transients condition (9) holds, i.e. the manifold is attractive, and provided that K is sufficiently large, $\max(\theta) \in \Omega_3$ during the transitory phase.

4.5 Dynamic model of the two frame system

It is easy to see that the policy introduced for the single frame in Section 4.1, in which X and Y were sent to $(0,0)$ when outside the stabilizability sector turns out to be useful for the falling policy. Indeed, the regulator applied to the frames is exactly the same as before. A similar reasoning as the one in section 4.4 can be applied to the compound triangle, to determine a suitable k . This k is used also when the two triangles are not locked. Thus the control law ensures that the dynamics of the single frame is not affected by the disturbance imposed by the other frame.

In Section 2, the biggest stabilization region for one isolated frame was shown to be $[\theta_m, \theta_M]$. A similar static analysis can be applied to the composite two frame system in the case when the frames are locked. Given the locking angle Δ , $M < \frac{3}{2} m \tan^2 \Delta$ is a sufficient condition for the new limit angle

$$\theta_{M_2} = \arctan \left(\frac{\tan \frac{\alpha}{2} + \tan \Delta \left(1 + \frac{2M}{3m} \right)}{1 + \frac{2M}{3m} - \tan \frac{\alpha}{2} \tan \Delta + \frac{2M}{3m \cos \Delta}} \right)$$

to be bigger than the one in (2). Thus, if this condition is satisfied, the composite system of two loosely coupled inverted pendulums have the potential to cooperate and operate over a zone of stability wider than that for the two pendulums operating in isolation.

5 Coordination and synchronisation logic

The tasks of Section 3 are translated into a series of different set points for the frames. Let θ_1 denote the angular position of $T1$ and with θ_2 the angular position of $T2$. Then:

$$\begin{aligned} S(\text{Load}_T1) &= \dot{\theta}_1 + c(\theta_1 - \theta_1) \\ S(\text{Unload}_T1) &= \dot{\theta}_1 + c(\theta_1 - \theta_2) \\ S(\text{Unload}_T2) &= \dot{\theta}_2 + c(\theta_2 - \theta_u) \\ S(\text{Load}_T2) &= \dot{\theta}_2 + c(\theta_2 - \theta_1) \end{aligned}$$

The system requirement contains also a series of rules that define, in general terms, the logic necessary to coordinate and synchronise the frames and forms the functional requirement of the discrete event part of the hybrid system. Such logic can be incorporated directly in the discrete event part of the system. To facilitate analysis and reasoning, the interfaces and discrete event system were modelled using Petri nets [5] which have a tangible graphical representation, constructs for describing asynchronous concurrent behaviour, and an underlying mathematical structure.

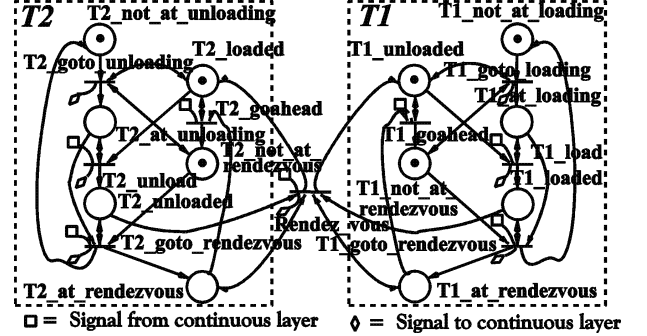


Figure 4

Using Petri nets the task sequences were modelled and the coordination and synchronisation logic designed as shown in Figure 4. Transducers are modelled using a \square symbol and an arc leading to a transition: when a signal from the continuous layer is received the event "tokenises" the square and thus enables the transition (assuming all input places are tokenised). Actuators or continuous layer controllers are modelled by the symbol \diamond and an arc leading from a transition: when the transition fires ("tokenises" the diamond) it causes a command to be sent to the continuous layer actuator or controller.

The Petri net of Figure 4 is live (free from deadlocks) and safe (has a single, realisable, instance of states) and the reachability graph has 24 states and 41 arcs.

6 Rescue Mode

As a final phase of the design process, the falling/rescue policy was integrated with the task sequence. The resulting Petri net, Figure 5, has been designed for the discrete part of the hybrid controller.

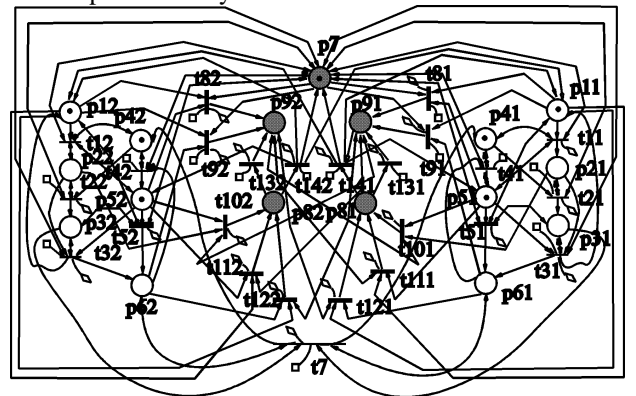


Figure 5

With respect to the Petri Net in Figure 4, the shaded places have been added along with new transitions. Place $p7$ is marked when none of the frames is in rescue mode. As soon as a 'falling down' signal is received from the continuous layer, the token in $p7$ is removed, and no transition in the sub-nets linked to the regular behaviour can fire.

Analysis of the Petri net of Figure 5, made with *Design/CPN_3.1.2* (University of Aarhus, Denmark), shows that its reachability graph has 35 states and 103 arcs. It has 4 deadlock configurations, which correspond, as expected, to the event of both frames falling. Analysis also shows that the Petri net contains a 'livelock' or 'dynamic deadlock', in which the two frames are cycling between the falling and rescuing states.

7 Implementation with Matlab Toolbox

The continuous system has been implemented as a Simulink (Vers.3) model. The models of the inverted pendulum have been simplified in the locked case to a composite pendulum model, and particular care has been devoted to the modeling of the lock/unlock conditions that permits switching between the composite model and the two separated models. The controlling Petri net has been translated into a Stateflow (Vers.2) diagram. Since Stateflow is not a verifiable tool, the translation from the Petri net to Statechart [4] was achieved by designing a Statechart which implements the Petri net reachability graph.

Figure 6 shows the angular position (left) and the angular velocities (right) of the arbors of the two synchronized frames for a regular sequence of movements when a second order sliding mode control is applied. In Figure 7 is shown the behaviour of the system with the same parameters when a first order sliding mode control is applied. It is apparent that with a first order sliding mode control the system is faster i.e. the loading/unloading cycle period is shorter. Also the velocity profile has less chatter. In Figure 8 are shown the arbor trajectories for the case that triangle $T2$ starts in an unstable position and the lock angle is not too big (i.e. less than $\alpha/2$). After an initial rescue phase, the falling frame enters the stable area and from that instant on, the two frames follows the regular sequence of set-points. The rescue is successful with both the control techniques.

8 Conclusions

This paper has examined the control of a set of interconnected nonlinear components comprising triangular-form inverted pendulums. It has demonstrated that hybrid sliding mode control can stabilise the inherently unstable pendulums and provide relatively complex supervisory control. The use of Petri nets to make the decision layer accessible to analysis and reasoning is shown to provide invaluable feedback concerning potential behaviour and facilitates the design of a supervisory

controller for the composite system. The paper demonstrates how these techniques can be used to design a composite system of linked non-linear components in which co-operative behaviour increases the stable performance envelope relative to that obtained by the system components acting in isolation.

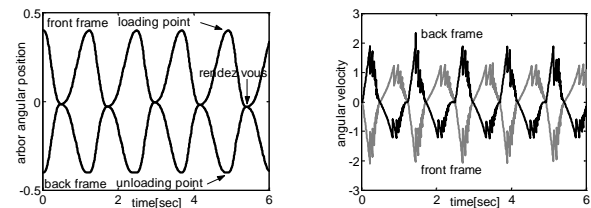


Figure 6. Normal operations with second order sliding mode control.

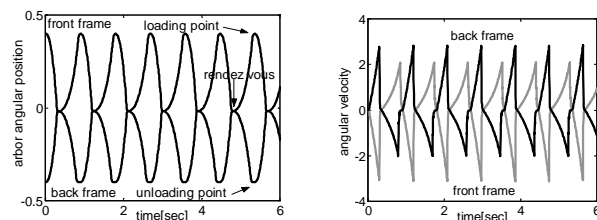


Figure 7 Normal operations with first order sliding mode control.

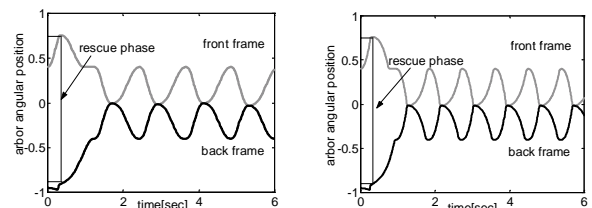


Figure 8. Rescue phase: second order sliding mode on the left, first order one on the right.

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