Non-Fragile PID Controller Design[1]

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Abstract

Recent results have provided a complete characterization of all stabilizing PID controllers for a given plant. This characterization has an elegant property: for a given fixed proportional gain, the set of all stabilizing integral and derivative gain values is the intersection of the admissible solutions of sets of linear inequalities. Based on this characterization, the design of non-fragile PID settings for enhancing the robustness of the closed-loop systems to perturbations in the controller coefficients is addressed.

1 Introduction

The PID controller is the most widely used control algorithm in industrial applications. Over the past five decades, many formulas have been proposed for PID controller settings, such as Ziegler-Nichols step response method [1], Ziegler-Nichols frequency response method [1], and Cohen-Coon method [2]. These formulas were obtained empirically based on extensive simulations of a large number of simple and stable plants. Hence, for arbitrary plants, these adhoc methods of design can not guarantee the closed-loop stability. In other words, these design formulas may give PID settings that are dangerously close to the verge of instability or even lead to closed-loop unstable.

A controller for which the closed-loop system is destabilized by small perturbations in the controller coefficients is referred to as a “fragile” controller. Recent results in [3] have brought attention to the fragility problem of controllers. In practice, many controllers are implemented digitally. Thus controller implementation is subject to round off errors and finite word length in numerical computations. Moreover, for any controller design, it is necessary to make manual tuning to obtain the desired performance of a control system. Therefore, the controller design must be able to tolerate some perturbations in its coefficients. As forememtioned, the classical PID controller design methods may give a very fragile controller or even cause the closed-loop instability. This is the motivation for us to design a non-fragile PID controller which not only stabilizes the closed-loop system but also allows to tolerate some controller uncertainties. Recent results [4] have provided a computationally efficient characterization of all stabilizing PID controllers for any given plant. The characterization for PID controllers involves the solution of a linear programming problem. These results were in turn based on a fundamental and new result generalizing the classical Hermite-Biehler Theorem [5] to the case of not necessarily Hurwitz polynomials. This characterization has an elegant property: for a given fixed proportional gain, the set of all stabilizing integral and derivative gain values is the intersection of the admissible solutions of sets of linear inequalities. Based on this characterization, a linear programming design procedure proposed in [4] allows us to enhance robustness of the closed-loop system to perturbations in the PID controller coefficients. This non-fragile controller design is to determine PID settings which give the maximum l_2 stability margin with respect to the plane of stabilizing integral and derivative gain values. The aim of this paper is to extend this earlier result to the case of the three-dimensional space of stabilizing proportional, integral and derivative gain values.

The paper is organized as follows. In Section 2, we state the results on the characterization of all stabilizing PID controllers. In Section 3, we show that the classical PID design methods may give a controller which is very fragile or even leads to the closed-loop instability. In Section 4, we propose an algorithm for setting a PID controller which not only stabilizes a given plant also is robust to perturbations in the controller coefficients. An illustrative example is also included. Finally, Section 5 contains some concluding remarks.

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2 The Results on the Characterization of All Stabilizing PID Controllers

In this section, we state the earlier results on the characterization of all stabilizing ideal PID controllers for a given plant. For details, the reader is referred to [4].

We first introduce the standard signum function $sgn : \mathcal{R} \to \{-1, 0, 1\}$ defined by

$$sgn[x] = \begin{cases} -1 \text{ if } x < 0 \\ 0 \text{ if } x = 0 \\ 1 \text{ if } x > 0 \end{cases}$$

Let $\delta(s) = \delta_0 + \delta_1 s + \cdots + \delta_n s^n$ be a given real polynomial of degree n. Let $\mathcal{C}^-$ denote the open left-half plane and $\mathcal{C}^+$ the open right-half plane. Let $l(\delta(s))$ and $r(\delta(s))$ denote the numbers of roots of $\delta(s)$ in $\mathcal{C}^-$ and $\mathcal{C}^+$ respectively.

To this end, consider the feedback system shown in Figure 1. Here r is the command signal, y is the output,

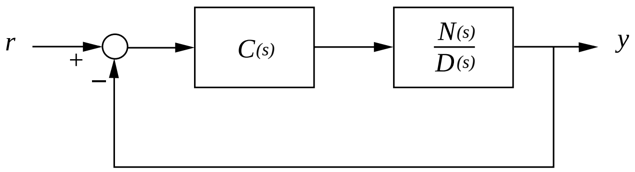

Figure 1: Feedback control system.

$$G(s) = \frac{N(s)}{D(s)}$$

is the plant to be controlled, $N(s)$ and $D(s)$ are coprime polynomials, and $C(s)$ is the controller to be designed. $C(s)$ is the ideal PID controller as:

$$C(s) = k_p + \frac{k_i}{s} + k_d s.$$

The closed loop characteristic polynomial becomes

$$\delta(s, k_p, k_i, k_d) = sD(s) + (k_i + k_d s^2)N(s) + k_p s N(s). \tag{1}$$

The problem of stabilization using a PID controller is to determine the values of k_p, k_i and k_d for which the closed loop characteristic polynomial $\delta(s, k_p, k_i, k_d)$ is Hurwitz. Consider the even-odd decompositions

$$\begin{aligned} N(s) &= N_e(s^2) + sN_o(s^2) \\ D(s) &= D_e(s^2) + sD_o(s^2). \end{aligned}$$

Define

$$N^*(s) = N(-s) = N_e(s^2) - sN_o(s^2).$$

Also let n, m be the degrees of $\delta(s, k_p, k_i, k_d)$ and $N(s)$ respectively. Now, multiplying $\delta(s, k_p, k_i, k_d)$ by $N^*(s)$ we have

$$\begin{aligned} \delta(s, k_p, k_i, k_d)N^*(s) = {} & [s^2(N_e(s^2)D_o(s^2) \\ & -D_e(s^2)N_o(s^2)) \\ & +(k_i + k_d s^2)(N_e(s^2)N_e(s^2) \\ & -s^2 N_o(s^2)N_o(s^2))] \\ & +s[D_e(s^2)N_e(s^2) \\ & -s^2 D_o(s^2)N_o(s^2) \\ & +k_p(N_e(s^2)N_e(s^2) \\ & -s^2 N_o(s^2)N_o(s^2))]. \end{aligned} \tag{2}$$

Substituting $s = j\omega$, we obtain

$$\delta(j\omega, k_p, k_i, k_d)N^*(j\omega) = p(\omega,\ k_i,\ k_d) + jq(\omega,\ k_p)$$

where

$$\begin{aligned} p(\omega,\ k_i,\ k_d) &= p_1(\omega) + (k_i - k_d\omega^2)p_2(\omega) \\ q(\omega,\ k_p) &= q_1(\omega) + k_p q_2(\omega) \\ p_1(\omega) &= -\omega^2(N_e(-\omega^2)D_o(-\omega^2) \\ &\quad -D_e(-\omega^2)N_o(-\omega^2)) \\ p_2(\omega) &= N_e(-\omega^2)N_e(-\omega^2) \\ &\quad +\omega^2 N_o(-\omega^2)N_o(-\omega^2)) \\ q_1(\omega) &= \omega(D_e(-\omega^2)N_e(-\omega^2) \\ &\quad +\omega^2 D_o(-\omega^2)N_o(-\omega^2)) \\ q_2(\omega) &= \omega(N_e(-\omega^2)N_e(-\omega^2) \\ &\quad +\omega^2 N_o(-\omega^2)N_o(-\omega^2)). \end{aligned}$$

Also, define

$$\begin{aligned} p_f(\omega, k_i, k_d) &= \frac{p(\omega, k_i, k_d)}{(1+\omega^2)^{\frac{m+n}{2}}} \\ q_f(\omega, k_p) &= \frac{q(\omega, k_p)}{(1+\omega^2)^{\frac{m+n}{2}}}. \end{aligned}$$

Before formally stating our main result on PID stabilization, we first introduce some definitions.

Definition 2.1 *Let m, n, $q_f(\omega, k_p)$ be as already defined. For a given fixed k_p, let $0 = \omega_0 < \omega_1 < \omega_2 < \cdots < \omega_{l-1}$ be the real, non-negative, distinct finite zeros of $q_f(\omega, k_p)$ with odd multiplicities*[1]*. Define a sequence of numbers $i_0, i_i, i_2, \cdots, i_l$ as follows:*
(i) If $N^*(j\omega_t) = 0$ for some $t = 1, 2, \cdots, l-1$, then define

$$i_t = 0;$$

(ii) If $N^*(s)$ has a zero of multiplicity k_n at the origin, then define

$$i_0 = sgn[p_{1_f}^{(k_n)}(0)]$$

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[1] Note that these zeros are independent of k_i or k_d.

where

$$p_{1_f}(\omega) := \frac{p_1(\omega)}{(1+\omega^2)^{\frac{(m+n)}{2}}}$$

$$p_{1_f}^{(k_n)}(0) := \frac{d^{k_n}}{d\omega^{k_n}}[p_{1_f}(\omega)]|_{\omega=0};$$

(iii) For all other $t = 0, 1, 2, \cdots, l$,

$$i_t \in \{-1, 1\}.$$

With $i_0, i_1, \cdots$ *defined in this way, we define the set* A_{k_p} *as*

$$A_{k_p} := \begin{cases} \{\{i_0, i_1, \cdots, i_l\}\} & \text{if } n+m \text{ is even} \\ \{\{i_0, i_1, \cdots, i_{l-1}\}\} & \text{if } n+m \text{ is odd.} \end{cases}$$

In other words A_{k_p} *is the set of all possible strings of 1's, 0's and -1's, whose length is* l *or* $l+1$ *depending on the value of* $n+m$*, and subject to the restrictions outlined in (i), (ii) and (iii).*

Next we introduce the set $A_{k_p}(\gamma)$ of strings in A_{k_p} with a prescribed "imaginary signature" γ. To do so, we first need to define the "imaginary signature" $\gamma(\mathcal{I})$ associated with any element $\mathcal{I} \in A_{k_p}$. This definition is motivated by Theorem 2.1 to follow.

Definition 2.2 *Let* m, n, $q(\omega, k_p)$, $q_f(\omega, k_p)$ *be as already defined. For a given fixed* k_p*, let* $0 = \omega_0 < \omega_1 < \omega_2 < \cdots < \omega_{l-1}$ *be the real, non-negative, distinct finite zeros of* $q_f(\omega, k_p)$ *with odd multiplicities. Also define* $\omega_l = \infty$*. For each string* $\mathcal{I} = \{i_0, i_1, \cdots\}$ *in* A_{k_p}*, let* $\gamma(\mathcal{I})$ *denote the "imaginary signature" associated with the string* $\mathcal{I}$ *defined by*

$$\gamma(\mathcal{I}) : = \begin{cases} \{i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{l-1}2i_{l-1} \\ +(-1)^l i_l\} \cdot (-1)^{l-1} sgn[q(\infty, k_p)] \\ \qquad \text{for } m+n \text{ even} \\ \\ \{i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{l-1}2i_{l-1}\} \\ \cdot(-1)^{l-1} sgn[q(\infty, k_p)] \\ \qquad \text{for } m+n \text{ odd} \end{cases} \tag{3}$$

Definition 2.3 *The set of strings in* A_{k_p} *with a prescribed imaginary signature* $\gamma = \psi$ *is denoted by* $A_{k_p}(\psi)$*. For a given fixed* k_p*, we also define the set of feasible strings for the PID stabilization problem as*

$$F^*_{k_p} = A_{k_p}(n - (l(N(s)) - r(N(s)))).$$

We are now ready to state the main result of this section.

Theorem 2.1 *(Main Result on PID Stabilization) The PID stabilization problem, with a fixed* k_p*, is solvable for a given plant with transfer function* $G(s)$ *if and only if the following conditions hold:*
(i) $F^*_{k_p}$ *is not empty where* $F^*_{k_p}$ *is as already defined,* i.e., *at least one feasible string exists*
and
(ii) There exists a string $\mathcal{I} = \{i_0, i_1, \cdots\} \in F^*_{k_p}$ *and values of* k_i *and* k_d *such that* $\forall\, t = 0, 1, 2, \cdots$ *for which* $N^*(j\omega_t) \neq 0$

$$p(\omega_t, k_i, k_d)i_t > 0. \tag{4}$$

where $p(\omega, k_i, k_d)$ *is as already defined. Furthermore, if there exist values of* k_i *and* k_d *such that the above condition is satisfied for the feasible strings* $\mathcal{I}_1, \mathcal{I}_2, \cdots, \mathcal{I}_s \in F^*_{k_p}$*, then the set of stabilizing* (k_i, k_d) *values corresponding to the fixed* k_p *is the union of the* (k_i, k_d) *values satisfying (4) for* $\mathcal{I}_1, \mathcal{I}_2, \cdots, \mathcal{I}_s$*.*

Remark 2.1 *It should be noted that since the constraint set is linear, the admissible set for (4) is either a convex polygon or an intersection of half planes, which is again a* convex set*. Therefore, for each fixed* k_p*, the region in the* $(k_i,\ k_d)$ *plane, if any, for which* $\delta(s, k_p, k_i, k_d)$ *is Hurwitz is a union of convex sets.*

3 Fragility of PID Settings Using Classical Tuning Methods

Over the past decades, several PID design methods have been developed for industrial use. Most of these design techniques require very little knowledge of the plant and also simple formulas are given for controller parameter settings. These formulas are obtained by extensive simulations of many stable and simple plants. In this section, using the characterization of all stabilizing PID controllers stated in the last section we assess the stability of PID controllers design using classical tuning methods. In particular, we focus on the Ziegler-Nichols frequency response design method [1].

The Ziegler-Nichols frequency response method is a closed-loop tuning method. This method first determines the point where the Nyquist curve of the plant $G(s)$ intersects the negative real axis. It can be obtained experimentally in the following way: Turn the integral and differential actions off and set the controller to be in the proportional mode only and close the loop. Slowly increase the proportional gain k_p and observe the system output response y. This action requires changing k_p in step increments and waiting for steady-state in the system output, before further increments in k_p are made. When a value of k_p leads to a periodic oscillation in the output, this critical value of k_p is called *ultimate gain* (k_u). The resulting period of the oscillation is referred to as the *ultimate period* (T_u). Based on k_u and T_u, the Ziegler-Nichols frequency response method gives simple formulas for setting PID controller parameters as follows:

$$\begin{aligned} k_p &= 0.6k_u \\ k_i &= \frac{1.2k_u}{T_u} \\ k_d &= 0.075k_uT_u\,. \end{aligned} \qquad (1)$$

Now we present two examples to show fragility of the Ziegler-Nichols frequency response design method using the results stated in the last section.

Example 3.1 *Consider the problem of choosing stabilizing PID gains for the plant* $G(s) = \frac{N(s)}{D(s)}$ *where*

$$\begin{aligned} D(s) &= 0.8s^2 + 4.2s + 1 \\ N(s) &= -4s + 1 \\ &\textit{and} \\ C(s) &= k_p + \frac{k_i}{s} + k_d s. \end{aligned}$$

The closed loop characteristic polynomial is

$$\delta(s, k_p, k_i, k_d) = sD(s) + (k_i + k_d s^2)N(s) + k_p s N(s).$$

The task is to determine those values of k_p, k_i *and* k_d, *if any, for which* $\delta(s, k_p, k_i, k_d)$ *is Hurwitz. To do so, first using the root locus ideas presented in [4], the range of* k_p *values over which the sweeping needs to be carried out was narrowed down to* $k_p \in (-1, 1.1)$. *Then by sweeping over* $k_p \in (-1, 1.1)$ *and using the results of the last section, we obtained the stabilizing set of* (k_p, k_i, k_d) *values sketched in Fig. 2.*

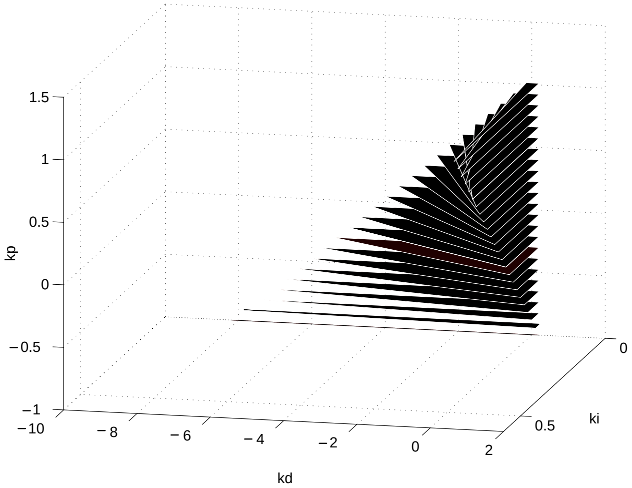

Figure 2: The stabilizing region of (k_p, k_i, k_d) values.

Now let us examine where in this plot, the parameters obtained from the Ziegler-Nichols frequency response design would be located. For the plant of this example, the ultimate gain $K_u = 1.05$ *and the ultimate period* $T_u = 3.925$. *Hence, using the Ziegler-Nichols frequency response formulas, we obtain* $k_p = 0.63$, $k_i = 0.321$, *and* $k_d = 0.3091$. *Now for* k_p *fixed at* 0.63, *the set of stabilizing* (k_i, k_d) *values can be obtained from Fig. 2. This set is sketched in Fig. 3. From Fig. 3 it is clear that for this example, the PID controller parameters obtained by the Ziegler-Nichols frequency response method are outside of the stabilizing region. It leads to the closed-loop instability.*

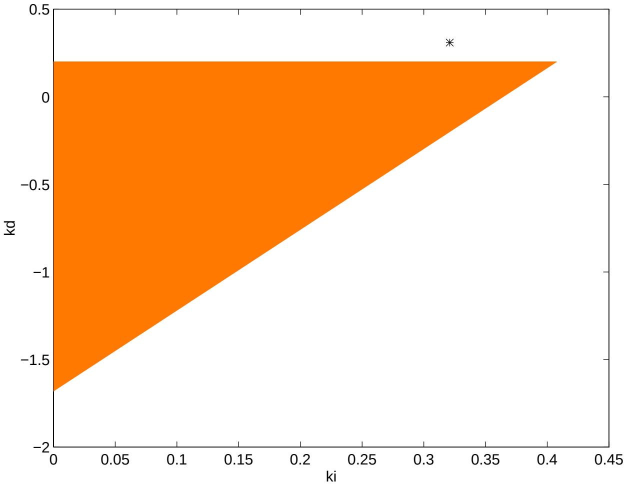

Figure 3: The stabilizing set of (k_i, k_d) values when $k_p = 0.63$; $*$ denotes the parameters corresponding to the Ziegler-Nichols frequency response method.

Example 3.2 *Again, we consider PID settings using the Ziegler-Nichols frequency response method for the plant* $G(s)$ *where*

$$G(s) = \frac{1}{s^4 + 9s^3 + 19s^2 + 7s + 6}.$$

For this plant, the ultimate gain $K_u = 8.1728$ *and the ultimate period* $T_u = 7.1246$. *Hence, using the Ziegler-Nichols frequency response formulas, we obtain* $k_p = 4.9037$, $k_i = 1.3765$, *and* $k_d = 4.3671$. *For* $k_p = 4.9037$, *using the results stated in the last section, we are able to obtain the set of all stabilizing* (k_i, k_d) *values. This stabilizing set is sketched in Fig. 4. From Fig. 4, it shows that the PID controller parameters obtained by the Ziegler-Nichols frequency response method are closed to the stability boundary. In this case, the resulting PID controller is, therefore, a fragile controller.*

4 Design of Non-Fragile PID Settings

From the previous section, we know that classical tuning methods may give PID controller parameters that

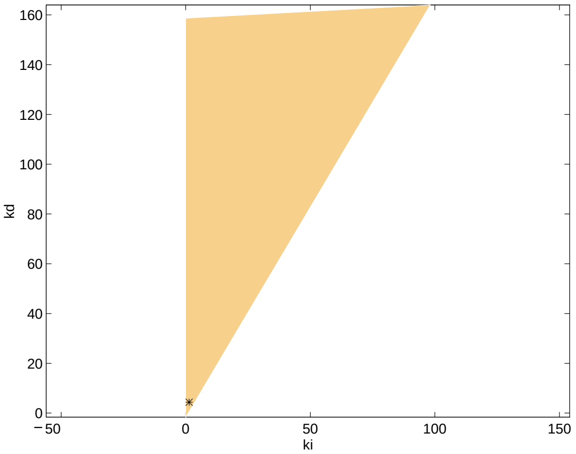

Figure 4: The stabilizing set of $(k_i,\, k_d)$ values when $k_p = 4.9037$; $*$ denotes the parameters corresponding to the Ziegler-Nichols frequency response method.

are closed-loop unstable or dangerously close to stability boundary. Since any controller that is to be practically implemented must necessarily be non-fragile so that (1) round off errors and finite word length during implementation do not destabilize the closed loop; and (2) the tuning of the parameters about the nominal design values is allowed. This motivates us to design non-fragile PID controllers for which the closed-loop systems are not destabilized by small perturbations in PID settings.

Before proceeding further, we should note that for a given plant and a fixed k_p, the set of stabilizing $(k_i,\, k_d)$ values are the admissible solutions of sets of linear inequalities in terms of k_i and k_d. Thus, for a fixed k_p, the stabilizing regions of $(k_i,\, k_d)$ values are either convex polygons or half planes. Based on this parametric structure, our objective now is to choose $(k_p,\, k_i,\, k_d)$ to be at the center of the ball of largest radius inscribed inside the stabilizing region. The radius of this ball is the maximum l_2 parametric stability margin to accommodate perturbations among the controller parameter space.

We first consider the problem of finding a circle inside a convex polygon. Consider the general m-sided polygon illustrated in Fig. 5. This polygon can be represented analytically by a set of linear inequalities:

$$\mathcal{P} = \{x | {a_i}^T x \le b_i, i = 1, ..., m\}$$

where each inequality generates the half plane containing one side of the polygon. Now consider a circle $\mathcal{C}$ with radius r. Then we have the following Lemma:

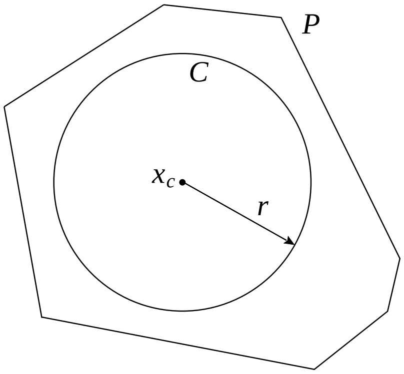

Figure 5: A circle $\mathcal{C}$ inside a polygon $\mathcal{P}$.

Lemma 4.1 *[6] Circle $\mathcal{C}$ will lie in $\mathcal{P}$ if and only if*

$${a_i}^T x_c + r\|a_i\| \quad \le \quad b_i,\ (i = 1, ..., m) \tag{1}$$

is feasible, where the feasible solution x_c is the corresponding center of $\mathcal{C}$.

From Lemma 4.1, we have that the problem of finding the largest circle $\mathcal{C}$ inside a polygon $\mathcal{P}$ is a Linear Programming (LP) problem:

$$\begin{aligned} &\text{maximize } r \\ &\text{subject to } {a_i}^T x_c + r\|a_i\| \le b_i,\ (i = 1, ..., m). \end{aligned} \tag{2}$$

Based on (2), a systematic procedure was proposed in [4] for determining PID settings which give the maximum l_2 stability margin with respect to the stabilizing k_i-k_d plane. It is of interest to extend this earlier result one step further to the one with the maximum l_2 stability margin in the stabilizing k_p-k_i-k_d space. To do so, we consider a ball $\mathcal{B}$ with radius r and center at $(x_{k_p},\, x_{k_i},\, x_{k_d})$. Denote $\mathcal{C}_\theta$ to be a circle with radius $r\cos\theta$, center at $(x_{k_p} + r\sin\theta,\, x_{k_i},\, x_{k_d})$ and parallel to the k_i-k_d plane. Circle $\mathcal{C}_\theta$ is as illustrated in Fig. 6. It is clear that

$$\mathcal{B} = \cup \mathcal{C}_\theta, \forall \theta \in [-\frac{\pi}{2},\, \frac{\pi}{2}]. \tag{3}$$

Now consider $\mathcal{C}_\theta$ with fixed x_{k_p} and θ so that $k_p = x_{k_p} + r\sin\theta$ is fixed. Let the stabilizing $(k_i,\, k_d)$ region associated with k_p be given by

$$\mathcal{P}_\theta = \{x | {a_{\theta_i}}^T x \le b_{\theta_i}, i = 1, ..., m_\theta\}.$$

Using Lemma 4.1, consequently we have $\mathcal{C}_\theta$ lies inside the stabilizing region $\mathcal{P}_\theta$ if and only if

$${a_{\theta_i}}^T x_c + r\cos\theta\|a_{\theta_i}\| \quad \le \quad b_{\theta_i},\ (i = 1, ..., m_\theta) \tag{4}$$

is feasible. We denote the set of the feasible solutions of (4) to be S_θ. From the geometrical structure, we know that for all $\theta \in [-\frac{\pi}{2},\, \frac{\pi}{2}]$ the set of $\mathcal{C}_\theta$ is concentric about k_p-axis. Since S_θ is the set of the $(k_i,\, k_d)$ coordinates

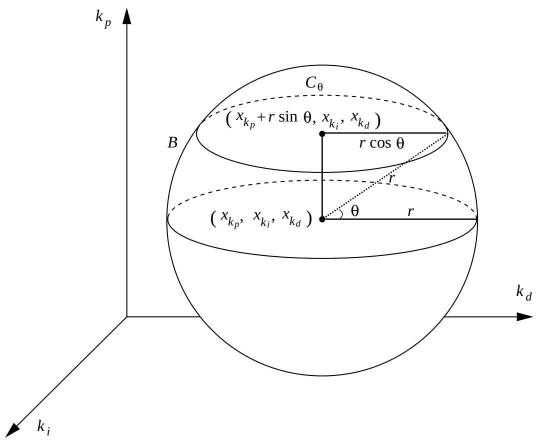

Figure 6: A ball $\mathcal{B}$ and the definition of circle $\mathcal{C}_\theta$.

of the centers associated with $\mathcal{C}_\theta$, it follows that $\mathcal{B}$ lies inside the stabilizing $(k_p,\ k_i,\ k_d)$ region if and only if

$$\cap S_\theta \quad \neq \quad \emptyset,\ \forall \theta \in [-\frac{\pi}{2},\ \frac{\pi}{2}].$$

Above observations suggest a bisection algorithm for determining the maximum l_2 parametric stability margin while k_p is fixed. Let r_{ub} be the upper bound for r. Since we have the complete characterization of all stabilizing $(k_p,\ k_i,\ k_d)$ values, we are able to determine the stabilizing range of k_p explicitly. Let us assume all stabilizing $k_p \in [k_{p_{min}},\ k_{p_{max}}]$. Thus for a fixed k_p, r_{ub} can be easily found by the following:

$$r_{ub} = \min(k_p - k_{p_{min}},\ k_{p_{max}} - k_p).$$

Then the bisection algorithm is given as follows:

Step 1: Set $r_L = 0$ and $r_U = r_{ub}$;

Step 2: Set $r = \frac{r_L + r_U}{2}$;

Step 3: Sweeping over all $\theta \in [-\frac{\pi}{2},\ \frac{\pi}{2}]$ and solving the feasibility problem of (4) for S_θ at each stage;

Step 4: If $\cap S_\theta \neq \emptyset,\ \forall \theta \in [-\frac{\pi}{2},\ \frac{\pi}{2}]$ then set $r_L = r$; otherwise set $r_U = r$;

Step 5: If $|r_U - r_L| \leq$ specified level then **STOP**; otherwise **GOTO Step 2**.

The above algorithm can be applied to determine the maximum l_2 parametric stability margin for any fixed k_p. Moreover, we can sweep over k_p and choose that value of k_p that gives the largest radius of the inscribed ball. Setting the $(k_p,\ k_i,\ k_d)$ values at the center of this ball will yield the maximum l_2 parametric stability margin with respect to $(k_p,\ k_i,\ k_d)$. The following example illustrates the steps involved.

Example 4.1 *Consider the same plant as the one used in Example 3.1, i.e.,*

$$G(s) = \frac{-4s+1}{0.8s^2+4.2+1}.$$

As in Example 3.1, we conclude that for the existence of stabilizing $(k_i,\ k_d)$ *values,* k_p *must lie in* $(-1,\ 1.1)$. *We then sweep over* $k_p \in (-1,\ 1.1)$ *and find the largest ball inscribed in the stabilizing region* $(k_p,\ k_i,\ k_d)$ *with a fixed* k_p *at each stage using the proposed bisection algorithm. Thereafter we can choose the ball with the largest radius. Following this procedure we determined that the ball with the largest radius occurs at* $k_p = 0.74488$ *and its center is located at* $k_p = 0.74488$, $k_i = 0.18537$, *and* $k_d = 0.01463$. *The radius of this ball is* $r = 0.18537$.

5 Concluding Remarks

The classical PID design methods may give PID settings which are either closed-loop unstable or dangerously close to instability. In this paper, a PID controller design which not only stabilizes the closed-loop system but can also tolerate perturbations in controller coefficients was presented. The design procedure was to extend an earlier result in [4] to determine PID settings where the maximum l_2 parametric stability margin occurs in the stabilizing k_p-k_i-k_d space. The extension of this result to the robust non-fragile PID controller design for plants with structured parametric uncertainties and controllers with coefficient perturbations is a topic for further investigation.

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