

Global Asymptotic Stability of Discrete-Time Recurrent Neural Networks

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Abstract

This paper presents new analytical results on the global asymptotic stability for the equilibrium states of a general class of discrete-time recurrent neural networks (DTRNNs) described by using a set of nonlinear difference equations. We provide a few sufficient conditions for the global asymptotic stability of DTRNNs. The resulting criteria include diagonal stability and nondiagonal stability. These stability conditions are less restrictive than the existing ones in literature.

1 Introduction

In recent years, the dynamical behavior and stability of neural networks are widely investigated. Research areas include local stability [1] and [7], global asymptotic stability, absolute stability, exponential stability [5] and [8], and so on. In particular, global asymptotic stability and absolute stability [2] and [3] are intensively studied.

When applying neural networks to solve many practical problems in the fields of optimization, control, and signal processing, we usually design neural networks to have a unique equilibrium point as well as to be globally asymptotically stable (or attractive), to avoid the risk of spurious responses or the problem of local minima. Hence, exploring the global asymptotic stability of neural networks is an important topic. For example, Liu and Michel [6] established some results for the global asymptotic stability of the equilibrium point of a discrete-time dynamical neural network with state saturation using the Lyapunov function method. The results

This work was supported in part by the Hong Kong Research Grants Council under Grant CUHK4150/97E

were used to design n th-order fixed point digital filters. They also pointed out that the global asymptotic stability of neural networks guarantees the nonexistence of limit cycles in such filters. Jin et al. [2] applied the diagonal Lyapunov function method to obtain several global asymptotic stability criteria for an analog neural network with an asymmetric weight matrix. They also emphasized that it is very difficult to search for nondiagonal Lyapunov functions.

This paper deals with the global asymptotic stability of a general class of discrete-time recurrent neural networks (DTRNNs). The DTRNN model discussed in our paper is more general than that in [3], and consequently is of more practical sense.

2 Preliminaries

2.1 Model Description

Consider a general class of discrete-time recurrent neural networks (DTRNNs) with continuous states and described by the following set of difference equations:

$$x_i(k+1) = \sum_{j=1}^n a_{ij}x_j(k) + \beta_i \sigma\left(\sum_{j=1}^n w_{ij}x_j(k) + s_i\right), \quad (1)$$

$i = 1, 2, \dots, n$; or equivalently in a vector form

$$x(k+1) = Ax(k) + B\sigma(Wx(k) + s) \quad (2)$$

where $x(k) = (x_1(k), x_2(k), \dots, x_n(k))^T$ is the neural state vector, $W = \{w_{ij}\}_{n \times n}$ is the real-valued connection weight matrix, $s = (s_1, s_2, \dots, s_n)^T$ is the input vector, $A = \{a_{ij}\}_{n \times n}$ with $|a_{ii}| \leq 1$ is the state feedback coefficient matrix, $B = \text{diag}[\beta_1, \beta_2, \dots, \beta_n]$ with $\beta_i \neq 0$ is the activation gain matrix, and $\sigma(Wx(k) + s) = (\sigma(\sum_{j=1}^n w_{1j}x_j(k) + s_1), \sigma(\sum_{j=1}^n w_{2j}x_j(k) + s_2), \dots, \sigma(\sum_{j=1}^n w_{nj}x_j(k) + s_n))^T$.

$s_2), \dots, \sum_{j=1}^n w_{nj}x_j(k) + s_n))^T$ is the vector-valued activation function. The first term in (2) is called the state feedback term or the linear term of the network.

We also consider the special case of Model (1) defined as follows:

$$x_i(k+1) = \alpha_i x_i(k) + \beta_i \sigma\left(\sum_{j=1}^n w_{ij}x_j(k) + s_i\right), \quad (3)$$

$i = 1, 2, \dots, n$; or equivalently in a vector form

$$x(k+1) = Ax(k) + B\sigma(Wx(k) + s) \quad (4)$$

where $A = \text{diag}[\alpha_1, \alpha_2, \dots, \alpha_n]$, $|\alpha_i| \leq 1$; x, W, s, B and $\sigma(Wx(k) + s)$ are same as in (2).

The DTRNN (3) is discussed in [3]. The DTRNN (1) involves the self-feedback terms as well as the other state feedback terms which make use of more state information in the application of such neural networks. Therefore, The DTRNN (1) is of more practical sense.

The function $\sigma(\cdot)$ may be chosen as a continuous and differentiable sigmoid function satisfying the following conditions: 1) $\sigma(u) \rightarrow \pm 1$ as $u \rightarrow \pm\infty$; 2) $\sigma(u)$ is bounded with the upper bound 1 and the lower bound -1 ; 3) $\sigma(u) = 0$ at a unique point $u = 0$; 4) $\sigma'(u) > 0$ and $\sigma'(u) \rightarrow 0$ as $u \rightarrow \pm\infty$; 5) $\sigma'(u)$ has a global maximal value one where $\sigma'(u) = d\sigma(u)/du$. In this paper, $\sigma(\cdot)$ is chosen as the hyperbolic tangent sigmoid function $\sigma(u) = \tanh(u)$. It is easy to verify $\|\sigma(Wx(k) + s)\| \leq \sqrt{n}$ where $\|\cdot\|$ represents the Euclidean norm throughout this paper.

Given an initial condition $x(0)$ of the DTRNN (2), the state solution of the network at the instant k can be expressed as $x(k) = A^k x(0)$

$$+ \sum_{i=0}^{k-1} A^{k-i-1} B\sigma(Wx(i) + s) \triangleq \phi(k, x(0), s). \quad (5)$$

It is well known that the equilibrium points of the DTRNN (2) are defined by $x(k+1) = x(k)$, $\forall k$.

For (2), we have the following lemma with respect to its equilibrium points.

Lemma 1: *Given any state feedback matrix A , weight matrix W , gain matrix B , and constant input $s \in R^n$, the DTRNN (2) has a unique equilibrium point, if the following conditions hold: i) A is a stable matrix; i.e., all the eigenvalues of A are located inside the unit circle in the complex plane; ii) $\det(I - A - H(k)BW) \neq 0$ for any $H(k)$ where $H(k) = \text{diag}[h_1(k), h_2(k), \dots, h_n(k)]$, $0 \leq h_i(k) \leq 1$ and I is the identity matrix throughout this paper.*

The detailed proof of Lemma 1 is omitted.

2.2 Global Equilibrium Point Stability

Definition 1: The DTRNN (2) is said to be globally asymptotically stable (GAS for short) if it has a unique stable equilibrium point x^* and the attractive region of x^* is the whole space or the state solution $\phi(k, x(0), s)$ for any initial state $x(0) \in R^n$ satisfies $x^* = \lim_{k \rightarrow +\infty} \phi(k, x(0), s)$.

Definition 2: The matrix M is said to be a stable matrix if all of the eigenvalues of M are located inside the unit circle in the complex plane. The matrix M is said to be a marginally stable matrix if all of the eigenvalues of M are not located outside the unit circle in the complex plane and there at least exists one eigenvalue on the unit circle in the complex plane.

To study the global asymptotic stability of the DTRNN (2) by applying the Lyapunov function method, we need to transform the DTRNN (2) into a new DTRNN where the origin is an equilibrium point. Let $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ be an equilibrium state of the DTRNN, and $z = (z_1, z_2, \dots, z_n)^T = x - x^*$ be a new state vector. Then, (2) can be expressed in terms of z as $z(k+1) = Az(k)$

$$+ B(\sigma(W(z(k) + x^*) + s) - \sigma(Wx^* + s)). \quad (6)$$

Applying the Lagrangian mean value theorem into (6) we have that the DTRNN (2) is equivalent to the following transformed DTRNN

$$z(k+1) = (A + \sigma'(\xi)BW)z(k) \triangleq (A + H(k)BW)z(k) \quad (7)$$

where $H(k) = \text{diag}[h_1(k), h_2(k), \dots, h_n(k)]$

$$\triangleq \text{diag}[\sigma'(\xi_1), \sigma'(\xi_2), \dots, \sigma'(\xi_n)] \triangleq \sigma'(\xi),$$

$\xi_i \in (a_i, b_i)$, $a_i = \min\{\sum_{j=1}^n w_{ij}(z_j(k) + x_j^*) + s_i, \sum_{j=1}^n w_{ij}x_j^* + s_i\}$ and $b_i = \max\{\sum_{j=1}^n w_{ij}(z_j(k) + x_j^*) + s_i, \sum_{j=1}^n w_{ij}x_j^* + s_i\}$, and $0 \leq h_i(k) \leq 1$,

$$H(k) \rightarrow \sigma'(Wx^* + s) \text{ as } \|z(k)\| \rightarrow 0. \quad (8)$$

According to Lemma 1, if A, B and W satisfy $\det(I - A - H(k)BW) \neq 0$ for any $H(k)$ where $0 \leq h_i(k) \leq 1$, then the transformed DTRNN (7) has a unique equilibrium point $z^* = 0$. In the following discussion, we will concentrate on the transformed DTRNN (7) if the DTRNN (2) has at least an equilibrium point.

3 Global Asymptotic Stability in the General Case

We first give the following two well-known lemmas.

Lemma 2: *For a sequence $\{x(k)\} \subset \Omega_0$ which is a bounded, convex, and closed set there must exist a subsequence $\{x(i_k)\}$ such that $\lim_{k \rightarrow +\infty} x(i_k) = x^* \in \Omega_0$.*

Lemma 3: (Lyapunov linearization theorem) *Consider a nonlinear DTRNN $z(k+1) = f(z(k))$ where $f(0) = 0$. If its linearized DTRNN $z(k+1) = Az(k)$ at the origin is stable, then the DTRNN $z(k+1) = f(z(k))$ is locally stable at the origin.*

Lemma 4: *Let A_1 and A_2 be two $n \times n$ stable matrices, λ_{i1} and λ_{i2} be the eigenvalues of A_1 and A_2 respectively, $i = 1, \dots, n$. Define $\lambda_{\max} = \max\{|\lambda_{11}|, \dots, |\lambda_{n1}|, |\lambda_{12}|, \dots, |\lambda_{n2}|\}$. Let λ_i be the eigenvalues of A_1A_2 . Define $\tilde{\lambda}_{\max} = \max\{|\lambda_1|, \dots, |\lambda_n|\}$. Then $\tilde{\lambda}_{\max} < \lambda_{\max}^2 < 1$.*

The proof of Lemma 4 is simple. To save space, we omit its proof here.

Theorem 1: *When $\|A\| < 1$. If $A + HBW$ is a stable matrix for any H where $H = \text{diag}[h_1, h_2, \dots, h_n]$ and $0 \leq h_i \leq 1$, then the DTRNN (2) is GAS.*

Based on Lemmas 2–4, the proof of Theorem 1 is easily given and omitted here.

Corollary 1: *Consider the DTRNN (4) where $|\alpha_i| < 1, i = 1, 2, \dots, n$. If each $A + HBW$ is a stable matrix for any H as in Theorem 1, the DTRNN (4) is GAS.*

Obviously, Corollary 1 is a direct result of Theorem 1.

In general, for a higher order network it is not easy to verify the condition in Theorem 1 and Corollary 1; that is, each $A + HBW$ is a stable matrix. In the following, we supply one special remark.

Remark 1: Consider the DTRNN (2) or (4) where $\|A\| < 1$ and $A + BW \triangleq \{m_{ij}\}_{n \times n}$ satisfies $\det(A + BW) = m_{11}m_{22} \dots m_{nn}$. If $|m_{ii}| < 1$ for $i = 1, 2, \dots, n$, then we easily see that each $A + HBW$ is a stable matrix for any H as in Theorem 1.

Since checking the condition in Theorem 1 and Corollary 1 is not easy for a high order DTRNN (2). We need to develop other methods to determine the global asymptotic stability of the DTRNNs (2) and (4). Sections 4 and 5 will present a few of such methods.

4 Diagonal Stability

For discrete-time linear systems, the following lemma is well known.

Lemma 5: *The matrix M is stable \iff there exist two positive definite matrices $P > 0$ and $Q > 0$ such that.*

$$M^T P M - P = -Q. \quad (9)$$

In (9), If P is a positive diagonal matrix, then the matrix M is said to be diagonally stable. If there only exists nondiagonal matrix P , then the matrix M is said to be nondiagonally stable. The diagonal stability implies the schur stability of the matrix. Moreover, if the tested matrix is a nonnegative or M -matrix, the diagonal stability is equivalent to the schur stability [4] and [9].

Lemma 6: *Given any $A = \{a_{ij}\}_{n \times n}, B$ and W .*

$\det(I - A - H(k)BW) \neq 0$ for any $H(k)$ where $H(k) = \text{diag}[h_1(k), h_2(k), \dots, h_n(k)]$ and $0 \leq h_i(k) \leq 1$, if there exist two positive diagonal matrices $F = \text{diag}[r_1, r_2, \dots, r_n] > 0$ and $P = \text{diag}[p_1, p_2, \dots, p_n] > 0$, and a positive definite matrix $Q > 0$ such that 1) $|a_{ii}| \leq \frac{1}{\sqrt{1+r_i}}, i = 1, \dots, n$; 2)

$$A^T(I+F)PA + W^T B(I+F^{-1})PBW - P = -Q. \quad (10)$$

Lemma 7: *The DTRNN (7) is GAS if there exist two positive diagonal matrices F and P as in Lemma 6 and a positive definite matrix $Q > 0$ such that (10) holds.*

The detailed proofs of Lemmas 6 and 7 are omitted.

The matrix F in Lemma 7 involves adjustable parameters r_1, \dots, r_n . If $F = rI_{n \times n}$ in Lemma 7, then we immediately have the following theorem.

Theorem 2: *The DTRNN (7) is GAS if there exist two positive diagonal matrices $F = rI_{n \times n} > 0$ and $P = \text{diag}[p_1, p_2, \dots, p_n] > 0$, and a positive definite matrix $Q > 0$ such that*

$$(1+r)A^T P A + (1 + \frac{1}{r})W^T B P B W - P = -Q \quad (11)$$

which can be represented equivalently by

$$rA^T P A - \frac{r}{1+r}P + W^T B P B W = -Q \quad (12)$$

where $r > 0$ is an adjustable parameter.

Lemma 8: *Given any constant matrices A, B and W . Let $\tilde{M} = \{a_{ij} + t_{ij}\beta_i w_{ij}\}_{n \times n}$ where $0 \leq t_{ij} \leq 1$. Let $M^* = \{m_{ij}^*\}_{n \times n} \triangleq \{|a_{ij} + t_{ij}^*\beta_i w_{ij}|\}_{n \times n}$ where t_{ij}^* satisfies the following conditions: $0 \leq t_{ij}^* \leq 1$ and*

$$|a_{ij} + t_{ij}^*\beta_i w_{ij}| = \max_{0 \leq t_{ij} \leq 1} |a_{ij} + t_{ij}\beta_i w_{ij}|. \quad (13)$$

If M^* is a stable matrix, then $\det(I - A - H(k)BW) \neq 0 \forall H(k)$ where $H(k) = \text{diag}[h_1(k), \dots, h_n(k)], 0 \leq h_i(k) \leq 1$.

The detailed proof of Lemma 8 is omitted.

Theorem 3: *The DTRNN (7) is GAS if M^* in Lemma 8 is a stable matrix; i.e., there exist a positive*

diagonal matrix $P = \text{diag}[p_1, p_2, \dots, p_n] > 0$ and a positive definite matrix $Q > 0$ such that

$$M^{*T} P M^* - P = -Q. \quad (14)$$

From Lemma 8, the proof of Theorem 3 easily follows and omitted here.

Theorem 4: *The DTRNN (7) is GAS if $|A| + |B||W| \triangleq \bar{M} \triangleq \{\bar{m}_{ij}\} \triangleq \{|a_{ij}| + |\beta_i| |w_{ij}|\}$ is a stable matrix; i.e., there exist a positive diagonal matrix $P = \text{diag}[p_1, p_2, \dots, p_n] > 0$ and a positive definite matrix $Q > 0$ such that*

$$(|A| + |B||W|)^T P (|A| + |B||W|) - P = -Q. \quad (15)$$

Noting $0 \leq m_{ij}^* \leq \bar{m}_{ij}$, Lemma 8 and Theorem 3, we can easily prove Theorem 4.

Remark 2: i) Consider the DTRNN (4) where $|\alpha_i| < 1, i = 1, \dots, n$. We may directly verify that the conditions in Theorems 3, 4, 7 and 8 in [3] guarantee $|A| + |B||W|$ to be a stable matrix. However, a stable matrix $|A| + |B||W|$ obviously can not guarantee the conditions in Theorems 3, 4, 7 and 8 in [3]. Therefore, the conditions in Theorems 3, 4, 7 and 8 in [3] are stronger than the condition that $|A| + |B||W|$ is a stable matrix. ii) Consider the DTRNN (4) where $|\alpha_i| = 1, i = 1, \dots, n$. The conditions in Theorems 5 and 6 in [3] guarantee the connection weight W is invertible. Thus, by noting Theorem 2 in [3], the proofs of Theorems 5 and 6 in [3] actually show that the DTRNN (3) in [3] has a unique equilibrium point which is GAS. iii) From Theorem 4, it follows that the conditions in Theorems 3, 4, 7 and 8 guarantee the DTRNN (3) in [3] has a unique equilibrium point and is GAS. Hence, noting ii) above, we easily know that [3] actually discusses the global asymptotic stability instead of the absolute stability as in Definition 2 of [3].

5 Nondiagonal Stability

Consider the DTRNN (7). Let

$$H(k) = \text{diag}[h_1(k), h_2(k), \dots, h_n(k)]$$

where $0 \leq h_i(k) \leq 1$. Let an $n \times n$ matrix

$$e_i = \begin{bmatrix} 0 & \cdots & \overset{i}{0} & \cdots & 0 \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}, \quad G_i = m_i e_i B W \quad (16)$$

where $i = 1, 2, \dots, n$ and $m_i \geq 1$.

Then the DTRNN (7) may be rewritten by

$$\begin{aligned} z(k+1) &= \left(\frac{1}{m_0} m_0 A + \frac{h_1(k)}{m_1} G_1 + \cdots + \frac{h_n(k)}{m_n} G_n \right) z(k) \\ &\triangleq (\tilde{h}_0 G_0 + \tilde{h}_1(k) G_1 + \cdots + \tilde{h}_n(k) G_n) z(k) \end{aligned} \quad (17)$$

where $0 < \tilde{h}_0 = \frac{1}{m_0} \leq 1$, $G_0 = m_0 A$.

If m_0, m_1, \dots, m_n satisfy $\sum_{i=0}^n \frac{1}{m_i} \leq 1$, then $\tilde{h}_0 + \tilde{h}_1(k) + \cdots + \tilde{h}_n(k) \leq 1$ by noting $0 \leq h_i(k) \leq 1$.

Lemma 9: *If P is a positive definite matrix such that $D_1^T P D_1 - P < 0$ and $D_2^T P D_2 - P < 0$ where $D_1, D_2, P \in R^{n \times n}$, then $D_1^T P D_1 + D_2^T P D_2 - 2P < 0$.*

$$\begin{aligned} \text{Proof:} \quad D_1^T P D_1 + D_2^T P D_2 - 2P &= -(D_1 - D_2)^T P (D_1 - D_2) + D_1^T P D_1 + D_2^T P D_2 - 2P \\ &= -(D_1 - D_2)^T P (D_1 - D_2) + D_1^T P D_1 - P + D_2^T P D_2 - P. \end{aligned}$$

Since $P > 0$, $-(D_1 - D_2)^T P (D_1 - D_2) \leq 0$. Therefore, the condition of the lemma follows.

Lemma 10: *The equilibrium point $z^* = 0$ of the DTRNN (17) is GAS if $\tilde{h}_0 + \tilde{h}_1(k) + \cdots + \tilde{h}_n(k) \leq 1$ and there exists a common positive definite matrix P such that*

$$G_i^T P G_i - P < 0, \quad i = 0, 1, \dots, n. \quad (18)$$

Proof of Lemma 10: Consider a Lyapunov function $V(z(k)) = z^T(k) P z(k)$. Compute $\Delta V(z(k)) = V(z(k+1)) - V(z(k)) = z^T(k+1) P z(k+1) - z^T(k) P z(k)$

$$= z^T(k) (\tilde{h}_0 G_0 + \sum_{i=1}^n \tilde{h}_i(k) G_i)^T P (\tilde{h}_0 G_0 + \sum_{i=1}^n \tilde{h}_i(k) G_i) z(k)$$

$$- z^T(k) P z(k) = z^T(k) ((\tilde{h}_0 G_0 + \sum_{i=1}^n \tilde{h}_i(k) G_i)^T P (\tilde{h}_0 G_0$$

$$\begin{aligned} &+ \sum_{i=1}^n \tilde{h}_i(k) G_i - P) z(k) \\ &\leq z^T(k) ((\tilde{h}_0 G_0 + \sum_{i=1}^n \tilde{h}_i(k) G_i)^T P (\tilde{h}_0 G_0 + \sum_{i=1}^n \tilde{h}_i(k) G_i) \\ &\quad - (\tilde{h}_0 + \sum_{i=1}^n \tilde{h}_i(k)) P (\tilde{h}_0 + \sum_{i=1}^n \tilde{h}_i(k))) z(k) \\ &= z^T(k) (\tilde{h}_0^2 (G_0^T P G_0 - P) + \sum_{i=1}^n \tilde{h}_0 \tilde{h}_i(k) (G_0^T P G_i \\ &\quad + G_i^T P G_0 - 2P) + \sum_{i=1}^n \tilde{h}_i^2(k) (G_i^T P G_i - P) \\ &\quad + \sum_{i < j}^n \tilde{h}_i(k) \tilde{h}_j(k) (G_i^T P G_j + G_j^T P G_i - 2P)) z(k) \\ &\triangleq z^T(k) F z(k). \end{aligned}$$

Noting $\tilde{h}_0 > 0$, $\tilde{h}_i(k) \geq 0$, (18), and Lemma 9, it easily follows $F < 0$. Hence, The equilibrium point $z^* = 0$ of (17) is GAS.

Noting Lemma 10 and the proof of Lemma 10, we easily obtain the following theorem.

Theorem 5: *The equilibrium $z^* = 0$ of the DTRNN (7) is GAS if the following two conditions hold: i) There exist $m_i \geq 1$ ($i = 0, 1, 2, \dots, n$) such that $\sum_{i=0}^n \frac{1}{m_i} \leq 1$; ii) There exists a common positive definite matrix P such that $G_i^T P G_i - P < 0$, $i = 0, 1, 2, \dots, n$.*

In view of the proof of Lemma 10, we easily have the following theorem.

Theorem 6: *The equilibrium $z^* = 0$ of the DTRNN (7) is GAS if the following conditions hold: i) Condition i) in Theorem 5; ii) There exists a common positive definite matrix P such that $G_i^T P G_i - P \leq 0$, $i = 0, 1, 2, \dots, n$; iii) Let $V(k) = z^T(k) P z(k)$, the equation $\Delta V(k) \triangleq V(k+1) - V(k) = 0$ has a unique solution $z(k) = 0$.*

6 An illustrative Example

We now present an example to use Corollary 1 and Remark 1. For space reason we omit applications of other results.

Example 1: Consider a DTRNN:

$$\begin{aligned}
 x_1(k+1) &= \alpha_1 x_1(k) + \beta_1 \sigma(w_{11} x_1(k) + s_1) \\
 x_2(k+1) &= \alpha_2 x_2(k) + \beta_2 \sigma(w_{21} x_1(k) + w_{22} x_2(k) + s_2) \\
 &\vdots \\
 x_n(k+1) &= \alpha_n x_n(k) + \beta_n \sigma(w_{n1} x_1(k) + \dots + w_{nn} x_n(k) + s_n)
 \end{aligned} \tag{19}$$

where $|\alpha_i| < 1, i = 1, \dots, n$. The architecture of this DTRNN can be found in Figure 1 of [2].

If $|\alpha_i + \beta_i w_{ii}| < 1 \forall i = 1, \dots, n$, then, the DTRNN (19) is GAS by noting Corollary 1 and Remark 1.

When $n = 3, \alpha_1 = -0.6, \beta_1 = 0.1, w_{11} = -3.8, \alpha_2 = -0.35, \beta_2 = 1.5, w_{21} = 1, w_{22} = -0.4, \alpha_3 = 0.5, \beta_3 = 2, w_{31} = -1, w_{32} = 1, w_{33} = -0.7$, a simulation result is depicted in Fig. 1.

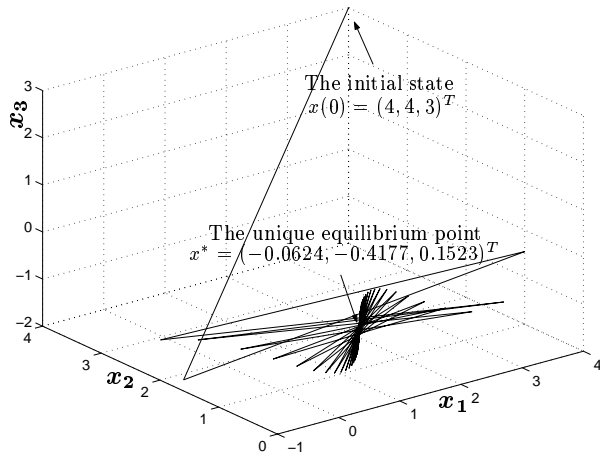


Fig. 1. The phase space trajectory of the states x_1, x_2 and x_3 of the DTRNN (19) with the input $s = (-4, -0.5, 0.5)^T$, and the initial state $x(0) = (4, 4, 3)^T$. The unique equilibrium point $x^* = (-0.0624, -0.4177, 0.1523)^T$.

Remark 3: i) In Example 1 of [2], a sufficient condition (i.e., $|\alpha_i| + |w_{ii}| \mu_i < 1$) is applied to determine the global asymptotic stability of the DTRNN. As shown in Example 1 above, the condition $|\alpha_i + w_{ii} \beta_i| < 1$ is obviously more relaxed than the existing condition. Moreover, the global asymptotic stability of the DTRNN (19) also has nothing to do with w_{ij} ($i > j$).

7 Conclusions

In this paper, several analytical results on the global asymptotic stability of a general class of discrete-time recurrent neural networks (DTRNNs) are discussed. The DTRNN model involved in this paper is more general than that in [3] and consequently is of more practical sense. We first present two conditions for the global asymptotic stability of a general DTRNN. In these conditions, the weight matrix may not be restricted to be symmetric or asymmetric. We then give several sufficient conditions which tackle with diagonal stability and non-diagonal stability. The resulting criteria are novel and less restrictive than the previous stability results.

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