Model Reduction of Singular Systems

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Abstract

In this paper, the model reduction problem for singular systems is investigated. Firstly, the previous model reduction algorithm reported in [4] is presented and proved to be wrong. Detail examination of the existing algorithm [4] will show that the difficulty of model reduction for singular systems is to retain its impulsive nature. Thus, based on this observation, we investigate the impulsive controllability and impulsive observability of singular systems and propose a new decomposition approach for singular systems. Then a new model reduction algorithm is designed based on the new decomposition via the machinery of Nehari's approximation algorithm. This new model reduction algorithm will retain the impulsive nature of the original system. Finally, an example is presented to illustrate the effectiveness of the proposed model reduction algorithm.

1 Introduction

Recently, Perev and Shafai [4] considered the problem of model reduction via balanced realizations for singular system and gave an algorithm for model reduction. Unfortunately, their method ignored the impulsive behavior and therefore produced inaccuarate approximation of the original singular system. Detail examination of the method [4] will show that the failure is due to loss of the impulsive nature of the reduced system.

In this paper, we will investigate the impulsive properties of singular systems and then propose a new decomposition technique for singular systems which is then used to propose the model reduction technique. The impulsive nature is retained in the truly fast subsystems in the decomposition. It is shown in this paper that the impulsive controllability of the truly fast system is equivalent to the controllability of a regular discrete system. A model reduction algorithm called Nehari's approximation algorithm is applied to the discrete system and a new reduced discrete system can be obtained. Then a reduced truly fast system can be constructed which can retain the impulsive nature of the original truly fast system. Finally, one numerical example is used to show the effectiveness of the proposed model reduction algorithm.

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2 Preliminary Results

Consider singular system

$$\begin{array}{l} E\dot{x}(t) = Ax(t) + Bu(t) \quad x(0-) = x_0 \\ y(t) = Cx(t) \end{array} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^m$ is the control vector and $y(t) \in \mathbf{R}^r$ is the output vector. $E \in \mathbf{R}^{n\times n}$, $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$, $C \in \mathbf{R}^{r\times n}$ are constant with E possibly singular. Assume that the matrix pair (E, A) is regular (i.e.,$|sE - A| \neq 0$).

Let us consider the following system

$$\begin{array}{l} \hat{E}\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}\hat{u}(t), \quad \hat{x}(0-) = \hat{x}_0 \\ \hat{y}(t) = \hat{C}\hat{x}(t) \end{array} \tag{2}$$

System (1) is called restricted equivalent to system (2) if there exist two nonsingular matrices P and Q such that

$$x = P\hat{x}, \quad QEP = \hat{E}, \quad QAP = \hat{A}, \quad QB = \hat{B}, \quad CP = \hat{C}$$

It is well-known from [1] that there exist two nonsingular matrices Q and P such that system (1) reduces to

$$\begin{array}{l} \dot{x}_1(t) = A_1x_1(t) + B_1u(t) \quad x_1(0-) = x_{1,0} \\ N\dot{x}_2(t) = x_2(t) + B_2u(t) \quad x_2(0-) = x_{2,0} \\ y(t) = C_1x_1(t) + C_2x_2(t) \end{array} \tag{3}$$

where $x_1(t) \in \mathbf{R}^{n_1}$, $x_2(t) \in \mathbf{R}^{n_2}$, $n_1 + n_2 = n$, $N \in \mathbf{R}^{n_2\times n_2}$ is nilpotent, and

$$\begin{array}{l} QEP = diag(I, N), \quad QAP = diag(A_1, I), \quad CP = [C_1 \;\; C_2] \\ P^{-1}x = [x_1^T \;\; x_2^T]^T, \quad QB = [B_1^T \;\; B_2^T]^T \end{array}$$

Rewriting system (3) in the slow-fast subsystems form, one can get

$$\begin{array}{l} \dot{x}_1(t) = A_1x_1(t) + B_1u(t) \\ y_1(t) = C_1x_1(t) \end{array} \tag{4}$$

$$\begin{array}{l} N\dot{x}_2(t) = x_2(t) + B_2u(t) \\ y_2(t) = C_2x_2(t) \end{array} \tag{5}$$

$$y(t) = y_1(t) + y_2(t) = C_1 x_1(t) + C_2 x_2(t) \quad (6)$$

It is also known from [1] that the state responses of the slow and fast sub systems are, respectively:

$$\begin{aligned} x_1(t) &= e^{A_1 t} x_{1,0} + \int_0^t e^{A_1(t-\tau)} B_1 u(\tau) d\tau \quad (7) \\ x_2(t) &= -\sum_{i=1}^{h-1} \delta^{(i-1)}(t) N^i x_{2,0} - \sum_{i=0}^{h-1} N^i B_2 u^{(i)}(t) \quad (8) \end{aligned}$$

where h is is the index of N ($N^m = 0,\ m \geq h;\ N^m \neq 0,\ 0 \leq m < h$), δ is the impulse function. It can be seen from (4)-(6) that system (1) has two parts: the slow part (4) is a regular linear time-invariant system and the fast part (5), in which state response has impulsive behavior for all initial conditions $x_{2,0} \neq 0$.

Lemma 2.1 *[1] 1.The following statements are equivalent:*

1. *System (1) is impulsive controllable.*
2. $$KerN + Im[B_2,\ NB_2, \cdots, N^{h-1}B_2] = \mathbf{R}^{n_2} \quad (9)$$
   where Ker and Im denote the null space and image space of corresponding operators.
3. $$rank \begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix} = n + rankE$$

2. *The following statements are equivalent:*

1. *System (1) is impulsive observable.*
2. $$ImN \bigcap Im[C_2^T,\ C_2^T N^T, \cdots, N^{h-1} C_2^T N^{T(h-1)}]^T = \{\mathbf{0}\} \quad (10)$$
3. $$rank \begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix} = n + rankE$$

These results will be used in the impulsive analysis conducted in section 4.

3 Perev and Shafai's Model Reduction Algorithm

Perev and Shafai proposed balanced realization concept for singular systems and presented a model reduction algorithm based on balanced realization idea in [4]. Their model reduction algorithm does not work in practice. In their paper authors have reduced a fifth order singular system (which is essentially a truly fast system) given by

$$\begin{aligned} N\dot{x}(t) &= x(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}$$

where

$$N = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 \\ 0.2 \\ 1.8 \\ 2.5 \\ 3.0 \end{bmatrix}, \quad C^T = \begin{bmatrix} 0.1 \\ 0.3 \\ 1.2 \\ 1.8 \\ 2.8 \end{bmatrix}$$

to a 3rd order system given below:

$$\begin{bmatrix} 0.5147 & 0.2445 & -0.0459 \\ -0.2445 & 0.2614 & 0.4158 \\ -0.0459 & -0.4158 & -0.5659 \end{bmatrix} \dot{x}_r(t) = x_r(t) + \begin{bmatrix} 3.9400 \\ 0.6512 \\ 0.1733 \end{bmatrix} u(t)$$
$$y_r(t) = [3.9400, -0.6512, 0.1733] x_r(t)$$

This is a regular state space system rather than a singular system due to the following fact

$$\det \begin{bmatrix} 0.5147 & 0.2445 & -0.0459 \\ -0.2445 & 0.2614 & 0.4158 \\ -0.0459 & -0.4158 & -0.5659 \end{bmatrix} = -0.0309 \neq 0$$

Therefore the model has no impulsive nature and furthermore, it can be shown that the reduced system is unstable. Hence the reduced order model does not approximate the original system, which proves that the method proposed in [4] does not work in practice.

The failure of their algorithm is due to the fact that the authors in [4] treated the fast system as a slow system. There are two important facts for fast systems: firstly the nilpotent structure N determines the impulsive behavior and one should retain this structure in the reduced system; secondly if *dim* $N = 1$ or $N = 0$, then the corresponding system will have no impulse responses which can be seen from (8). Therefore, desired reduced model for the fast system should retain the impulsive nature of the original system. This is the motivation for the new method proposed in the next section.

4 Main Results

In this section, we will investigate the structure of matrix N and then present some results on impulsive controllability and impulsive observability for singular systems. Based on these properties, we then propose a new decomposition of singular systems and design a new model reduction algorithm via Nehari's approximation algorithm. This algorithm will retain the impulsive nature of the singular systems.

4.1 Impulsive Controllability and Impulsive Observability

In this section we study the properties of a fast system (5) and find that the fast system (5) can be further decomposed into a truly fast system and a static system.

Consider the transfer function of the fast system (5)

$$C_2(sN-I)^{-1}B_2 = -C_2(I+sN+s^2N^2+\cdots+s^{h-1}N^{h-1})B_2$$

It is a polynomial matrix, which is quite different from regular state space systems. Now observe that

$$C_2(sN-I)^{-1}B_2 = -\frac{1}{s}C_2(\frac{1}{s}I-N)^{-1}B_2 = -\frac{1}{s}G_f(\frac{1}{s}) \tag{11}$$

where $G_f(s)$ is the transfer function matrix of the following state space system

$$\begin{aligned} \dot{x}_f(t) &= Nx_f(t) + B_2u(t) \\ y_f &= C_2x_f(t) \end{aligned} \tag{12}$$

It can be seen from (11) that the transfer function matrix of system (5) can be decomposed into a product of two transfer function matrices $G_1(s) = -\frac{1}{s}$ and $G_f(\frac{1}{s})$. Since $G_1(s)$ is an integrator, one can conclude that much of the information for the system (5) is contained in $G_f(\frac{1}{s})$. The following theorem describes the relationship between systems (5) and (12).

Theorem 4.1 *1. If system (12) is controllable, then system (5) is impulsive controllable.*

2. Let

$$N = \begin{bmatrix} J_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_p \end{bmatrix}$$

and

$$J_i = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} \times \\ b_1 \\ \times \\ b_2 \\ \vdots \\ \times \\ b_p \end{bmatrix}$$

b_i = the vector in B corresponding the last row of J_i

where $\times$ is some matrix whose elements are not important for the analysis. If system (5) is impulsive controllable and the following holds

$$rank \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \end{bmatrix} = p \tag{13}$$

Then system (12) is controllable.

3.

$$rank[sI-N, B] \geq rankN, \quad \forall\ s\ \in \mathbf{C}$$

where $\mathbf{C}$ denotes the complex plane.

Proof 1. If system (12) is controllable, Then

$$\text{rank}\left[B_2, NB_2, \cdots N^{h-1}B_2\right] = n_2$$

which implies

$$\text{Im}\left[B_2, NB_2, \cdots N^{h-1}B_2\right] = \mathbf{R}^{n_2}$$

In this case, it is obvious that

$$\text{Ker}N + \text{Im}\left[B_2, NB_2, \cdots N^{h-1}B_2\right] = \mathbf{R}^{n_2}$$

According to Lemma 2.1, system (5) is impulsive controllable.

2. Note that system (12) is controllable if and only if

$$\text{rank}\,[sI-N,\ B_2] = n_2, \quad \forall s\ \in \mathbf{C} \tag{14}$$

$\Longleftrightarrow$

$$\text{rank}\begin{bmatrix} sI-J_1 & 0 & \cdots & 0 & \begin{bmatrix} \times \\ b_1 \end{bmatrix} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & sI-J_p & \begin{bmatrix} \times \\ b_p \end{bmatrix} \end{bmatrix} = n_2, \quad \forall s\ \in \mathbf{C} \tag{15}$$

With the special structure of N, one can see that (14) is true for all $s \neq 0$. For $s = 0$, one can see from (15) that controllability of system (12) is equivalent to the validity of (13).

3. This is obvious. □

This theorem states the relationship of the controllability of a regular system and the impulsive controllability of a fast (singular) system. Further, one can have

Corollary 4.1 *If there is only one block in N and dim $J > 1$, i. e.,*

$$N = J_1$$

Then system (12) is controllable if and only if system (5) is impulsive controllable.

Proof In this case,

$$\mathrm{Ker}N = \begin{bmatrix} x \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad x \in \mathbf{R} \tag{16}$$

$$B_2 = \begin{bmatrix} \times \\ b_1 \end{bmatrix}, \; NB_2 = \begin{bmatrix} \times \\ 0 \end{bmatrix}, \cdots N^{h-1}B_2 = \begin{bmatrix} \times \\ 0 \end{bmatrix} \tag{17}$$

If system (5) is impulsive controllable, it can be seen from Lemma 2.1 that

$$\mathrm{Ker}N + \mathrm{Im}[B_2, NB_2, \cdots, N^{h-1}B_2] = \mathbf{R}^{n_2}$$

which, from (16) and (17), leads to

$$b_1 \neq 0$$

Then by Theorem 4.1, system (12) is controllable. □

In general case, the following theorem holds

Theorem 4.2 *Assume that for all J_i, dim$J_i > 1$, $i = 1, 2, 3, \cdots, p$. Then system (5) is impulsive controllable if and only if system (12) is controllable.*

Proof From part one of Theorem 4.1, it is sufficient to prove that if system (5) is impulsive controllable, then system (12) is controllable.

Assume that system (5) is impulsive controllable, it follows from Lemma 2.1 that

$$\mathrm{rank} \begin{bmatrix} N & 0 & 0 \\ I & N & B_2 \end{bmatrix} = \mathrm{rank}N + n_2 \tag{18}$$

This is equivalent to

$$\mathrm{rank}\left[-N^2, -NB_2\right] = \mathrm{rank}N$$

$\Longleftrightarrow$

$$\mathrm{rank} \begin{bmatrix} J_1^2 & \cdots & 0 & \begin{bmatrix} \times \\ b_1 \\ 0 \end{bmatrix} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & J_p^2 & \begin{bmatrix} \times \\ b_p \\ 0 \end{bmatrix} \end{bmatrix} = \mathrm{rank}N \tag{19}$$

Noting that

$$J_i^2 = \begin{bmatrix} 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} O & I_{dimJ_i-2} \\ 0 & O \end{bmatrix}$$

and rearranging equation (19), one can get

$$\mathrm{rank} \begin{bmatrix} A_w & \times \\ 0 & \begin{bmatrix} b_1 \\ \vdots \\ b_p \end{bmatrix} \\ 0 & 0 \end{bmatrix} = \mathrm{rank}N \tag{20}$$

where matrix A_w has rank$N - p$. It is obvious from (20) that

$$\mathrm{rank} \begin{bmatrix} b_1 \\ \vdots \\ b_p \end{bmatrix} = p$$

By Theorem 4.1, this implies that system (12) is controllable. □

Remark 4.1 *All above results for controllability have their dual results in observability.*

The above results indicate that if *dim* $N_i > 1$, the controllability of the regular system is equivalent to the impulsive controllability of the corresponding fast system. Also recall from (8) that the fast system will have no impulsive behavior when *dim* $N_i = 1$. Therefore, one can further decompose the fast system (5). Thus system (1) is restricted equivalent to the following system

$$\begin{array}{l} \dot{x}_1(t) = A_1 x_1(t) + B_1 u(t) \\ N_1 \dot{x}_2(t) = x_2(t) + B_2 u(t) \\ 0 = x_3(t) + B_3 u(t) \\ y(t) = C_1 x_1(t) + C_2 x_2(t) + C_3 x_3(t) \end{array} \tag{21}$$

where the slow system

$$\begin{array}{l} \dot{x}_1(t) = A_1 x_1(t) + B_1 u(t) \\ y_1(t) = C_1 x_1(t) \end{array} \tag{22}$$

is a regular state space system and the *truly fast system* is defined as

$$\begin{array}{l} N_1 \dot{x}_2(t) = x_2(t) + B_2 u(t) \\ y_2(t) = C_2 x_2(t) \end{array} \tag{23}$$

where

$$N_1 = \begin{bmatrix} \bar{J}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \bar{J}_p \end{bmatrix} \tag{24}$$

with $dim(\bar{J}_i) > 1$, and the third is a static system

$$\begin{array}{l} 0 = x_3(t) + B_3u(t) \\ y_3(t) = C_3x_3(t) \end{array} \tag{25}$$

It should be noted that the new decomposition separate the static system from the fast system. This will help us understand the impulsive nature of the singular system which is actually generated by the truly fasy system. By Theorem 4.2, the impulsive controllability of the truly fast system (23) is equivalent to the controllability of the following state space system

$$\begin{array}{l} \dot{x}_2(t) = N_1x_2(t) + B_2u(t) \\ y_2(t) = C_2x_2(t) \end{array} \tag{26}$$

4.2 The Proposed Model Reduction Method

The method proposed here is based on system decomposition discussed in last section. First one decompose the original singular system into slow, truly fast and static subsystems. The slow and truly fast subsystems are reduced separately and the static subsystem is kept unchanged to form the reduced order model. The slow subsystem is reduced using balanced truncation as proposed in [4]. However, for the truly fast system, one can use Nehari Shuffle algorithm [2]. This algorithm was proposed in [2] for approximating a desired infinite impulse response (IIR) or finite impulse response (FIR) with a FIR. Detail discussion about the algorithm and its properties can be found in the appendix in this paper or [2]. The advantage of this method is that the reduced model can retain the impulsive nature of the original systems.

Now the proposed model reduction algorithm can be summarized as follows:

1. Decompose the original system into slow, truly fast and static subsystems. **Slow subsystem**

$$\begin{array}{rcl} \dot{x}_1(t) &=& A_1x_1(t) + B_1u(t) \\ y_1(t) &=& C_1x_1(t) \end{array}$$

Truly fast subsystem

$$\begin{array}{rcl} J\dot{x}_2(t) &=& x_2(t) + B_2u(t) \\ y_2(t) &=& C_2x_2(t) \end{array}$$

$$J = \begin{bmatrix} \tilde{J}_1 & 0 & \dots & 0 \\ 0 & \tilde{J}_2 & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & \dots & \tilde{J}_p \end{bmatrix} \quad \text{and}$$

$$\tilde{J}_i = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

and ith subsystem of the truly fast subsystem is given by

$$\begin{array}{rcl} \tilde{J}_i(t)\dot{\tilde{x}}_i &=& \tilde{x}_i(t) + \tilde{B}_iu(t) \\ \tilde{y}_i(t) &=& \tilde{C}_i\tilde{x}_i(t) \end{array}$$

where

$$\begin{array}{rcl} B_2 &=& \begin{bmatrix} \tilde{B}_1 & \tilde{B}_2 & \dots & \tilde{B}_p \end{bmatrix}^T \\ C_2 &=& \begin{bmatrix} \tilde{C}_1 & \tilde{C}_2 & \dots & \tilde{C}_p \end{bmatrix} \end{array}$$

Static system

$$\begin{array}{rcl} 0 &=& x_3(t) + B_3u(t) \\ y_3(t) &=& C_3x_3(t) \end{array}$$

2. Reduce the slow subsystem system using the balanced truncation method [3] to obtain an intermediate reduced model:

$$\begin{array}{l} \dot{x}_{1r}(t) = A_rx_r(t) + B_{1r}u(t) \\ y_1(t) = C_{1r}x_r(t) \end{array}$$

3. If $dimJ_i > 2$ in the truly fast system, one can reduce the ith subsystem of the truly fast system given by

$$\begin{array}{l} \tilde{J}_i\dot{x}_2(t) = x_2(t) + \tilde{B}_2u(t) \\ y_2(t) = \tilde{C}_2x_2(t) \end{array}$$

by reducing the following discrete-time system:

$$\begin{array}{l} x_2(k+1) = \tilde{J}_ix_2(k) + \tilde{B}_2u(k) \\ y_2(k) = \tilde{C}_2x_2(k) \end{array} \tag{27}$$

using Nehari shuffle algorithm [2] to obtain the following reduced order model:

$$\begin{array}{l} x_2(k+1) = \hat{J}_ix_2(k) + \hat{B}_2u(k) \\ y_2(k) = \hat{C}_2x_2(k) \end{array}$$

where $\hat{J}_i$ can be guranteed to keep the nilpotent structure from the Nehari's approximation algorithm. Then construct the continuous time reduced order model (for the truly fast system) as follows:

$$\begin{array}{l} \hat{J}\dot{x}_2(t) = x_2(t) + \hat{B}_2u(t) \\ y_2(t) = \hat{C}_2x_2(t) \end{array}$$

4. Combine the reduced order models obtained in step 2 and 3 with the static system to obtain the final reduced model:

$$\begin{array}{l} \dot{x}_{1r}(t) = A_rx_r(t) + B_{1r}u(t) \\ y_1(t) = C_{1r}x_r(t) - C_3B_3u(t) \end{array}$$

$$\begin{array}{l} \hat{J}\dot{x}_2(t) = x_2(t) + \hat{B}_2u(t) \\ y_2(t) = \hat{C}_2x_2(t) \end{array}$$

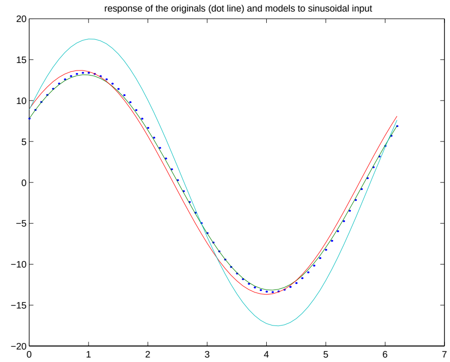

Figure 1: Responses for the sinusoidal input.

In step 3 of the proposed method, Nehari shuffle algorithm [2] is used for the reduction of truly fast system. Since this algorithm [2] was first proposed to reduce a FIR filter to low order FIR filter, the reduced order discrete-time model obtained has all the eigenvalues at the origin (Detail reference for this algorithm can be found in the Appendix). This guarantees that the constructed continuous-time reduced order model is also a truly fast system which means the impulsive nature of the original system is retained in the reduced order model. This is important to the singular systems since keeping the impulsive nature in the reduced order system is vital if the original system has impulsive property. Another issue in the proposed algorithm is the requirement of dimension for $\tilde{J}_i$ in the truly fast systems. We require that the minimum order of $\tilde{J}_i$ is 2 since there will be no impulsive behaiour for the corresponding truly fast systems if $\mathrm{d}im \ \ \tilde{J}_i = 1$. It is not suitable to approximate a singular system with impulsive behavior with a regular state system. Example in next section will show the effectiveness of the proposed algorithm.

5 Illustrative Example

Consider the 5th order system used in the previous example. Note that this system in fact has only truly fast part. There is no slow or static component in the system. Using the proposed model reduction method, the reduced order models are derived.

The following figures 1, 2, and 3 show the responses of the original system and the reduced order models for a sinusoidal input.

It can be seen from these simulations that the reduced models nearly have similar system responses. Specifically, for the sinusoidal input, the reduced models has similar shape responses and the difference of amplitude between the original system and reduced system becomes larger when the order of reduced systems becomes smaller. For the ramp input and parabolic input, the responses of the original system and the reduced systems are nearly the same due to the fact that the high oder derivatives of the these inputs will be zero. These simulation results showed that the proposed model reduction algorithm works well in practice.

6 Conclusions

In this paper, the model reduction problem for singular systems was investigated. First, the proposed model reduction algorithm for singular systems reported in [4] was examined and proved to be partly wrong. In order to design new model algorithms for singular systems, we found that the central point is to cope with its impulsive behavior. To this end, we proposed a new decomposition technique for singular systems and further investigated some properties for the truly fast subsystems. Based on this decomposition and its corresponding properties of the truly fast subsystems, we developed a new model reduction algorithm for singular systems via Nehari's approximation method. Finally, simulation results showed the effectiveness of the proposed algorithm.

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