Robust Adaptive Control of Bilinear Plants with High-Order Perturbation Uncertainties

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Abstract: The robust adaptive control issue is considered for a class of bilinear plants with uncertainties of high-order unmodelled dynamics and bounded disturbances. The generalized minimum variance control strategy is first employed to give a basic optimal control law, followed by modification of introducing the modelling error estimate to the control law. Modified least-squares scheme with a relative dead zone is developed to work with the control law to form a novel robust adaptive control algorithm, without much priori knowledge of the plant. The resulting closed loop system is proven theoretically to be robustly stable to high-order unmodelled dynamics and bounded disturbances.

Keywords: robust adaptive control, bilinear system, unmodelled dynamics

1. Introduction

Attempts to invent, design and build systems capable of controlling unknown plants or adapting to unpredictable changes in the environment resulted in the emergence of adaptive control in the 1950's, and since then adaptive control has been in the mainstream of control research and development with numerous papers and books published and a number of successful applications every year [1-4]. Significant contributions [4-6] have been made with the stability establishment of adaptive control theory. They, however, have been limited to the ideal cases, such as linear plants of disturbance-free, random noises, etc. A stable adaptive control algorithm may not be necessarily robust stable [7,8], and the disturbances in an adaptive control system may be inherently related to the plant inputs and outputs [9]. This led to the recent research of interest of the robust stability issues for adaptive control systems. In the presence of nonlinear uncertainties, unmodelled dynamics and/or bounded disturbances, it has been shown for the linear plants that robust stability can be ensured by combining the normalization, σ-modification and relative dead zone parameter estimation algorithms with the control strategies of minimum variance, generalized minimum variance or pole placement [10,11,12]. For the SISO and MIMO bilinear systems, stability has been studied recently with bounded external disturbances [13,14,15], but the robustness issue has not been considered in the presence of unmodelled dynamics.

This paper examines the robust adaptive control problem of the bilinear system in the presence of unmodelled plant uncertainties and bounded disturbances. The considered plant is a class of high-order bilinear systems with unknown and perturbed parameters. By means of minimizing a generalized variance function, a basic control law is first derived and then modified by introducing model error feedback. A novel self-tuning controller is proposed by combining the control law with a modified least squares parameter estimator with relative dead zone of bilinearity. With the proposed self-tuning controller it is proved that robust stability for the bilinear system can be ensured with respect to the high-order unmodelled nonlinear dynamics and bounded disturbances.

2. The plant description

Consider a class of bilinear plants with

$$y(t) = q^{-d} G_1 u(t) + q^{-d} G_2 y(t) u(t) + \nu(t) \qquad (1)$$

$$G_1 = \frac{B(1+\mu B')}{A(1+\mu A')} \qquad (2)$$

$$G_2 = \frac{C(1+\mu C')}{A(1+\mu A')} \qquad (3)$$

where y and u are the scalar output and input, respectively, ν is the bounded output disturbances, $d \ge 1$ is the plant delay, A, A', B, B', C and C' are polynomials of delay operator q^{-1} of orders $n_A, n_{A'}$, $n_B, n_{B'}, n_C$ and $n_{C'}$, respectively, and $\mu \ge 0$ is a singular uncertain perturbation scalar, which leads to the high-order unmodelled dynamics . In fact from (1)-(3):

$$y(t) = \frac{B}{A}u(t-d) + \frac{C}{A}u(t-d)y(t-d) + \eta_P(t) \qquad (4)$$

$$\eta_P(t) = \mu\frac{B(B'-A')}{A(1+\mu A')}u(t-d) +$$
$$+\mu\frac{C(C'-A')}{A(1+\mu A')}u(t-d)y(t-d) + v(t) \qquad (5)$$

Substituting (4) into (5) results in:

$$\eta_P(t) = \mu\frac{B'-A'}{1+\mu B'}y(t) + \mu\frac{C(C'-B')}{A(1+\mu B)}u(t-d)y(t-d) +$$
$$+\frac{1+\mu A'}{1+\mu B'}v(t) \qquad (6)$$

Then a singular perturbation from $\mu > 0$ to $\mu = 0$ results in the reduced-order model

$$y(t) = \frac{B}{A}u(t-d) + \frac{C}{A}u(t-d)y(t-d) + v(t) \qquad (7)$$

The designer is assumed to be given only the reduced-order model (7), and this without knowledge of the coefficients of *A*, *B* and *C*. The modelling error $\eta_P(t)$ thus includes the high-order unmodelled dynamics related to $u(t)$ and $y(t)$. Model representation of (7) has been effectively used to model a combustion process with one input (flow of air) and one output (Oxygen content) in discrete time [15]. The following assumptions are made for the model polynomials *A*, *B* and *C*.

Assumption 1: *A is monic and coprime with B.*

Assumption 2: n_A, n_B, n_C *and the delay d are known.*

Remark 1: Assumption 1 implies that the reduced-model is controllable. Assumption 2 provides a necessary parameter frame for the designer to construct a self-tuning controller.

3. A novel self-tuning controller

The objective is to design a self-tuning controller based on the reduced-order model, or knowledge of *A*, *B* and *C*, but to apply it to the plant (1)-(3) such that the closed loop system tracks the desired output with robust stability, in the presence of the unmodelled dynamics and bounded disturbances.

Let *P* be an arbitrary polynomial in q^{-1} of order n_P. Introduce the polynomial identity

$$P = AF + q^{-d}G \qquad (8)$$

where *F* and *G* are polynomials in q^{-1} of orders $n_F = d-1$ and $n_G = \max\{n_A - 1, n_P\}$, respectively. Multiplying (4) by *AF* gives

$$Py(t+d) = Gy(t) + FBu(t) + FCu(t)y(t) + AF\eta_P(t+d) \qquad (9)$$

Define

$$\phi(t) = Py(t)$$

$$X^T(t) = (y(t),\ldots,y(t-n_G), u(t),\ldots,u(t-n_B-d+1),$$
$$u(t)y(t),\ldots,u(t-n_C-d+1)y(t-n_C-d+1)) \qquad (10)$$

$$\eta(t) = AF\eta_P(t) \qquad (11)$$

then a regression form of (9) can be given as follows

$$\phi(t+d) = \theta^T X(t) + \eta(t+d) \qquad (12)$$

where θ is the parameter vector composed of the coefficients of *G*, *FB* and *FC*. It should be noted that though the plant is modelled linearly in θ, the nonlinearity exists in the multicity of measured inputs and outputs, and the high-order unmodelled dynamics $\eta(t)$. The following lemmas are given to establish a relative upper bound of $\eta(t)$.

*Lemma 1***:** Let $D(q^{-1})$ be a polynomial in q^{-1} with finite order n_D. For arbitrary $\sigma \in (0,1)$ there exists $\mu_0 > 0$ such that $D_\mu(z^{-1}) = 1 + \mu D(z^{-1}) \neq 0$ for any $|z| > \sigma$ and $\mu \in [0, \mu_0]$, that is $D_\mu(q^{-1})$ is strictly *Hurwitz* uniformly in $\mu \in [0, \mu_0]$.

The proof is given in [12].

*Lemma 2***:** There exist non-negative constants K_1 and K_2 independent of μ and μ_1 such that for any $\mu \in [0, \mu_1]$

$$|\eta(t)| \le \mu K_1\{\max_{0\le\tau\le t}|y(\tau)| + \max_{0\le\tau\le t-d}|u(\tau)y(\tau)|\} + K_2 \qquad (13)$$

Proof: From (11) and (6) one obtains

$$\eta(t) = \mu\frac{AF(B'-A')}{1+\mu B'}y(t) + \mu\frac{FC(C'-B')}{1+\mu B'}u(t-d)y(t-d)$$
$$+\frac{AF(1+\mu A')}{1+\mu B'}v(t) \qquad (14)$$

The result follows by applying Lemma 1 to $B_\mu^{/} = 1 + \mu B'$ and referring to the proof of Lemma 2 in [12].

Suppose that $\{\eta(t)\}$ is a white noise sequence, the generalized minimum variance control strategy of Clarke-Gawthrop type [10,11] then may be employed to achieve a basic optimal control law, minimizing the following quadratic cost function with respect to *u*(*t*):

$$J = E\{P(y(t+d) - y^*(t+d)) + Qu(t)\}^2 \qquad (15)$$

where *P* and *Q* are constant weighting polynomials in q^{-1}, and $y^*(t)$ is the bounded desired output. It follows from (12) that

$$J = E\{(\theta^T X(t) - Py^*(t+d)) + Qu(t)\}^2 + D\eta \qquad (16)$$

An optimal control law is given by

$$\theta^T X(t) + Qu(t) = Py^*(t+d) \tag{17}$$

The preceding control law (17) is not suitable for our purpose, as $\eta(t)$ includes unmodelled dynamics. For a self-tuning case, replacing θ in (17) with its estimate $\hat{\theta}(t)$ and then applying (17) to (12) one obtains

$$P(y(t+d) - y^*(t+d)) = (\theta - \hat{\theta}(t))^T X(t) - Qu(t) + \eta(t+d) \tag{18}$$

From (17), it is clear that due to the existence of $\eta(t)$ the tracking error $e(t) = y(t) - y^*(t)$ will not become zero even when the parameter estimates $\hat{\theta}(t)$ converge to their true values. To remove the effect of unmodelled dynamics, the control law (3.10) is modified by introducing an estimate of $\eta(t)$:

$$\hat{\eta}(t) = \phi(t) - \hat{\theta}(t)^T X(t-d)\,, \tag{19}$$

Then a modified self-tuning control law is given by

$$\hat{\theta}(t)^T X(t) + Qu(t) = Py^*(t+d) - \hat{\eta}(t) \tag{20}$$

The parameters are estimated with the following modified least squares scheme

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \lambda(t)L(t)\varepsilon(t) \tag{21}$$

$$\begin{aligned} W(t-1) = {} & W(t-2) - \lambda(t)L(t)[\alpha + \\ & + X^T(t-d)W(t-2)X(t-d)]L^T(t) \end{aligned} \tag{22}$$

$$L(t) = \frac{W(t-2)X(t-d)}{\alpha + X^T(t-d)W(t-2)X(t-d)} \tag{23}$$

$$\varepsilon(t) = \phi(t) - \hat{\theta}(t-1)^T X(t-d) \tag{24}$$

where

$$\begin{aligned} \lambda(t) = 0, \quad \mathit{if}\ \left|\varepsilon(t)\right| \le 2\beta[\mu^*(\max_{0\le\tau\le t}\left|y(\tau)\right| + \\ + \max_{0\le\tau\le t-d}\left|u(\tau)y(\tau)\right|) + 1] \end{aligned} \tag{25}$$

$$\lambda(t) = \gamma,\ \mathit{otherwise};\ \gamma \in [\sigma_0, 3(1-\sigma_0)/4],\ 0 < \sigma_0 < 3/7 \tag{26}$$

α and β are positive user adjustable parameters with $\beta \ge \max\{K_1, K_2\}$ (see (13)), and $\{W(t)\}$ is a matrix sequence with arbitrary initial $W(-1) > 0$.

Remark 2: It is shown that for linear plant (*e.g. C* equals to zero in (1)) the quadratic cost function (15) is equivalent to the generalized minimum variance function of the Clarke-Gawthrop type [12]

$$J = E\{[P(y(t+d) - y^*(t+d)]^2 + [Qu(t)]^2\} \tag{27}$$

Remark 3: Despite of the bilinear term appearing in the control law, *u*(*t*) is always solvable from (20) by choosing proper $\lambda(t)$ and/or α. Hence the singularity problem in solving *u*(*t*) from (20) can have been avoided.

Remark 4: The condition $\beta \ge \max\{K_1, K_2\}$ is not crucial. In practice, one can start with a large initial value, and then reduce β when the closed-loop system approaches the steady state, to improve control accuracy.

The following assumption is made on *P* and *Q*.

Assumption 3: *Off-line choices of P and Q are such that*

$$f(q^{-1}) = P(q^{-1})B(q^{-1}) + Q(q^{-1})A(q^{-1}) \tag{28}$$

is stable, that is $f(z) \ne 0$, $\left|z\right| \le 1$.

Remark 5: This condition is often used for the linear systems with the pole placement control. However, here it is made for the reduced-order model with the consideration of the robust adaptive control issues to high-order bilinear unmodelled dynamics.

Regarding unmodelled dynamics the only assumption is made as the following.

Assumption 4: *A sufficiently small upper bound* μ^* *of* μ *is available.* (*The meaning of 'sufficiently small' will be elaborated later.*)

Remark 6: This condition was used in literature for constructing relative dead zone adaptation algorithms to solve the robust adaptive control problem of linear systems [7,12]. Extension has been made here to yield a bilinear relative dead zone method.

4. Robust stability analysis

The following lemmas are given for the robust stability of the resulting closed loop system.

Lemma 3: If μ^* is sufficiently small such that $\mu^* \le \mu_1$, the application of parameter estimation scheme (21)-(26) to (12) for any $\mu \in [0, \mu^*]$ has the following properties.

$$\text{(i)}\ \lim_{t\to\infty} \frac{\lambda(t)^{1/2}\varepsilon(t)}{[\alpha + X^T(t-d)W(t-2)X(t-d)]^{1/2}} = 0 \tag{29}$$

$$\begin{aligned} &\text{(ii)}\ \left|[\hat{\theta}(t-1) - \hat{\theta}(t-d)]^T X(t-d)\right| \le h(t)\left\|X(t-d)\right\|, \\ &\quad h(t) \to 0\ \text{as}\ t \to \infty \end{aligned} \tag{30}$$

where $\|\cdot\|$ denotes the vector-Euclidean norm.

(iii) $\hat{\theta}(t)$ is bounded.

The proof can be referred to [12], and is thus omitted here.

Lemma 4: Tracking error and input dynamics satisfy

$$\begin{aligned} (PB+QA)e(t) = {} & CQy(t-d)u(t-d) + AQ\eta_P(t) + \\ & + B\Delta_d\varepsilon(t) + \delta_1(t) \end{aligned} \tag{31}$$

$$\begin{aligned} (PB+QA)u(t-d) = {} & -CPy(t-d)u(t-d) - AP\eta_P(t) + \\ & + A\Delta_d\varepsilon(t) + \delta_2(t) \end{aligned} \tag{32}$$

where $\Delta_d = 1 - q^{-d}$ and

$$\begin{aligned}\delta_1(t) &= B[\hat\theta(t-1) - \hat\theta(t-d)]^T X(t-d) + \\ &+ B[\hat\theta(t-d) - \hat\theta(t-d-1)]^T X(t-2d) - AQy^*(t)\end{aligned} \tag{33}$$

$$\begin{aligned}\delta_2(t) &= A[\hat\theta(t-1) - \hat\theta(t-d)]^T X(t-d) + \\ &+ A[\hat\theta(t-d) - \hat\theta(t-d-1)]^T X(t-2d) + APy^*(t)\end{aligned} \tag{34}$$

Proof: Using (20) and (24) gives

$$\begin{aligned}Pe(t) &= \phi(t) - \hat\eta(t-d) - \hat\theta(t-d)^T X(t-d) - Qu(t-d) \\ &= \Delta_d \varepsilon(t) + [\hat\theta(t-1) - \hat\theta(t-d)]^T X(t-d) + \\ &+ [\hat\theta(t-d) - \hat\theta(t-d-1)]^T X(t-2d) - Qu(t-d)\end{aligned} \tag{35}$$

From (4) one obtains

$$Ae(t) = Bu(t-d) + Cu(t-d)y(t-d) + A\eta_P(t) - Ay^*(t) \tag{36}$$

A summation of (36) multiplied by Q and (35) multiplied by B results to (31) with $\delta_1(t)$ of (33). In the same fashion, a summation of (36) multiplied by P and (35) multiplied by -A leads to (32) with $\delta_2(t)$ of (34).

Lemma 5: There exist sufficiently small $\mu^* > 0$ and non-negative constants C_1', C_2', C_3' and C_1'', C_2'', C_3'' independent of μ such that for all $\mu \in [0, \mu^*]$

$$\max_{t_0 \le \tau \le t-d} |u(\tau)| \le C_1' \max_{t_0 \le \tau \le t-d} |u(\tau) y(\tau)| + C_2' \max_{t_0 \le \tau \le t} |\varepsilon(\tau)| + C_3' \tag{37}$$

$$\max_{t_0 \le \tau \le t} |y(\tau)| \le C_1'' \max_{t_0 \le \tau \le t-d} |u(\tau) y(\tau)| + C_2'' \max_{t_0 \le \tau \le t} |\varepsilon(\tau)| + C_3'' \tag{38}$$

Proof: From (31) and (32) one obtains

$$\begin{aligned}e(t) &= \frac{CQ}{PB+QA} y(t-d)u(t-d) + \frac{AQ}{PB+QA}\eta_P(t) + \\ &+ \frac{B\Delta_d}{PB+QA}\varepsilon(t) + \frac{1}{PB+QA}\delta_1(t)\end{aligned} \tag{39}$$

$$\begin{aligned}u(t-d) &= \frac{-CP}{PB+QA} y(t-d)u(t-d) + \frac{-AP}{PB+QA}\eta_P(t) + \\ &+ \frac{A\Delta_d}{PB+QA}\varepsilon(t) + \frac{1}{PB+QA}\delta_2(t)\end{aligned} \tag{40}$$

Introduce the notations

$$X_y^T(t) = (y(t), \ldots, y(t-n_G))$$

$$X_u^T(t) = (u(t), \ldots, u(t-n_B-d+1))$$

$$X_{uy}^T(t) = (u(t)y(t), \ldots, u(t-n_C-d+1))$$

Then $X^T(t) = (X_y^T(t), X_u^T(t), X_{uy}^T(t))$. On the basis of Assumption 3, Lemma 2 (ii), (39) and (40), it follows that there exist sufficiently small $\mu^* > 0$ and non-negative constants C_1, C_2, C_3 such that for any $\mu \in [0, \mu^*]$

$$\begin{aligned}\max_{t_0 \le \tau \le t} \|X_y(\tau)\| + \max_{t_0 \le \tau \le t-d} \|X_u(\tau)\| &\le C_1 \max_{t_0 \le \tau \le t-d} |u(\tau)y(\tau)| + \\ &+ C_2 \max_{t_0 \le \tau \le t} |\varepsilon(\tau)| + C_3\end{aligned} \tag{41}$$

The results readily follow.

To ensure the robust stability of resulting closed loop system the following assumption is made on C and Q, and then the robust stability result is given in the sequel.

Assumption 5: *C and Q are Hurwitz polynomials.*

Remark 7: If the reduced-order model of the plant is stable and minimum phase, the assumptions can be easily satisfied. Unstable and non-minimum phase system will be considered in the future.

Theorem 1: Subject to Assumptions 1-5, there exists sufficiently small $\mu^* > 0$ such that the application of self-tuning control algorithm (20)-(26) to plant (1) ensures that

(i) the resulting closed loop system is globally robust stable in the sense that u and y are bounded for arbitrary bounded initial conditions and all $\mu \in [0, \mu^*]$

(ii) if input $u(t)$ is taken as incremental form, *i.e.* $u(t) = \Delta\tilde u(t)$, the tracking error satisfies

$$\lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N} e(t) = 0 \tag{42}$$

Proof: (i) Define sets

$$S_1 = \{ t \mid \lambda(t) \ne 0 \} \tag{43}$$

$$S_2 = \{ t \mid \lambda(t) = 0 \} \tag{44}$$

It is shown first that set S_1 is finite, this implies that with the parameter estimation scheme (21)-(26) there exists a finite time $T > 0$ such that for all $t \ge T$, $\hat\theta(t) = \hat\theta(T)$. If this were not true, then $S_1 = \{t_n\}_{n=1}^{\infty}$ and along $\{t_n\}$ from (26), (37) and (38) one obtains

$$\begin{aligned}&\frac{\lambda(t_n)^{1/2} |\varepsilon(t_n)|}{[\alpha + X(t_n-d)^T W(t_n-2) X(t_n-d)]^{1/2}} \\ &\ge \frac{\sigma_0^{1/2} |\varepsilon(t_n)|}{[\alpha + \lambda_{\max}(W(-1)) \|X(t_n-d)\|^2]^{1/2}} \\ &\ge \sigma_0^{1/2} |\varepsilon(t_n)| \big/ [\alpha + \lambda_{\max}(W(-1))(\|X_u(t_n-d)\|^2 + \\ &\qquad + \|X_y(t_n-d)\|^2 + \|X_{uy}(t_n-d)\|^2]^{1/2}\end{aligned}$$

$$\geq \frac{2\beta\sigma_0^{1/2}}{[1+\lambda_{\max}(W(-1))K]^{1/2}} > 0$$

for some constant $K>0$, where $\lambda_{\max}(W(-1))$ denotes the maximum eigenvalue of $W(-1)$. This contradicts (29), suggesting that S_1 is a finite set. This implies that when $t \geq T$, $t \in S_2$. It is observed from (33) and (34) that $\delta_i(t)$ ($i=1,2$) are bounded, and one obtains

$$(PB+QA)e(t) = CQy(t-d)u(t-d) + AQ\eta_P(t) + \\ +B\Delta_d\varepsilon(t) - AQy^*(t) \qquad (45)$$

On the basis of *Assumption* 5, (6) and (46) one obtains

$$y(t-d)u(t-d) = \frac{PB+AQ}{CQ}e(t) - \frac{A}{C}\eta_P(t) - \\ -\frac{B\Delta_d}{CQ}\varepsilon(t) + \frac{A}{C}y^*(t) \qquad (46)$$

$$\frac{A}{C}\eta_P(t) = \mu\frac{A(B'-A')}{C(1+\mu B')}y(t) + \mu\frac{C'-B'}{1+\mu B'}u(t-d)y(t-d) + \\ +\frac{A(1+\mu A')}{C(1+\mu B')}v(t) \qquad (47)$$

Hence it follows from (25) that for $t_0 > T$ there exist sufficiently small $\mu^* > 0$ and non-negative constants C_4 and C_5 such that for all $\mu \in [0, \mu^*]$

$$\max_{t_0 \leq \tau \leq t-d} |y(\tau)u(\tau)| \leq C_4 \max_{t_0 \leq \tau \leq t} |y(\tau)| + C_5 \qquad (48)$$

The boundedness of $\{u(t)\}$ and $\{y(t)\}$ can, therefore, be obtained by referring to [12].

(ii) It follows from (35) that

$$\lim_{N\to\infty}\frac{1}{N}\sum_{t=0}^{N}e(t) = \lim_{N\to\infty}\frac{1}{N}[\varepsilon(N)-\varepsilon(0)] - \\ -\lim_{N\to\infty}\frac{1}{N}[Q\tilde{u}(N)-Q\tilde{u}(0)] = 0$$

It leads to (42).

5. Conclusions

A new self-tuning control algorithm has been developed In this paper for a class of bilinear systems with uncertain perturbation and bounded disturbances. The robust stability of the resulting closed loop system has been established with respect to high-order unmodelled dynamics related to the plant input and output, and to bounded disturbances. It has been shown theoretically that under relatively small uncertainties the bilinear plant can be ensured robust stable with adaptive control technology.

Acknowledgement

The authors would like to acknowledge the support of National Natural Science Foundation of China (No. 69825106), and Hong Kong Research Grant Council (HKUST RIG95/96. EG03).

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