Stability Analysis of Decentralized Adaptive Fuzzy Logic Control for Robot Arm Tracking

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Abstract

The global ultimate stability of a decentralized adaptive fuzzy controller for trajectory tracking of robot manipulators is presented. Employing a PD control and a cubic feedback to ensure the global stability for robot tracking, an adaptive fuzzy logic scheme is incorporated to reduce the effects of interconnections, frictions, gravity force and other uncertainties. It shows that with a very limited knowledge on the sizes of interconnection terms, the controller guarantees the global ultimate boundedness of tracking errors. As the overall controller is based on a decentralized controller structure, it results in very simple fuzzy rules which can be implemented in most robot systems without hardware alternation. The simulation results are included for verification.

1 Introduction

For the trajectory tracking control of robot manipulators, a numerous approaches based on the methodologies of feedback linearization, adaptive control and robust control have been developed and some of them showed excellent tracking performance with global asymptotically convergence of tracking errors.

However, belonging to a kind of model-based or model-structure-based approaches, the schemes need a fairly good knowledge of robot dynamics at least their model structures. For instance, to apply the scheme proposed in [14], we need to know the regression matrix to have a linear-in-parameter expression for equations of motion, which can be a very tough task when the DOFs of robots increase. Another problem is that the schemes need a centralized controller structure which is not compatible to most robot systems in which the decentralized (or so-called independent-joint) controller structures are adopted. Thus, to apply these approaches, the controller hardware configurations have to be altered.

In last twenty years or so, the theory and application of fuzzy logical control have made significant progress. Using the result obtained in [15] that under certain conditions a fuzzy system can approximate a nonlinear function in an arbitrary precision, significant efforts have been made to apply fuzzy logical for controlling complicated non-linear systems. The general explorations on fuzzy controller designs and the stability features of various fuzzy controllers been reported in [12], [16], [8], [4] [13] and [7], etc. A number of studies on fuzzy based robot control have also been reported, e.g., see [11] and [17].

In the case of robot control, due to the multi-input and multi-output nature of robot systems, a centralized fuzzy logic control rule for an n joint robot will automatically result in a $2n^2$ dimensional fuzzy mapping rule including $2n$ inputs (n position errors and n velocity errors) and n (control torque) outputs. For large n, the design and on-line implementation of fuzzy rule can be dramatically difficult tasks.

It has been shown in [9] that the decentralized PD and cubic feedback control ensures the global convergence of robot tracking errors into a bounded residual set. Even though in theory the residual set can be made as small as possible by enlarging the feedback gains, it is unpractical since the larger gains will cause high frequency vibration due to the un-modeled high frequency modes in robot dynamics. To further reduce the tracking errors without enlarging the feedback gains, in this paper, we consider the decentralized control which combines the PD and cubic feedback control given in [9] with fuzzy logical control. Using the former control as a basic stabilizer the fuzzy control is to reduce the effects of interconnection, un-modeled dynamics, frictions, gravity forces and other uncertainties and therefor to improve the tracking performance. Compared with the model-based approaches, the scheme does not rely on good knowledge on robot models except some estimates on the upper-bounds of interconnection strengths. Further, since the controller is based on a decentralized structure in which each joint of robot is controlled by its own controller using local feedback only, the fuzzy rules can be significantly simplified. This also allows the scheme to be implemented in most robot systems without hardware alteration.

In Section 2, we review the robot dynamics and its decentralized subsystem structure. Section 3 shows the basic configuration of fuzzy logical systems. The decentralized PD, cubic feedback and fuzzy control scheme is presented in Section 4. We give stability results for the scheme in Section 5. In Section 6, we present simulation results and finally, in Section 7, conclusions are given.

2 Decentralized Robot Dynamics

We consider the equations of motion of rigid robot arms with n joints:

$$M(q)\ddot{q} + H(q,\dot{q})\dot{q} + g(q) + f(\dot{q}) = \tau. \tag{1}$$

In (1), $q = q(t) \in \Re^n$ is the robot arm joint angle vector; $\tau \in \Re^n$, the input torque vector; $M(q) = M^T(q) \in \Re^{n\times n}$, the positive definite inertia matrix; $H(q,\dot{q})\dot{q} = (I_n \otimes \dot{q}^T)H_v(q)\dot{q} \in \Re^n$ in which $H_v(q) = [H_1^T(q) \cdots H_n^T(q)]^T \in \Re^{n^2\times n}$, the centrifugal and Coriolis torque; $g(q) \in \Re^n$, the gravity torque; and $f(\dot{q}) \in \Re^n$, the friction torque which is upper-bounded by $c_1 + c_2\|\dot{q}\|$ where $c_1, c_2 > 0$ are constants.

It is well-known that among many possible expressions of H there exists an $H_m = H$ such that matrix $S = \dot{M}/2 - H_m$ is a skew-symmetrical matrix holding

$$\dot{q}^T(\dot{M}/2 - H_m)\dot{q} = 0, \quad \forall r \neq 0 \tag{2}$$

which implies that (1) defines a passive mapping from $(\tau - f(\dot{q}))$ to $\dot{q}$ [2] [14].

Let $q_d = q_d(t) \in C^2$ be a class of twice differentiable reference trajectories, we define $e = q_d - q$ and $\dot{e} = \dot{q}_d - \dot{q}$ as the position and velocity error vectors respectively. We further introduce a generalized tracking error (or error output vector) [14]:

$$r = \Phi e + \dot{e} = C\bar{e} = [\Phi, I]\begin{bmatrix} e \\ \dot{e} \end{bmatrix} \tag{3}$$

where $C = [\Phi, I]$ is a constant output matrix, $\Phi = diag\{\phi_1, \cdots, \phi_n\} > 0$ is a constant matrix and $\bar{e} = [e^T, \dot{e}^T]^T$ is the generalized tracking error state vector.

Once Φ and q_d are given, (1) can be rewritten as

$$M(q)\dot{r} + H_m(q,\dot{q})r = -\tau + \chi \tag{4}$$

where

$$\chi = M(q)(\ddot{q}_d + \Phi\dot{e}) + H_m(q,\dot{q})(\dot{q}_d + \Phi e) + g(q) + f(\dot{q}). \tag{5}$$

It can be shown, e.g., see [9], that there exist some positive constants α_i $(i = 1, \cdots, 4)$, determined by the robot dynamic parameters and reference trajectories given, such that χ in (5) is upper-bounded in such a way that

$$\|\chi\| \leq \alpha_1 + \alpha_2\|e\| + \alpha_3\|\dot{e}\| + \alpha_4\|e\|\|\dot{e}\|. \tag{6}$$

Further, in view of (3), it follows that $e^2 + \dot{e}^2 = \bar{e}^2$, which results in that $\|e\| \leq \|\bar{e}\|$ and $\|\dot{e}\| \leq \|\bar{e}\|$. Applying them to (6) yields

$$\|\chi\| \leq \beta_1 + \beta_2\|\bar{e}\| + \beta_3\|\bar{e}\|^2 \tag{7}$$

in which $\beta_1 = \alpha_1$, $\beta_2 = \alpha_2 + \alpha_3$ and $\beta_3 = \alpha_4$.

To be fitted into the joint-independent control configuration adopted by most robot systems, our algorithm is based on a decentralized control configuration. In this configuration, the motion of each joint of a robot arm is independently controlled by its own controller. That is, τ_i, the control toque of joint i, is merely a function of feedback signals q_i and $\dot{q}_i$ and feed-forward signals q_{di}, $\dot{q}_{di}$ and $\ddot{q}_{di}$. To formulate the decentralized tracking error dynamics, we split the overall error dynamics in (4) and (5) into n joint-torque-based subsystems. The i-th error dynamic system is then given by

$$\sum_{j=1}^{n} m_{ij}(q)\dot{r}_j + \dot{q}^T H_i(q)r + \tau_i = \chi_i \tag{8}$$

where $m_{ij}(q)$ is the i-j-th entry of $M(q)$ and χ_i, the i-th component of (5), is given by

$$\begin{aligned}\chi_i \;=\; & \sum_{j=1}^{n} m_{ij}(q)(\ddot{q}_{dj} + \phi_j\dot{e}_j) + \dot{q}^T H_i(q)(\dot{q}_d + \Phi e) \\ & + g_i(q) + f_i(\dot{q}).\end{aligned} \tag{9}$$

The left hand side of (8) can be regarded as the dominate portion of sub-system i while χ_i, given by (9), is the interconnection term acting on subsystem i. In fact, χ_i can also be expressed as

$$\chi_i = \xi_i(\bar{e}_i) + \delta_i(\bar{e}) \tag{10}$$

where $\xi_i(\bar{e}_i)$ is the local portion (related to the tracking error and parameters of subsystem i only) and $\delta_i(\bar{e})$ is the global portion (related to the other subsystems) of the interconnection, respectively. Following the same argument on the bounded feature shown in (7), it can be deduced that there exist positive constants β_{i1}, β_{i2} and β_{i3} such that χ_i is bounded by the same fashion as (7), i.e.,

$$|\chi_i| \leq \beta_{i1} + \beta_{i2}\|\bar{e}\| + \beta_{i3}\|\bar{e}\|^2. \tag{11}$$

For decentralized tracking error system shown by (4) and (5) whose upper-bound satisfies (11), the local L_∞ stability of PD control has been studied in [1] and [9]. As an extension of that, a decentralized cubic feedback control has been proposed in [9] and [10], which improved the local stability of PD control to global stability. In this study, in addition to the PD and cubic feedback controls mentioned, we introduce a decentralized adaptive fuzzy logical control (AFLC) component to further improve robots' tracking performances.

3 Fuzzy Logical Systems

It should be noticed that to develop a centralized multi-input and multi-output fuzzy logical controller of robots may be a very difficult task. For instance, for an n-joint robot the centralized fuzzy system will have $2n$ inputs (position and velocity measurements for n joint) and n outputs (fuzzy control torque vector) which might therefore lead to extremely complicated fuzzy roles especially for large n and large numbers of linguistic terms. To overcome this problem, in this study, we consider the utilizing of decentralized control structure in which n independent fuzzy logical controllers are used. Since each controller has only 2 inputs (e_i and $\dot{e}_i$) and one output (ξ_i), the fuzzy rules can be significantly simplified.

Let $\bar{e}_i = (e_i, \dot{e}_i)^T = (\bar{e}_{i1}, \bar{e}_{i2})^T \in U_i$ and $\xi_i \in V_i$ be the input and output state linguistic variables of subsystem i respectively, $U_i \subset \Re^2, V_i \subset \Re$, the universes of discourse of the input and output variables respectively, $(E_{i1}, E_{i2}) \subset U_i$ and $\Xi_i \subset V_i$, the labels in linguistic terms of input and output fuzzy sets. Using Mamdani fuzzy rules [12], the k-th fuzzy IF-THEN rule of the i-th fuzzy system controlling joint i has a form:

$$\begin{aligned} R_i^k \quad : \quad & \text{IF } (e_i = \bar{e}_{i1} \text{ is } E_{i1}^{k_l}) \text{ AND } (\dot{e}_i = \bar{e}_{i2} \text{ is } E_{i2}^{k_p}) \\ & \text{THEN } \xi_i \text{ is } \Xi_i^k \end{aligned}$$

where $k = 1, 2, \ldots, M_i$, $k_l = 1, 2, \ldots, L_i$ and $k_p = 1, 2, \ldots, P_i$ are the numbers of output sets and input sets for joint i. Further, using triangular membership functions, algebraic products for logical AND operations, product-sum inference and Center of Area (COA) defuzzification method, the output of the i-th fuzzy system is given by:

$$\xi_i(\bar{e}_i) = \frac{\sum_{k=1}^{M} \prod_{j=1}^{2} \mu_{E_{ij}^k}(\bar{e}_{ij}) y_{ik}}{\sum_{k=1}^{M} \prod_{j=1}^{2} \mu_{E_{ij}^k}(\bar{e}_{ij})} \qquad (12)$$

where $\mu_{E_{ij}^k}(\bar{e}_{ij})$ denotes the membership function of the k-th input fuzzy set for the j-th input variable and y_{ik} the k-th fuzzy singleton output. If we view the y_{ik} as adjustable parameters, then (12) can be rewritten as

$$\xi_i(\bar{e}_i, \theta_i) = \psi_i^T(\bar{e}_i)\theta_i \qquad (13)$$

where $\theta_i = (y_{i1}, \ldots, y_{iM})^T$ is the parameter vector and $\psi_i(x) = (\psi_{i1}(\bar{e}_i), \ldots, \psi_{iM}(\bar{e}_i))^T$ is the regression vector whose the l-th element is given by:

$$\psi_{il}(\bar{e}_i) = \frac{\prod_{j=1}^{2} \mu_{E_{ij}^l}(\bar{e}_{ij})}{\sum_{k=1}^{M} \prod_{j=1}^{2} \mu_{E_{ij}^k}(\bar{e}_{ij})}.$$

Using the results given in [15], fuzzy system (13) is a universal approximator in the sense that for any given real continuous function $\xi_i(\bar{e}_i)$ defined on a compact set U_i and arbitrary $\epsilon_i > 0$, there exists a fuzzy logic system $\hat{\xi}_i$ in the form of (12) such that:

$$\sup_{\bar{e}_i \in U_i} |\xi_i(\bar{e}_i, \theta_i) - \hat{\xi}_i(\bar{e}_i, \hat{\theta}_i)| < \epsilon_i. \qquad (14)$$

4 PD and Cubic Feedback based Fuzzy Logical Control

Consider a fuzzy system (13) with a parameter vector $\theta^* \in \Omega_i$:

$$\xi_i(\bar{e}_i, \theta_i^*) = \psi_i^T(\bar{e}_i)\theta_i^*$$

as an approximator of the local portion $\chi_i(\bar{e})$ in (10). That is, express $\xi_i(\bar{e}_i)$ as

$$\xi_i(\bar{e}_i) = \xi_i(\bar{e}_i, \theta_i^*) + \bar{w}_i$$

in which θ_i^* holds

$$\theta_i^* = \arg \min_{\theta_i \in \Omega_i} [\sup_{\bar{e}_i \in U_i} |\xi_i(\bar{e}_i, \theta_i) - \xi_i(\bar{e}_i)|] \qquad (15)$$

for all possible θ_i in parameter set Ω_i, and $\bar{w}_i$ is the approximation error. With such a treatment, the χ_i in (10) can be further rewritten as

$$\begin{aligned} \chi_i &= \xi_i(\bar{e}_i) + \delta_i(\bar{e}) + \xi_i(\bar{e}_i, \theta_i^*) - \xi_i(\bar{e}_i, \theta_i^*) \\ &= \xi_i(\bar{e}_i, \theta_i^*) + \delta_i(\bar{e}) + \bar{w}_i \\ &= \xi_i(\bar{e}_i, \theta_i^*) + w_i \qquad (16) \end{aligned}$$

where $w_i = \delta_i(\bar{e}) + \bar{w}_i$. Let $W = [w_i, \ldots, w_n]^T$ be the overall approximation error vector, then clearly in view of (7), (11), (14) and (15), there exists some positive constants β_1', β_2' and β_3' such that

$$||W|| \le \beta_1' + \beta_2'||\bar{e}|| + \beta_3'||\bar{e}||^2. \qquad (17)$$

For the decentralized tracking error dynamics given by (8) and (10), we propose the following decentralized PD+nonlinear+adaptive-fuzzy-logic-control (PD+NL+AFLC) law:

$$\tau_i = k_i r_i + \rho_i r_i^3 + \xi_i(\bar{e}_i, \hat{\theta}_i) \qquad (18)$$

where $k_i, \rho_i > 0$ are constant PD and nonlinear control (cubic feedback) gains and

$$\xi_i(\bar{e}_i, \hat{\theta}_i) = \psi_i^T(\bar{e}_i)\hat{\theta}_i \qquad (19)$$

is an adaptive fuzzy based approximation (or estimate) of $\xi_i(\bar{e}_i, \theta_i^*)$.

Substituting (18) into (8) and taking (16) and (19) into account we have that

$$\begin{aligned} & \sum_{j=1}^{n} m_{ij}(q)\dot{r}_j + \dot{q}^T H_i(q) r + k_i r_i + \rho_i r_i^3 \\ = \; & \psi_i^T(\bar{e}_i)(\theta_i^* - \hat{\theta}_i) + w_i \\ = \; & \psi_i^T(\bar{e}_i)\varphi_i(t) + w_i \qquad (20) \end{aligned}$$

where $\varphi_i(t) = \theta_i^* - \hat{\theta}_i$ is estimation error of the fuzzy system. The fuzzy adaptation is carried out by updating $\hat{\theta}_i$ in (18) using:

$$\dot{\varphi}_i = -\dot{\hat{\theta}}_i = -\Gamma_i \psi_i(\bar{e}_i) r_i + \sigma_i \Gamma_i \hat{\theta}_i \tag{21}$$

where $\Gamma_i = diag\{\gamma_{i1}, \ldots, \gamma_{iM}\} > 0$ is a constant matrix and $\sigma_i > 0$ is a constant. The vector versions of (20) and (21) are given by

$$M\dot{r} + Hr + Kr + \tau_c = \Psi^T(\bar{e})\varphi + W \tag{22}$$

and

$$\dot{\varphi} = -\Gamma\Psi r + \Sigma\Gamma\hat{\theta} \tag{23}$$

where $K = diag\{k_1, \ldots, k_n\}$, $\tau_c = [\rho_1 r_1^3, \cdots, \rho_n r_n^3]^T$, $\varphi = [\varphi_1^T, \cdots, \varphi_n^T]^T$, $\Gamma = diag\{\Gamma_1, \cdots, \Gamma_n\}$ and $\Psi = diag\{\psi_1, \cdots, \psi_n\}$.

5 Main Results

Primarily, we need the following lemma in the stability analysis follows:

Lemma 1 *Consider vector $v = [v_1 \;\; v_2 \;\; \cdots \;\; v_n]^T \in \Re_n$. For $\forall v \neq 0$, there exists a constant $\alpha \geq n > 0$ such that*

$$\sum_{i=1}^{n} v_i^4 \geq \frac{1}{\alpha}||v||^4 \tag{24}$$

in which $||\cdot||$ is the Euclidean norm.

Proof: Clearly, (24) holds if $\alpha \to +\infty$. We now prove that $\alpha \geq n$. Rewrite (24) as

$$\begin{aligned}\sum_{i=1}^{n} v_i^4 - \frac{1}{\alpha}\left(\sum_{i=1}^{n} v_i^2\right)^2 &= \sum_{i=1}^{n} v_i^4 - \frac{1}{\alpha}\sum_{i=1}^{n} v_i^2 \sum_{i=1}^{n} v_i^2 \\ &= \frac{1}{\alpha}\left((\alpha - 1)\sum_{i=1}^{n} v_i^4 - 2\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} v_i^2 v_j^2\right) \\ &= \frac{1}{\alpha} v^{oT} A v^o \geq 0\end{aligned}$$

where $v^o = [v_1^2 \;\; v_2^2 \;\; \cdots \;\; v_n^2]^T \in \Re^n$ and A is a symmetrical matrix given by

$$A = \{a_{ij}\} = \begin{bmatrix} \alpha - 1 & -1 & \cdots & -1 \\ -1 & \alpha - 1 & \cdots & -1 \\ \cdots & \cdots & \ddots & \cdots \\ -1 & -1 & \cdots & \alpha - 1 \end{bmatrix}_{n \times n}.$$

According to the diagonally dominant theorem [3], if $a_{ii} = |a_{ii}| \geq \sum_{j=1, j\neq i}^{n} |a_{ij}|$ holds, i.e., in our case, $\alpha \geq n$, then $A > 0$. On the other hand, we let $v_i = 1, i \in n$, (24) then becomes $n \geq n^2/\alpha$. This means (24) $\Rightarrow \alpha \geq n$ which implies that the condition is also necessary. □□□

The main result of this paper is the following theorem asserting the ultimate stability of control law (18) and (21):

Theorem 1 *Consider (8) and (9), in which τ_i is given by (18) and (21). Let $k_{1i} > 0$ and $k_{2i} > 0$ be two constants holding $k_i = k_{1i} + k_{2i}$. If $k_1' = min_i k_{1i} > 0$, $k_2' = min_i k_{2i} > 0$ and $\bar{\rho} = min_i \rho_i > n\bar{\beta}_3/(2||C||^4)$ then the tracking error r and fuzzy estimation error φ of the overall system globally converges to a residual set given by*

$$\Omega = \{(r, \varphi) : V(r, \varphi) \leq \eta/\zeta\}$$

where

$$\eta = \frac{\beta_1'}{4k_2'} + \frac{n(2\beta_2' + (\beta_3'||C||)^2)||C||}{8(2\bar{\rho} - n\beta_3'||C||)} + \frac{\bar{\sigma}}{2}||\theta^*||^2 \tag{25}$$

$$\zeta = min\{\frac{2k_1'||C||^2}{\lambda_{\bar{M}}}, \quad \bar{\sigma}\bar{\gamma}\} \tag{26}$$

$\lambda_{\bar{M}}$ is the maximum eigen-value of $\bar{M} = C^T M C$, $\bar{\sigma} = \min_i \sigma_i$ and $\bar{\gamma}$ is the minimum entry of Γ.

Proof: Consider the Lyapunov function candidate

$$V(r, \varphi) = \frac{1}{2} r^T M r + \frac{1}{2}\varphi^T \Gamma^{-1} \varphi. \tag{27}$$

Notice that the first term in the right hand side can also be written as

$$\frac{1}{2} r^T M r = \frac{1}{2}\bar{e}^T C^T M C \bar{e} = \frac{1}{2}\bar{e}^T \bar{M} \bar{e} \tag{28}$$

where $\bar{M} = C^T M C$. Taking (2) and (18) into account, the total derivative of $V(r, \varphi)$ along the solution of (1) is $\dot{V}(r, \varphi) = \dot{V}_1 + \dot{V}_2$ where, in view of (22) and (23),

$$\dot{V}_1 = -r^T K r - \sum_{i=1}^{n} \rho_i r_i^4 + r^T W$$

and

$$\dot{V}_2 = \sum_{i=1}^{n} (\dot{\varphi}_i^T \Gamma_i^{-1} + r_i \psi_i^T)\varphi_i.$$

Let $K_1 = diag\{k_{11}, \cdots, k_{1n}\} > 0$ and $K_2 = diag\{k_{21}, \cdots, k_{2n}\} > 0$ be two constant matrices holding $K = K_1 + K_2$ and apply Lemma 1 to $\dot{V}_1$, it follows that

$$\begin{aligned}\dot{V}_1 &= -r^T (K_1 + K_2) r - \sum_{i=1}^{n} \rho_i r_i^4 + r^T W \\ &\leq -k_1'||r||^2 - k_2'||r||^2 - \frac{\rho_{min}}{n}||r||^4 + r^T W\end{aligned}$$

where k_i' is the minimum entry of K_i. In view of (3) we have that $||r|| \leq c_\phi ||\bar{e}||$ where $c_\phi = ||C||$. Applying it and recalling (17) we obtain

$$\begin{aligned}\dot{V}_1 \leq\; & -\bar{k}_1||\bar{e}||^2 - \bar{k}_2||\bar{e}||^2 - \frac{\bar{\rho}}{n}||\bar{e}||^4 \\ & + \bar{\beta}_1||\bar{e}|| + \bar{\beta}_2||\bar{e}||^2 + \bar{\beta}_3||\bar{e}||^3\end{aligned}$$

where $\bar{k}_1 = k_1' c_\phi^2$, $\bar{k}_2 = k_2' c_\phi^2$, $\bar{\rho} = \rho_{min} c_\phi^4$, $\bar{\beta}_i = \beta_i' c_\phi$ ($i = 1, 2, 3$). Applying $\|\bar{e}\|^3 \le \left(\|\bar{e}\|^2 + \|\bar{e}\|^4\right)/2$, to the last term in the right hand side, we then have that

$$\begin{aligned} \dot{V}_1 &\le -\bar{k}_1\|\bar{e}\|^2 + \left(\bar{\beta}_2 + \frac{\bar{\beta}_3}{2}\right)\|\bar{e}\|^2 - \frac{\bar{\rho}\bar{\beta}_3}{2n}\|\bar{e}\|^4 \\ &\quad + \bar{\beta}_1\|\bar{e}\| - \bar{k}_2\|\bar{e}\|^2. \end{aligned}$$

Taking the completing square to the second and third terms and the forth and fifth terms, respectively, we finally have that

$$\dot{V}_1 \le -\bar{k}_1\|\bar{e}\|^2 + \frac{\bar{\beta}_1^2}{4\bar{k}_2} + \frac{n(2\bar{\beta}_2 + \bar{\beta}_3^2)}{8(2\bar{\rho} - n\bar{\beta}_3)}.$$

Applying (21) to $\dot{V}_2$ we have

$$\begin{aligned} \dot{V}_2 &= \sum_{i=1}^{n} \sigma_i(\theta_i^* - \varphi_i)^T \varphi_i \le \bar{\sigma}\|\theta^*\|\|\varphi\| - \bar{\sigma}\|\varphi\|^2 \\ &= -\frac{\bar{\sigma}}{2}(\|\theta^*\| - \|\varphi\|)^2 + \frac{\bar{\sigma}}{2}\|\theta^*\|^2 - \frac{\bar{\sigma}}{2}\|\varphi\|^2 \\ &\le \frac{\bar{\sigma}}{2}\|\theta^*\|^2 - \frac{\bar{\sigma}}{2}\|\varphi\|^2 \end{aligned}$$

where $\bar{\sigma} = \min_i \sigma_i$. Thus,

$$\dot{V}(r, \varphi) = \dot{V}_1 + \dot{V}_2 \le -\bar{k}_1\|\bar{e}\|^2 - \frac{\bar{\sigma}}{2}\|\varphi\|^2 + \eta$$

where η is given by (25). Since $\|\bar{e}\|^2 \le \bar{e}^T \bar{M}\bar{e}/\lambda_{\bar{M}} = r^T M r/\lambda_{\bar{M}}$ and $\|\varphi\|^2 \le \bar{\gamma}\varphi^T\Gamma^{-1}\varphi$, where $\lambda_{\bar{M}}$ is the maximum eigen-value of $\bar{M} = C^T M C$ and $\bar{\gamma} = \min_i \gamma_i$, we have that

$$\dot{V}(r, \varphi) \le -\frac{2\bar{k}_1}{\lambda_{\bar{M}}}\frac{r^T M r}{2} - \bar{\sigma}\bar{\gamma}\frac{\varphi^T\Gamma^{-1}\varphi}{2} + \eta.$$

In view of (26) we then obtain

$$\dot{V}(r, \varphi) \le -\zeta V(r, \varphi) + \eta$$

which implies that V is globally ultimately bounded by η/ζ and the theorem is proven. □□□

6 Simulation Results

A simulation study for the simplified version of the algorithm mentioned in the previous section has been carried out. Since the details have been presented in [6] only a brief description is given here. Without loss of generality, the simulation was subjected to a two-joint planar robot arm. The reference trajectories for joints q_{d1} and q_{d2} to follow were:

$$\begin{aligned} q_{d1}(t) &= 1.57 + 1.0\sin\omega t \\ q_{d2}(t) &= 0.5 + 1.2\sin\omega t \end{aligned}$$

where $\omega = 1.26$ rad/sec. The robot dynamic parameters used are shown in Table 1. Table 2 gives the controller parameters such as PD gains, cubic feedback gains, fuzzy control gains and $\Phi = diag\{\phi_1, \phi_2\}$.

During the simulation, the decentralized fuzzy control for each joint used a fuzzy rule shown in Table 3. That is, for each of two inputs e_i and $\dot{e}_i$, 7 fuzzy terms were used. The fuzzy output also employed 7 terms as shown in the table. As mentioned before, the triangular membership functions with equal individual sizes were used.

The performance was mainly evaluated by comparing the tracking errors of PD and that of PD+NL+AFLC. To test the robustness of the approach a step change of the payload from 3kg to 6kg occurred at time $t = 50$ seconds was also included in the simulation.

The position tracking error profiles for two joints are shown in Figures 1 and 2, respectively. It can be seen that the position tracking errors are reduced gradually at the beginning due to the parameter adaptation in fuzzy logic controllers.

After a step jump of payload at time $t = 50$, the PD control resulted in larger constant levels of position tracking errors for both joint. While, due to the adaptation effect of fuzzy controllers the position tracking errors were reduced and finally improved by about 50% compared with the errors of PD control. The parameter estimates and the control torques were also bounded as shown in [6].

Joint No.	Mass(kg)	Length(m)	Inertia(kgm^2)
Shoulder	5	0.5	1.2
Elbow	3	0.4	0.8

Table 1: Robot model parameters

Joint	PD	NL	AFLC
Shoulder	$k_1 = 30$ $\phi_1 = 0.35$	$\rho_1 = 0.8$	$\alpha_1 = 0.998$ $\gamma_1 = 0.05$
Elbow	$k_2 = 20$ $\phi_2 = 0.3$	$\rho_2 = 0.8$	$\alpha_2 = 0.995$ $\gamma_2 = 0.12$

Table 2: PD+NL+AFLC control parameters

	NB	NM	NS	Z	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	Z
NM	NB	NB	NB	NM	NS	Z	PS
NS	NB	NB	NM	NS	Z	PS	PM
Z	NB	NM	NS	Z	PS	PM	PB
PS	NM	NS	Z	PS	PM	PB	PB
PM	NS	Z	PS	PM	PB	PB	PB
PB	Z	PS	PM	PB	PB	PB	PB

Table 3: Fuzzy knowledge base rule for each joint

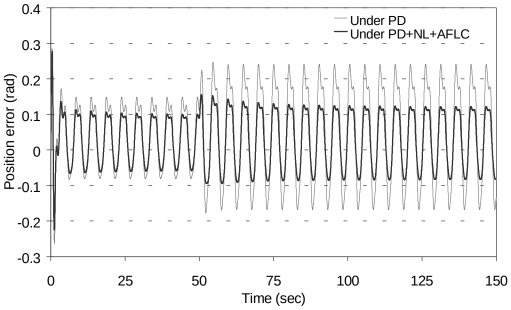

Figure 1: Position tracking error of joint 1 under PD and PD+NL+AFLC control.

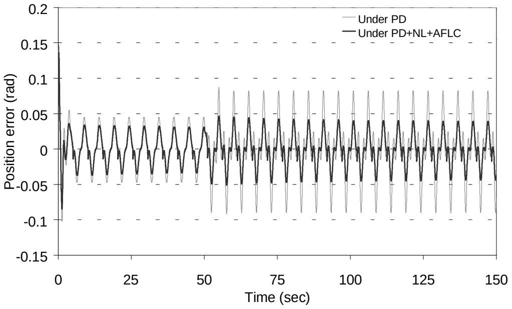

Figure 2: Position tracking error of joint 2 under PD and PD+NL+AFLC control.

7 Conclusion

The trajectory tracking control of robot manipulators using decentralized PD, cubic feedback and fuzzy control laws was studied. The stability conditions on the global ultimate boundedness of tracking errors were obtained using Lyapunov analysis. The non-fuzzy control was used to ensure the global stability while the fuzzy part was to further improve the performance. The decentralized configuration of the overall controller resulted in a very feasible fuzzy control algorithm. Simulation results presented verified the validity and effectiveness of the scheme.

References

[1] S. Arimoto and F. Miyazaki. “On the stability of PID feedback with sensory information”, In *Robotics Research*, eds. M. Brady and R. P. Paul, Cambridge: MIT Press, 1984.

[2] D. Hill and P. Moylan. “The stability of nonlinear dissipative systems”, *IEEE Trans. Automatic Control*, vol. AC-21, pp. 708-711, 1976.

[3] R. A. Horn and C. A. Johnson. *Matrix Analysis,* New York: Cambridge Press, 1985.

[4] Y. T. Kim and Z. Bien. “Robust self learning fuzzy logic controller”, *IEEE inter. conf. on robotics and Automation*, pp. 1172, 1995.

[5] E. Kwan and M. Liu. “Robot manipulators tracking using hybrid fuzzy logic controller”, *Australian Conf. on Robotics on Automation*, pp. 155, 1999.

[6] E. Kwan and M. Liu. “An adaptive fuzzy approach for robot manipulator tracking”, in *Proc. IEEE Inten. Sym. on Computational intelligence in Robotics and Automation*, USA, pp. 53-58, 1999.

[7] C. C. Lee. “Fuzzy logic in control system: fuzzy controller - part I and II”, *IEEE Trans. Syst. Man and Cybern.*, SMC-20, pp. 404-435, Nov. 1990.

[8] C. M. Lim and T. Hiyama. “Application of fuzzy logical control to manipulators”, *IEEE Trans. Robot. Automa.*, vol. 7, no. 5, pp. 688-691, 1991.

[9] M. Liu. “Decentralized PD and robust nonlinear control for robot manipulators”, *Journal of Intelligent and Robotics Systems*, vol. 20, no. 2-4, pp 319-332, 1997.

[10] M. Liu. “Decentralized control of robot manipulators: nonlinear and adaptive approaches”, *IEEE Trans. on Automat. Contr.*, vol. 44, no. 2, pp 357-363, 1999.

[11] N. J. Mandic, E.M. Scharf, and E. H. Mamdani. “Practical application of a heuristic fuzzy rule-based controller to the dynamic control of a robot arm”, *IEE Proc.*, vol. 132, Pt. D, No.4, pp. 190-203, 1985.

[12] E. H. Mamdani and S. Assilian. “An experimental in linguistic synthesis with a fuzzy logic controller”, *International Journal of Man-Machine Studies*, vol. 7, No.1:1-13, 1975.

[13] J. M. Mendel. “Fuzzy logic systems for engineering: a tutorial”, *IEEE Proc.*, vol. 83, no. 3, pp. 245-377, 1995.

[14] J. J. Slotine and W. Li. “On the adaptive control of robot manipulators”, *Int. Journal of Robotic Research*, vol. 6, pp. 49-59, 1987.

[15] L. X. Wang. “Fuzzy systems are universal approximators”, In *Proc. of IEEE International Conf. on Fuzzy Systems*, pp. 1153-1162, San Diego, CA, 1992.

[16] L. X. Wang. “Stable adaptive fuzzy controllers with application to inverted pendulum tracking”, *IEEE Trans. Syst. Man and Cybern.*, vol. 26, No. 5, pp. 677, 1996.

[17] S. Y. Yi and M. J. Chung. ”A robust fuzzy logic controller for robot manipulators with uncertainties”, *IEEE Trans. Syst. Man and Cybern*, vol. 27, No. 4, pp. 706, 1997.