On the Persistent Excitation Conditions of Adaptive Fuzzy Systems in Nonlinear Identifications[1]

Feng Wan and Li-xin Wang
eefwan@ee.ust.hk eewang@ee.ust.hk
Department of Electrical and Electronic Engineering
The Hong Kong University of Science and Technology
Clear Water Bay, Kowloon, Hong Kong, P.R.China
Fax: (852)2358-1485

Abstract

This paper addresses the persistent excitation conditions of adaptive fuzzy systems in the identifications of nonlinear functions and nonlinear dynamical systems. The adaptive fuzzy system is constructed as a standard fuzzy system and the parameters in the fuzzy system are tuned on-line by the orthogonal projection algorithm. We first give the conditions under which the parameters in the fuzzy system can be uniquely determined, and then propose methods to design input signals with the persistent excitation property for adaptive fuzzy systems in the identifications of nonlinear moving average and auto-regressive moving average systems.

1 INTRODUCTION

During the past two decades fuzzy systems have been widely used in the modeling and control of nonlinear dynamical systems. In most of this study, fuzzy systems are constructed to represent nonlinear functions which describe the dynamic behaviors of the systems under consideration. To capture the strong nonlinearities caused by the operation point variation and time-varying factors of the systems, fuzzy systems are usually tuned on-line so that the modeling or control schemes work in an adaptive manner [6], [9], [12].

Usually more attention is paid to the approximation accuracy of the fuzzy systems; however, in many circumstances it is important to ensure that the parameter estimates converge to their true values. For example, in the stability analysis of the closed-loop control system, the input signal is always required to be persistently exciting (PE) in order to guarantee the boundedness of the parameter estimation errors of the fuzzy systems. To avoid the complicated PE problem in nonlinear identifications, it is usually assumed that the input is either rich enough (random and sufficient) [12] or, simply, a signal with the PE property [6]. In view of industrial application, these assumptions are restrictive and unrealistic.

Recently, the study on the persistent excitation conditions (PEC) for identification of nonlinear systems has been reported in the literature. In [11] it is shown that, for the radial basis function (RBF) networks, the inputs with the PE property should consecutively coincide with the node center of the RBF network. This result is very restrictive. [4] gives a more relaxed result where the persistent excitation is achieved provide that the input variables belong to certain neighborhoods of the RBF network nodes. However, this condition, in practical terms, is still difficult to check.

The aim of this paper is to give persistent excitation conditions for adaptive fuzzy systems (AFS) which are easier to check for practical applications, and then design the input signal with the PE property. In Section 2 we will construct the fuzzy system to be used in the identification and introduce the orthogonal projection algorithm to tune the parameters in the fuzzy system. In Section 3 we will first study the persistent excitation conditions of the adaptive fuzzy systems used in function approximations and nonlinear system identifications, including both nonlinear deterministic moving average (N-DMA) and nonlinear deterministic auto-regressive moving average (N-DARMA) models. Then we will design input signals with the persistent excitation property for N-DMA and N-DARMA identifications. Finally, Section 4 concludes the paper.

2 ADAPTIVE FUZZY SYSTEMS AND TUNING ALGORITHM

2.1 AFS Construction

The fuzzy system which will be used in the identification is a standard fuzzy system $f(\cdot)$ (see [13]) which is a nonlinear mapping from $X \in \mathcal{D} \subset R^s$ to $f(X) \in R$. It is constructed through the following steps:

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Step 1: Let $\mathcal{D} = [\alpha_1, \beta_1] \times \cdots \times [\alpha_s, \beta_s]$. For every j ($j = 1, 2, \ldots, s$), define N_j fuzzy sets in $[\alpha_j, \beta_j]$ with the following equally-spaced triangular membership functions:

$$\begin{aligned} \mu_{A_j^1}(x_j) &= \mu(x_j; d_j^1, d_j^1, d_j^2), \\ \mu_{A_j^{N_j}}(x_j) &= \mu(x_j; d_j^{N_j-1}, d_j^{N_j}, d_j^{N_j}), \\ \mu_{A_j^r}(x_j) &= \mu(x_j; d_j^{r-1}, d_j^r, d_j^{r+1}) \end{aligned}$$

where $r = 2, 3, \ldots, N_j - 1$, $\alpha_j = d_j^1 < d_j^2 < \cdots < d_j^{N_j} = \beta_j$ and

$\mu(x; a, b, c) =$

$$\begin{cases} (x-a)/(b-a), & if \quad a \neq b \; and \; x \in [a, b], \\ (c-x)/(c-b), & if \quad b \neq c \; and \; x \in [b, c], \\ 1 \quad , & if \quad a = b \; and \; x \leq a, \\ 1 \quad , & if \quad b = c \; and \; x \geq c, \\ 0 \quad , & otherwise. \end{cases} \tag{1}$$

Step 2: Construct $N = \prod_{j=1}^{s} N_j$ fuzzy IF-THEN rules in the following form:

$$\begin{aligned} Rule^{i_1 \cdots i_s} \quad : \quad & \text{IF } x_1 \; is \; A_1^{i_1} \; and \ldots and \; x_s \; is \; A_s^{i_s}, \\ & \text{THEN } y \; is \; B^{i_1 \cdots i_s} \end{aligned} \tag{2}$$

where $i_1 = 1, 2, \ldots, N_1; \ldots; i_s = 1, 2, \ldots, N_s$.

Step 3: Construct fuzzy system $f(X)$ from the rules of (2) using the product inference engine, the singleton fuzzifier and the center average defuzzifier [13]:

$$f(X) = \frac{\sum_{i_1=1}^{N_1} \cdots \sum_{i_s=1}^{N_s} \theta^{i_1, \ldots, i_s} [\prod_{j=1}^{s} \mu_{A_j^{i_j}}(x_j)]}{\sum_{i_1=1}^{N_1} \cdots \sum_{i_s=1}^{N_s} [\prod_{j=1}^{s} \mu_{A_j^{i_j}}(x_j)]} \tag{3}$$

where $\theta^{i_1, \cdots, i_s}$ is the center of $B^{i_1 \cdots i_s}$.

2.2 Tuning Algorithm

Collect $\theta^{i_1, \cdots, i_s}$ into $\theta \in R^N$ in a natural order for $i_1 = 1, 2, \ldots, N_1; \ldots; i_s = 1, 2, \ldots, N_s$ as

$$\begin{aligned} \theta \quad = \quad & [\theta^{1,1,\ldots,1}, \ldots, \theta^{1,1,\ldots,N_s}, \\ & \ldots, \theta^{N_1,1,\ldots,1}, \ldots, \theta^{N_1,N_2,\ldots,N_s}]^T \end{aligned}$$

and the regression vector

$$\phi(X) = [\phi_1(X), \ldots, \phi_N(X)]^T$$

with the corresponding fuzzy basis functions defined as

$$\phi_l(X) = \frac{\prod_{j=1}^{s} \mu_{A_j^{i_j}}(x_j(t))}{\sum_{i_1=1}^{N_1} \cdots \sum_{i_s=1}^{N_s} [\prod_{j=1}^{s} \mu_{A_j^{i_j}}(x_j(t))]}$$

where l represents the array index $i_1, \cdots, i_s$. Then (3) can be rewritten into the so-called linear-in-the-parameter form:

$$f(X) = \phi^T(X)\theta(t) \tag{4}$$

where the time index t in $\theta(t)$ indicates that θ will change on-line. In this paper, $\theta(t)$ is tuned by the following orthogonal projection (OP) algorithm [3]:

$$\begin{aligned} \theta(t+1) &= \theta(t) \\ & \quad + \frac{P(t-1)\phi(X(t))}{\phi^T(X(t))P(t-1)\phi(X(t))} e(t) \quad (5) \\ e(t) &= y(t+1) - \phi^T(X(t))\theta(t) \quad (6) \end{aligned}$$

where

$$P(t) = P(t-1) - \frac{P(t-1)\phi(X(t))\phi^T(X(t))P(t-1)}{\phi^T(X(t))P(t-1)\phi(X(t))} \tag{7}$$

with the initial $\theta(t)$ given and $P(0) = I$.

2.3 Some Properties of the AFS

The following lemma, which gives the approximation accuracy of the fuzzy system (3), was proven in [14] and [13].

Lemma 1: Let $g(\cdot)$ be an arbitrary function on D with bounded derivatives. Then, by properly choosing the $\theta^{i_1, \cdots, i_s}$ in the fuzzy system $f(\cdot)$ of (3), we have

$$\sup_{X \in \mathcal{D}} |g(X) - f(X)| \leq ||\frac{\partial g}{\partial x_j}||_\infty h_j \tag{8}$$

where $||\frac{\partial g}{\partial x_j}||_\infty = \sup_{X \in \mathcal{D}} |\frac{\partial g}{\partial x_j}|$ and $h_j = \max_{1 \leq r \leq N_j - 1} |d_j^{r+1} - d_j^r|$.

Lemma 2 shows the decomposition property of the adaptive fuzzy system constructed in subsection 2.1.

Lemma 2: If the fuzzy system $f(X)$ is constructed through the steps in subsection 2.1, then for any given input $X(t) \in D \subset R^s$, it can be simplified into a local form:

$$\begin{aligned} f(X(t)) &= \sum_{i_1=i_1^t}^{i_1^t+1} \cdots \sum_{i_s=i_s^t}^{i_s^t+1} \theta^{i_1, \ldots, i_s} [\prod_{j=1}^{s} \mu_{A_j^{i_j}}(x_j(t))] \\ &= \overline{\phi}^T(X(t))\bar{\theta}(t) \end{aligned} \tag{9}$$

where the local parameter vector

$$\bar{\theta}(t) = [\theta^{i_1^t, \ldots, i_s^t}, \ldots, \theta^{i_1^t+1, \ldots, i_s^t+1}]^T$$

and the local regression vector

$$\overline{\phi}(X(t)) = [\phi_{i_1^t, \ldots, i_s^t}(X), \ldots, \phi_{i_1^t+1, \ldots, i_s^t+1}(X)]^T$$

with the index $i_j^t \in \{1, 2, \ldots, N_j\}$.

Proof: Note that from Step 1 in the design of $f(X)$, the domain $\mathcal{D}$ can be decomposed as

$$\mathcal{D} = \bigcup_{i_1=1}^{N_1} \cdots \bigcup_{i_s=1}^{N_s} \mathcal{D}^{i_1,\cdots,i_s}$$

where the local domain $\mathcal{D}^{i_1,\cdots,i_s} = [d_1^{i_1}, d_1^{i_1+1}] \times \cdots \times [d_s^{i_s}, d_s^{i_s+1}]$. Because of the localizing ability of the equally-spaced triangular membership function, for any $X(t) \in \mathcal{D}$, of all $\mu_{A_j^{i_j}}(x_j(t))$, $j = 1, 2, \ldots, s$, at most two but at least one value is nonzero while all others are zero. Therefore, $X(t)$ should belong to one of these patches of $\mathcal{D}$, say $\mathcal{D}^{i_1^t,\cdots,i_s^t}$. Moreover, we know that

$$\mu_{A_j^{i_j}}(x_j(t)) + \mu_{A_j^{i_j+1}}(x_j(t)) = 1,$$

for any $j \in \{1, 2, \ldots, s\}$ and $i_j \in \{1, 2, \ldots, N_j - 1\}$. This means that the denominator in (3) is always constant 1. Hence, the fuzzy system (3) can be simplified as (9). ■

Remark 1: (i) From (9) it can be seen that for any $X(t)$, only 2^s parameters in $\bar{\theta}(t)$ appear in $f(X(t))$ and thus will be tuned. The selection of the parameters to be tuned is automatically accomplished through the nonzero elements in $\phi(X(t))$. (ii) The decomposition property of the fuzzy system provides the advantage that the parameter convergence problem can be considered locally.

3 PERSISTENT EXCITATION CONDITIONS FOR ADAPTIVE FUZZY SYSTEMS

Consider a group of 2^s vectors $X(k)$, $k = 1, 2, \ldots, 2^s$ where

$$X(k) = [x_1(k), \ldots, x_s(k)].$$

For short, we denote it by $\{X(k)\}_{k=1}^{2^s}$. Let $V(k)$ be the vector:

$$\begin{aligned} V(k) &= [\prod_{j=1}^{s} x_j(k) \ \prod_{j=1}^{s-1} x_j(k) \ \ldots \ \prod_{j=2}^{s} x_j(k) \ \ldots \\ &\quad x_s(k)x_{s-1}(k) \ldots x_1(k)x_2(k) \ x_s(k) \ldots x_1(k) \ldots 1] \end{aligned}$$

where each element in $V(k)$ is the product of the elements selected from $X(k)$ if we assume the product of one element is itself and the product of the null selection is 1. Obviously, the dimension of $V(k)$ is $\sum_{i=0}^{s} C_s^i = 2^s$.

Definition 1: A group of input vectors $\{X(k)\}_{k=1}^{2^s}$ are called *complete of order s* if $span\{V(1), \ldots, V(2^s)\} = R^{2^s}$.

It can be seen later that $\{V(1), \ldots, V(2^s)\}$ derived from $\{X(k)\}_{k=1}^{2^s}$ collect all basis of the space spanned by all regression vectors in the short linear-in-the-parameter form (9) of the fuzzy system when $\{X(k)\}_{k=1}^{2^s}$ are the inputs to the fuzzy system.

3.1 PEC of AFS in Function Approximation

To model the following unknown mapping:

$$y(t+1) = g(x_1(t), \ldots, x_s(t)), \tag{10}$$

we construct an s-input-one-output fuzzy system $f(\cdot)$ through the steps in Section 2. The parameters in the fuzzy system are tuned using the orthogonal projection algorithm (5)-(7).

Theorem 1: The parameters $\theta^{i_1,\cdots,i_s}$, $\theta^{i_1+1,i_2,\cdots,i_s}$, $\ldots, \theta^{i_1+1,\cdots,i_{s-1}+1,i_s}$ and $\theta^{i_1+1,\cdots,i_s+1}$ ($i_j \in \{1, 2, \ldots, N_j - 1\}$) can be uniquely determined, if and only if in the local domain $\mathcal{D}_1^{i_1} \times \cdots \times \mathcal{D}_s^{i_s}$ there are at least one group of input points $\{X(t_k)\}_{k=1}^{2^s}$ which are complete of order s.

Proof: Suppose there are a group of inputs $\{X(t_k)\}_{k=1}^{2^s}$ inside the local domain $\mathcal{D}_1^{i_1} \times \cdots \times \mathcal{D}_s^{i_s}$, i.e., $x_j(t_k) \in [d_j^{i_j}, d_j^{i_j+1}]$ for $j = 1, 2, \ldots, s$ and $k = 1, 2, \ldots, 2^s$. From Lemma 2, using the decomposing property of the fuzzy system, we have the following 2^s equations ($k = 1, 2, \ldots, 2^s$):

$$\begin{aligned} \hat{y}(t_k+1) &= f(x_1(t_k), \ldots, x_s(t_k)) \\ &= \sum_{i_1}^{i_1+1} \cdots \sum_{i_s}^{i_s+1} \theta^{i_1,\cdots,i_s} [\prod_{j=1}^{s} \mu_{A_j^{i_j}}(x_j(t))] \\ &= \theta^{i_1,\cdots,i_s} \frac{x_1(t_k) - d_1^{i_1}}{\Delta d_1^{i_1}} \cdots \frac{x_s(t_k) - d_s^{i_s}}{\Delta d_s^{i_s}} \\ &\quad + \theta^{i_1+1,i_2,\cdots,i_s} \frac{d_1^{i_1+1} - x_1(t_k)}{\Delta d_1^{i_1}} \cdots \frac{x_s(t_k) - d_s^{i_s}}{\Delta d_s^{i_s}} \\ &\quad + \cdots \\ &\quad + \theta^{i_1+1,i_2+1,\cdots,i_s+1} \\ &\quad \cdot \frac{d_1^{i_1+1} - x_1(t_k)}{\Delta d_1^{i_1}} \cdots \frac{d_s^{i_s+1} - x_s(t_k)}{\Delta d_s^{i_s}}. \end{aligned} \tag{11}$$

For notation simplicity, denote

$$\mu_j(t_k) = (x_j(t_k) - d_j^{i_j}) / \Delta d_j^{i_j},$$

then (11) can be rewritten as

$$\begin{aligned} \hat{y}(t_k+1) &= \theta_1^{i_1,\cdots,i_s} \mu(t_k) \cdots \mu_s(t_k) \\ &\quad + \theta^{i_1+1,i_2,\cdots,i_s} (1 - \mu_1(t_k)) \mu_2(t_k) \cdots \mu_s(t_k) \\ &\quad + \cdots \\ &\quad + \theta^{i_1+1,i_2+1,\cdots,i_s+1} \\ &\quad \cdot (1 - \mu_1(t_k)) \cdots (1 - \mu_s(t_k)). \end{aligned} \tag{12}$$

Let

$$A_s = \begin{bmatrix} \mu_1(t_1)\cdots\mu_s(t_1) & (1-\mu_1(t_1))\mu_2(t_1)\cdots\mu_s(t_1) & \cdots & (1-\mu_1(t_1))\cdots(1-\mu_s(t_1)) \\ \cdots & \cdots & \cdots & \cdots \\ \mu_1(t_{2^s})\cdots\mu_s(t_{2^s}) & (1-\mu_1(t_{2^s}))\mu_2(t_{2^s})\cdots\mu_s(t_{2^s}) & \cdots & (1-\mu_1(t_{2^s}))\cdots(1-\mu_s(t_{2^s})) \end{bmatrix}.$$

The following elementary operations do not change the rank of A_s:

$$\begin{aligned} A_s &\rightarrow \begin{bmatrix} \prod_{j=1}^{s}\mu_j(t_1) & \cdots & \mu_{s-1}(t_1)\mu_s(t_1) & \cdots & \mu_1(t_1)\mu_2(t_1) & \mu_s(t_1) & \cdots & \mu_1(t_1) & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \prod_{j=1}^{s}\mu_j(t_{2^s}) & \cdots & \mu_{s-1}(t_{2^s})\mu_s(t_{2^s}) & \cdots & \mu_1(t_{2^s})\mu_2(t_{2^s}) & \mu_s(t_{2^s}) & \cdots & \mu_1(t_{2^s}) & 1 \end{bmatrix} \\ &\rightarrow \begin{bmatrix} \prod_{j=1}^{s}x_j(t_1) & \cdots & x_{s-1}(t_1)x_s(t_1) & \cdots & x_1(t_1)x_2(t_1) & x_s(t_1) & \cdots & x_1(t_1) & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \prod_{j=1}^{s}x_j(t_{2^s}) & \cdots & x_{s-1}(t_{2^s})x_s(t_{2^s}) & \cdots & x_1(t_{2^s})x_2(t_{2^s}) & x_s(t_{2^s}) & \cdots & x_1(t_{2^s}) & 1 \end{bmatrix} \\ &= [V(t_1), \ldots, V(t_{2^s})]^T. \end{aligned}$$

This means

$$rank(A_s) = rank(span\{V(t_1), \ldots, V(t_{2^s})\}).$$

From the linear equation theory the system of (12) has only one solution if and only if its coefficient matrix is invertible. Therefore, $\theta^{i_1,\cdots,i_s}$, $\theta^{i_1+1,i_2,\cdots,i_s}, \cdots, \theta^{i_1+1,\cdots,i_{s-1}+1,i_s}$ and $\theta^{i_1+1,\cdots,i_s+1}$ are unique if and only if $\{X(t_k)\}_{k=1}^{2^s}$ are complete of order s. ■

3.2 PEC of AFS in N-DMA Model Identification

For the identification of nonlinear system we first consider the following nonlinear deterministic moving average (N-DMA) model:

$$y(t+1) = g(u(t), \ldots, u(t-n+1)).$$

The corresponding fuzzy system is constructed as:

$$\begin{aligned} \hat{y}(t+1) &= f(u(t), \ldots, u(t-n+1)) \\ &= \frac{\sum_{i_1=1}^{N_1}\cdots\sum_{i_n=1}^{N_n}\theta^{i_1,\cdots,i_n}[\prod_{j=1}^{n}\mu_{A_j^{i_j}}(u_j(t))]}{\sum_{i_1=1}^{N_1}\cdots\sum_{i_n=1}^{N_n}[\prod_{j=1}^{n}\mu_{A_j^{i_j}}(u_j(t))]} \\ &= \varphi^T(t)\theta. \end{aligned} \quad (13)$$

Since the output of the system is explicitly independent on the past outputs, the result in subsection A is directly applicable to the N-DMA case as shown in the following theorem.

Theorem 2: The input sequence $\{u(t)\}$ is persistently exciting for the identification model (13), if for every local domain, it contains at least one group of vectors $\{U(t_k)\}_{k=1}^{2n}$ with $U(t_k) = [u(t_k), \ldots, u(t_k-n+1)]$ which are complete of order n in this local domain.

Proof: The proof is immediate from Theorem 1. ■

Remark 2: In [11] the input signal with the PE property needs to consecutively coincide with the node center. In contrast here, we do not require the inputs in the complete group to be consecutive.

For the special case where $n = 1$ we have the following corollary.

Corollary 1: For the case that

$$y(t+1) = g(u(t)) \quad (14)$$

(i) if there are two distinct inputs in an interval $\mathcal{D}^i$, $(i = 1, \ldots, N)$, then θ^i and θ^{i+1} are identifiable; (ii) if there exists a $u(t_1) = d^i$, then θ^i is identifiable; (iii) If the fuzzy system has N fuzzy rules and consequently N parameters, then the minimum number of inputs exciting all parameters is N.

Design a PE Input Signal for Adaptive Fuzzy Systems: N-DMA case

In identification of the N-DMA system, we design a PE input sequence satisfying Theorem 2 for the purpose of parameter convergence. Specifically, for the local domain $\mathcal{D}_1^{i_1} \times \cdots \times \mathcal{D}_n^{i_n}$, select $u(t_k) \in [d_n^{i_n}, d_n^{i_n+1}]$, $\ldots$, $u(t_k+n) \in [d_1^{i_1}, d_1^{i_1+1}]$ for $j = 1, 2, \ldots, s$ and $k = 1, 2, \ldots, 2^n$, then check whether the completeness condition is satisfied. Normally this requirement can be met if we choose distinct values for $u(t_k)$ in $[d_n^{i_n}, d_n^{i_n+1}]$, for $k = 1, 2, \ldots, 2^n$.

3.3 PEC of AFS in N-DARMA Model Identification

Consider the following nonlinear deterministic autoregressive moving average model (N-DARMA):

$$\begin{aligned} y(t+1) &= g(y(t), \ldots, y(t-m+1); \\ &\quad u(t), \ldots, u(t-n+1)). \end{aligned} \quad (15)$$

As we did before, to identify $g(\cdot)$ we construct a fuzzy system $f(\cdot)$ as:

$$\hat{y}(t+1) = f(Z(t))$$

where

$$Z(t) = [y(t), \ldots, y(t-m+1); u(t), \ldots, u(t-n+1)]$$

is the state of the nonlinear system (15) which is of $s = m + n$ dimension.

In terms of the state of the system we have the following theorem:

Theorem 3: The parameters $\theta^{i_1,\cdots,i_s}$, $\theta^{i_1+1,i_2,\cdots,i_s}$, $\ldots, \theta^{i_1+1,\cdots,i_{s-1}+1,i_s}$ and $\theta^{i_1+1,\cdots,i_s+1}$ ($i_1 \in \{1,\ldots,N_1-1\}$; $\ldots; i_s \in \{1,\ldots,N_s-1\}$) can be uniquely determined, if at least one group of state vectors $\{Z(t_k)\}_{k=1}^{2^s}$ are complete of order s in the local domain $\mathcal{D}_1^{i_1} \times \cdots \times \mathcal{D}_s^{i_s}$.

The proof of Theorem 3 is similar to the proof of Theorem 1, so it is omitted here.

Remark 3: To ensure that all parameters converge, it is necessary to explore enough representative states of the system. That is, an input signal with the PE property should sufficiently excite the system to be identified.

Design a PE Input Signal for Adaptive Fuzzy Systems: N-DARMA case

To simplify the notation we consider the following simple nonlinear deterministic auto-regressive moving average model:

$$y(t+1) = g(y(t), u(t)) \tag{16}$$

where the system state is $Z(t) = [y(t), u(t)]$.

A key difficulty now is to relate the completeness condition on the states of the system to condition on the input signal $\{u(t)\}$ alone. Since the state $Z(t)$ consists of the output and the input, we have to explore the relation between y and u. The following global controllability assumption on the nonlinear system $g(\cdot)$ is a necessity.

Assumption 1: The nonlinear system $g(\cdot)$ is assumed to be globally controllable, i.e., an admissible control input u exists so that any state z of the system can be reached from any initial state within a finite time [5], [8].

This assumption provides the possibility of controlling the state through the input. However, for the design purpose, we need to find such an input which can push the state into a specified local domain. Next, we assume that the fuzzy system $f(\cdot)$ constructed is accurate enough to describe the relation between $y(t+1)$ and $u(t)$.

Assumption 2: The fuzzy system $f(\cdot)$ is designed such that the approximation error is small enough in comparison with $\Delta d_y^{i_y} = d_y^{i_y+1} - d_y^{i_y}$.

Assumption 2 is practically possible, in fact from Lemma 1 it can be seen that if we know the upper bounds of the derivatives of the unknown function $g(\cdot)$, then we can construct a fuzzy system which can approximate $g(\cdot)$ with any required accuracy.

Since $Z(t)$ now is two dimensional, two control actions $u(t)$ and $u(t+1)$ are needed to push the system state $Z(t+1) = [y(t+1), u(t+1)]$ to a local sub-domain $\mathcal{D}_y^{i_y} \times \mathcal{D}_u^{i_u}$. The following steps give the $u(t)$ and $u(t+1)$ so that $Z(t+1) \in \mathcal{D}_y^{i_y} \times \mathcal{D}_u^{i_u}$:

Step 1: Find $u(t)$, such that $y(t+1) \in \mathcal{D}_y^{i_y} = [d_y^{i_y}, d_y^{i_y+1}]$.

Using Assumption 2 and expression (9), the control $u(t)$ should be selected such that

$$\begin{aligned} &\frac{d_y^{i_y} - \sum_{l_y=l_y^t}^{l_y^t+1} [\theta^{l_y,l_u+1} \mu_{A^{l_y^t}}(y(t))]}{\sum_{l_y=l_y^t}^{l_y^t+1} [(\theta^{l_y,l_u} - \theta^{l_y,l_u+1}) \mu_{A^{l_y^t}}(y(t))]} \\ &\leq u(t) \\ &\leq \frac{d_y^{i_y+1} - \sum_{l_y=l_y^t}^{l_y^t+1} [\theta^{l_y,l_u+1} \mu_{A^{l_y^t}}(y(t))]}{\sum_{l_y=l_y^t}^{l_y^t+1} [(\theta^{l_y,l_u} - \theta^{l_y,l_u+1}) \mu_{A^{l_y^t}}(y(t))]}. \end{aligned}$$

Step 2: Select a $u(t+1)$, such that $u(t+1) \in \mathcal{D}_u^{i_u} = [d_u^{i_u}, d_u^{i_u+1}]$.

The remaining design procedure is similar to the N-DMA case. The interesting thing here is, in general, the completeness holds if the relation between $y(t+1)$ and $u(t)$ is nonlinear. That is to say, if the inputs $\{U(t_k)\}_{k=1}^4$ with $U(t_k) = [u(t_k+1), u(t_k)]$ are complete of order 2, then *in most cases*, under Assumption 1, the resulting states $\{Z(t_k)\}_{k=1}^4$ are also complete of order 2. The proof is roughly stated as follows:

The completeness of $\{U(t_k)\}_{k=1}^4$ means the matrix

$$A_2^U = \begin{bmatrix} 1 & u(t_1+1) & u(t_1) & u(t_1+1)u(t_1) \\ 1 & u(t_2+1) & u(t_2) & u(t_2+1)u(t_2) \\ \cdots & \cdots & \cdots & \cdots \\ 1 & u(t_4+1) & u(t_4) & u(t_4+1)u(t_4) \end{bmatrix}$$

has full rank. Consider the matrix associated with $\{Z(t_k)\}_{k=1}^4$:

$$A_2^Z = \begin{bmatrix} 1 & u(t_1+1) & y(t_1+1) & u(t_1+1)y(t_1+1) \\ 1 & u(t_2+1) & y(t_2+1) & u(t_2+1)y(t_2+1) \\ \cdots & \cdots & \cdots & \cdots \\ 1 & u(t_4+1) & y(t_4+1) & u(t_4+1)y(t_4+1) \end{bmatrix}.$$

In most cases, since there is no reason that $y(t_k+1)$ should be related to $u(t_k+1)$, it can be seen that the columns of A_2^Z are linearly independent and thus A_2^Z is also full rank. This ensures, for the most part, the PE property of the design method.

4 CONCLUDING REMARKS

In this paper we studied the persistent excitation conditions of adaptive fuzzy systems used in identification of nonlinear functions and nonlinear dynamical systems. We gave the conditions under which the parameters can be uniquely determined. Under certain assumptions the input signals with the persistent excitation property were designed, for the identifications of both nonlinear deterministic moving average and auto-regressive moving average systems.

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